An extended dynamic IS-LM model of exchange rate adjustments and movements

Peijie Wang
University of Plymouth and IÉSEG
peijie.wang@plymouth.ac.uk

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Abstract: This paper proposes a model of exchange rate adjustments in an extended IS-LM analytical framework. It makes inquiries into the adjustment and evolution path of the exchange rate towards its new long-run equilibrium level following a change in money supply. Traditional monetary models of exchange rate determination and adjustments play primarily with the LM component of the IS-LM framework in discrete steps. The new model incorporates the IS component that is extended to deal with an external sector in an open economy evolving continuously. Effects of interest rate parity and purchasing power parity are then scrutinized, assuming neither flexible nor sticky prices.

JEL No: F31, F37

Key words: exchange rate, monetary policy, interest rate parity, purchasing power parity

1. Introduction

The foreign exchange market is crucial to international economic co-operations. It facilitates international trade and transactions. Nonetheless, disturbances generated in one part of this interlinked global economy can also be transmitted and magnified through the foreign exchange market as one of the direct and major channels to impact on the financial markets around the world. Exchange rate behavior, which is central to constitution of an orderly foreign exchange market, should therefore be scrutinized to better our understanding of exchange rate determination and adjustments.

Monetary models of exchange rate determination have prevailed for the floating exchange rate period under the setting that prices are not fixed. Major monetary models of exchange rate determination include the flexible price model of Frenkel (1976), the sticky price model of Dornbusch (1976) and the real interest rate differential model of Frankel (1979). One of the most famous features of the Dornbusch (1976) model is overshooting of the exchange rate in its adjustment process towards the new equilibrium pertinent to the new and changed economic fundamentals. Due to its prominence and influence, the overshooting proposition has been tested empirically over time. Although Dornbusch (1976) covers both money market and goods market, overshooting can be generated by incorporating uncovered interest rate parity and a kind of exchange rate expectations formation into the LM equation. That is, all traditional monetary models of exchange rate determination and adjustments play primarily with the LM component of the IS-LM framework.

This present study puts forward a model of exchange rate adjustments in an extended and dynamic IS-LM analytical framework. It incorporates the IS component that is extended to deal with an external sector in an open economy that evolves continuously. Effects of interest rate parity (IRP) and purchasing power parity (PPP) can be scrutinized in this framework as time goes by. The model sets no new equilibrium exchange rate explicitly, and it assumes neither flexible nor sticky prices. The goods price moves and adjusts naturally, corresponding with the movements and adjustments in the interest rate and the exchange rate. The rest of the paper is organized as follows. The next section presents the construct and paradigm of this study. Section 3 provides an illustrating case and calibrates the model, while Section 4 summarizes this study.

2. The model

We propose a model of exchange rate adjustments in an extended IS-LM analytical framework to deal with an external sector in an open economy. This involves an exchange rate-interest rate plane, in addition to an income-interest rate plane. The IS-LM analytical framework compromises of the following two equations for clearing the money market and goods market:

$$m_t - p_t = \eta y_t - \lambda r_t \tag{1}$$

$$\alpha y_t + \gamma r_t = g_t + \beta q_t \tag{2}$$

where m_t is demand for money that is equal to money supply in equilibrium, p_t is price of goods, y_t is income, r_t is interest rate, g_t is government spending, and q_t is real exchange rate, and φ, λ, α, β, and γ are positive coefficients. All the variables are in logarithms. In particular, λ is the interest rate semi-elasticity of the demand for money, γ is the aggregate interest rate semi-elasticity of savings and investment, and β is the real exchange rate sensitivity of trade balance. The real exchange rate, by definition, is:

$$q_t = e_t - p_t + p^f \tag{3}$$

where e_t is exchange rate and p^f is foreign price of goods in logarithms.

The IS-LM system in the interest rate-income plane treats the real exchange rate, alongside demand for money and government expenditure, as exogenous or as a policy instrument for altering the level of interest rates and income. We extend and map the IS-

LM system to the interest rate-exchange rate plane, so the real exchange rate, as well as the nominal exchange rate and price, becomes endogenous. We depart further from the conventional IS-LM analysis in the interest rate-income plane here. In the conventional IS-LM framework, an increase in money supply by dm would reduce the interest rate by $\frac{dm}{\lambda}$ in equation (1), with no change in price and income. Correspondingly in equation (2), q_t is required to be reduced by $\frac{\gamma}{\beta\lambda}dm$, given no change in income and government spending, which in turn indicates e_t is reduced by the same degree, given no immediate change in price. A reduction in q_t or e_t means appreciation of the domestic currency, which is implausible. We question this judiciousness. A domestic interest rate that is lower than the world level of interest rates would result in the domestic currency to appreciate. But it would take one year for the domestic currency to appreciate by $\frac{dm}{\lambda}$, according to interest rate parities, further assuming that the domestic interest rate remains unchanged at this lower level for one year.

We propose that, trade balance is affected by not only the exchange rate but also the velocity of change in exchange rates, i.e.:

$$ay_t + \gamma r_t = g_t + \beta q_t + \delta \frac{dq_t}{dt} \quad (2')$$

where δ is a positive coefficient. So, it is not required for q_t to be reduced by $\frac{\gamma}{\beta\lambda}dm$ immediately to attain the new temporary equilibrium in the goods market, q_t could increase as well. The shift is effected by the velocity of exchange rate changes, which makes the system dynamic meanwhile. Bringing equation (3) into equation (2') leads to:

$$\beta e_t + \delta \frac{de_t}{dt} - \gamma r_t = \beta p_t + \delta \frac{dp_t}{dt} - \beta p^f + ay_t - g_t \quad (4)$$

Assume that the system is in equilibrium at $t = 0$, with $m_0 = \overline{m}$, $p_0 = \overline{p}$, $y_0 = \overline{y}$, $r_0 = \overline{r}$, $e_0 = \overline{e}$ and $g_0 = \overline{g}$; and the equilibrium interest rate is set to equal the world level of interest rates $\overline{r} = r^*$. Given an increase in money supply at time 0^+, the money market equilibriums before and after the increase in money supply are:

$$m_0 - p_0 = \eta y_0 - \lambda r_0 \tag{1a}$$

$$m_0 + dm - p_{0^+} = \eta y_{0^+} - \lambda r_{0^+} \tag{1b}$$

The domestic interest rate, corresponding to the monetary expansion, is reduced by:

$$r_{0^+} - r_0 = -\frac{dm}{\lambda} \tag{5}$$

according to equation (1), since the price is fixed in the short-term and output is not supposed to be affected, i.e., $p_{0^+} = p_0$, $y_t \equiv y_0$. The domestic interest rate assumes a function form with which it rises gradually in reverting[1] to the world level of interest rates:

$$r_t = r^* - \frac{dm}{\lambda} e^{-\varphi t} \tag{6}$$

Then the price of goods rises in the same way as follows:

$$p_t = p_0 + \left(1 - e^{-\varphi t}\right) dm \tag{7}$$

for the money market to clear continuously. The goods market equilibriums before and after the increase in money supply are:

$$\beta e_0 - \gamma r_0 = \beta p_0 - \beta p^f + \alpha y_0 - g_0 \tag{4a}$$

$$\beta e_t + \delta \frac{de_t}{dt} - \gamma r_t = \beta p_t + \delta \frac{dp_t}{dt} - \beta p^f + \alpha y_0 - g_0 \tag{4b}$$

with $y_t \equiv y_0$ and $g_t \equiv g_0$. Subtracting equation (4a) from equation (4b) yields:

$$\delta \frac{de_t}{dt} = -\beta\left(e_t - e_0\right) + \gamma\left(r_t - r^*\right) + \beta\left(p_t - p_0\right) + \delta \frac{dp_t}{dt} \tag{8}$$

Given equation (6), equation (7) and $\frac{dp_t}{dt} = \varphi e^{-\varphi t} dm$, the above can be re-arranged to:

$$\begin{aligned} \delta \frac{de_t}{dt} &= -\beta e_t + \beta e_0 - \frac{\gamma}{\lambda} e^{-\varphi t} dm + \beta\left(1 - e^{-\varphi t}\right)dm + \delta\varphi e^{-\varphi t} dm \\ &= -\beta e_t + \beta e_0 + \beta dm + \left(\delta\varphi - \beta - \frac{\gamma}{\lambda}\right)e^{-\varphi t} dm \end{aligned} \tag{9}$$

which is a first-order linear differential equation and has a general solution of:

$$\begin{aligned} e_t &= e^{\int\left(-\frac{\beta}{\delta}\right)dt}\left\{\int\left[\frac{\beta}{\delta}e_0 + \frac{\beta}{\delta}dm + \frac{1}{\delta}\left(\delta\varphi - \beta - \frac{\gamma}{\lambda}\right)e^{-\varphi t} dm\right]e^{-\int\left(-\frac{\beta}{\delta}\right)dt} dt + C\right\} \\ &= e_0 + dm - \frac{\frac{\beta}{\delta} - \varphi + \frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi} dm \cdot e^{-\varphi t} + Ce^{-\frac{\beta}{\delta}t}, \forall t \geq t_{0^+} \end{aligned} \tag{10}$$

C is solved by taking the boundary conditions into consideration:

$$\begin{aligned} C &= e_{0^+} - e_0 + \left(\frac{\frac{\beta}{\delta} - \varphi + \frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi} - 1\right)dm \\ &= e_{0^+} - e_0 + \left(\frac{\frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi}\right)dm \end{aligned} \tag{11}$$

Conclusively, the exchange rate moves and evolves as follows:

$$\begin{aligned} e_t &= e_0 + dm + \left(e_{0^+} - e_0\right)e^{-\frac{\beta}{\delta}t} + \left(\frac{\frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi}\right)dm \cdot e^{-\frac{\beta}{\delta}t} - \frac{\frac{\beta}{\delta} - \varphi + \frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi} dm \cdot e^{-\varphi t} \\ &= e_0 + \left(1 - e^{-\frac{\beta}{\delta}t}\right)dm + \left(e_{0^+} - e_0\right)e^{-\frac{\beta}{\delta}t} + \frac{\frac{\beta}{\delta} - \varphi + \frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi} dm\left(e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}\right), \forall t \geq t_{0^+} \end{aligned} \tag{12}$$

It is apparent that $e_t = e_0 + dm, t \to \infty$. The features of the model and each of its elements are discussed in the next section.

3. The features of the model

Prior to discussing the features of the model and each of its elements, we address the issue in modeling e_{0^+} which is left unsolved in the above equation. We focus on the evolution path of the exchange rate after the shock in this study, and leave e_{0^+} unsolved and subject it to actual figures. Findings in the empirical literature imply that e_{0^+} is not predictable or can't be modeled. For example, Levin (1994) finds a monetary expansion that initially lowers interest rates can produce either overshooting or undershooting of the exchange rate. He has documented exchange rate undershooting in an earlier study (Levin 1989). Cavaglia (1991) also contradicts the exchange rate overshooting hypothesis. On the other hand, Kiguel and Dauhajre (1988) show the exchange rate is likely to overshoot in their cases. Bjørnland (2009) finds that a contractionary monetary policy shock has a strong effect on the exchange rate. The domestic currency appreciates on impact but it takes 1-2 quarters for the effect to maximize. In addition to the argument on overshooting or undershooting, a number of studies have indicated that the overshooting model is outperformed by random walk models in exchange rate forecasts. Hwang's (2003) results suggest that the random walk model outperforms the Dornbusch and Frankel models at every forecasting horizon. Similarly, Zita and Cupta (2008) find that naïve models outperform the Dornbusch model. Furthermore, many have departed far away from the original setting of immediate responses upon an increase in money supply. For example, Verschoor and Wolff (2001) investigate the effect at the 3-, 6-, and 12-month horizons to find out whether exchange rates overshoot using the Mexican data. Mussa (1982) has inspected exchange rate movements over 6 months to see if the currency over deprecated. In Bjørnland' (2009) case, the domestic currency appreciates on impact but it takes 1-2 quarters for the effect to maximize. Such departure is typified to a magnified degree by Heinlein and Krolzig (2012) who have detected delayed overshooting 2-3 years after a monetary policy shock.

The second term on the right hand side of equation (12) increases gradually from 0 at $t=0$ to *dm* when $t \to \infty$. The third term is the initial shock effect, which fades away eventually as $t \to \infty$. Given the initial effect is domestic currency depreciation to varied degrees, as evident in the above reviewed studies, $e_{0^+} - e_0 > 0$. Therefore, the third term decreases over time. The fourth term starts at 0 at $t=0$ and approaches 0 when $t \to \infty$. It is concave or has a minimum value, which can be proved as follows. $\frac{\beta}{\delta} - \varphi + \frac{\gamma}{\delta\lambda} > 0$, given that the elasticity or sensitivity parameters are smaller than unity. Then, $\frac{\frac{\beta}{\delta} - \varphi + \frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi} > 0$ if $\frac{\beta}{\delta} > \varphi$; and $\frac{\frac{\beta}{\delta} - \varphi + \frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi} < 0$ if $\frac{\beta}{\delta} < \varphi$. So, it is required to prove that $e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}$ is concave or has a minimum for $\frac{\beta}{\delta} > \varphi$ and $e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}$ is convex or has a maximum for $\frac{\beta}{\delta} < \varphi$. Technical proofs are as provided in Appendix A. Given the proofs, the fourth term decreases first and then increases, featuring initial appreciation of the domestic currency after the monetary shock.

{Figure 1}

{Figure 2}

Finally, we inspect the overall pattern in exchange rate evolutions, combining the features of individual elements. The domestic currency would appreciate initially after the shock, and then depreciate towards its new long-run equilibrium rate if $\frac{\beta}{\delta} > \varphi$. The exchange, starting at e_{0^+} , decreases and reaches its minimum e_{min} at t_m , shown in Appendix B. That is, the domestic currency would appreciate initially after the shock, and then depreciate towards its new long-run equilibrium rate. The domestic currency would further depreciate from $e_{0^+} < e_{max}$ after the initial shock, and then appreciate towards its new long-run equilibrium rate if $\frac{\beta}{\delta} < \varphi$. The exchange rate reaches its maximum e_{max} at t_m, featuring delayed overshooting in pro-overshooting empirical studies where the time horizons range from one quarter to four quarters. Previously it is stated that $\beta < \delta$ is required for the exchange rate to converge. But beyond mathematical modeling, β won't be much smaller that δ , given that they are both sensitivities of trade balance to exchange rate variables. Hence, $\frac{\beta}{\delta} > \varphi$ would prevail in the extended IS-LM framework, while not ruling out $\frac{\beta}{\delta} < \varphi$ completely. So the dominant pattern in exchange rate evolutions after the shock is that the domestic currency appreciates initially prior to gradual depreciation towards its new long-run equilibrium level $e_0 + dm$. These are exhibited in Figure 1 with the patterns in each element of equation (12) and the overall pattern in exchange rate adjustments and evolutions.

The above demonstrated pattern of exchange rate evolution is consistent with that in a recent study of Wang (2013). Upon an increase in money supply, the interest rate falls with IRP taking effect initially, with which the currency appreciates, and then the sticky price rises gradually from the medium-term and over the long-run, in which the currency depreciates. He has demonstrated three cases that initially reversely shoots, over shoots and under shoots respectively. All of them make reverse movements after the initial shock in the short-term, be the initial response overshooting, undershooting or reverse shooting. Unlike Wang (2013) who assumes a short-term exchange rate target in addition to a long-run equilibrium exchange rate, our model sets no short-term target at all and no new long-run equilibrium exchange rate explicitly. It lets the open economy evolves itself. The design of our model is also coherent with, but extends, the joint dynamics of exchange rates and interest rates of Anderson *et al.* (2010) who apply the affine class of term structure models to exchange rate movements as diffusion processes. The evolution path of exchange rates in our model goes beyond the horizon when IRP effects have diminished to a negligible extent.

4. The features of the extended IS-LM framework and IRP and PPP effects

The model and its features are derived from the extended IS-LM framework. This section demonstrates the path of exchange rate adjustments and movements on the exchange rate-interest rate plane in the extended IS-LM framework, together with the corresponding adjustments and movements on the traditional income-interest rate plane. These are exhibited in Figure 2. The right side panel is the traditional IS-LM analysis on the income-interest rate plane. LX is the LM curve mapped to the exchange rate-interest rate plane. When mapping the LM curve onto the exchange rate-interest rate plane, the curve is horizontal since the exchange rate is not a variable for the LM curve. IX is the IS curve mapped to the exchange rate-interest rate plane. There are two horizontal axes; one is for the exchange rate, and the other for the velocity of exchange rate changes. The velocity axis has two sections, one for $\frac{de_t}{dt} < 0$ and one for $\frac{de_t}{dt} > 0$. The curve is flatter when r_t is far away from r^* and steeper when r_t is closer to r^*, reflecting the IRP effect. For $r_t - r^* < 0$, it means the larger the interest rate differential, the greater the decrease in the exchange rate, or the appreciation of the domestic currency. Note that a shift of the IX curve is by changes in p_t, y_t, g_t and $\frac{dp_t}{dt}$. With $y_t \equiv y_0$ and $g_t \equiv g_0$, the IX curve will shift only if there is a change in p_t or $\frac{dp_t}{dt}$. Given $p_{0^+} = p_0$ and $\left.\frac{dp_t}{dt}\right|_{t=t_{0^+}} = \varphi \cdot dm$, the IX curve shifts leftwards by $\delta \cdot \varphi \cdot dm$ upon the shock. $\left.\frac{dp_t}{dt}\right|_{t\to\infty} = 0$ and $p_t|_{t\to\infty} = p_0 + dm$ as $t \to \infty$, the total distance the IX curve has traveled leftwards is dm when the system has settled down at the new equilibrium. The cross points of the corresponding LX and IX curves in their shifts exhibit the path of exchange rate adjustments and movements. The shift of the IS curve on the income-interest rate plane is by changes in $\beta q_t + \delta \frac{dq_t}{dt}$, which can also be expressed in the nominal exchange rate and prices:

$$\beta q_t + \delta \frac{dq_t}{dt} = \beta e_t - \beta p_t + \beta p^f + \delta \frac{de_t}{dt} - \delta \frac{dp_t}{dt} \tag{13}$$

At $t = t_{0^+}$, $e_{0^+} - p_{0^+} + p^f = e_{0^+} - e_0 + e_0 - p_0 + p^f = e_{0^+} - e_0 + q_0$, $\frac{de_t}{dt} = -\frac{dm}{\lambda}$, and $\frac{dp_t}{dt} = \varphi \cdot dm$, so the shift of the IS curve upon the shock is:

$$\beta q_0 + \beta\left(e_{0^+} - e_0\right) - \delta \frac{dm}{\lambda} - \delta\varphi \cdot dm \tag{14}$$

The downwards movement of the IS curve is enabled by the negative figures of $-\delta \frac{dm}{\lambda} - \delta\varphi \cdot dm$, allowing the domestic currency to depreciate upon the shock, i.e., allowing the nominal exchange rate to increase.

While φ measures the swiftness of money market adjustments to attain the new equilibrium, $\frac{\beta}{\delta}$ reflects the dynamics in goods market adjustments in moving to the temporal equilibrium and then attaining the new equilibrium. If $\delta = 0$, goods market adjustments respond to the level of the exchange rate only. A sizeable δ relative to β

feeds dynamics into the system – the velocity of exchange rate changes moves the goods market too. Moreover, $\delta > 0$ guarantees continuity in goods market adjustments. The exchange rate would otherwise be required to jump in the wrong direction immediately. The money market parameter φ plays a role for the IRP effect; φ and the goods market parameters $\frac{\beta}{\delta}$ jointly play a part for the PPP effect. The domestic currency would appreciate to a greater extent and the appreciation period would last longer with a smaller φ. This is because the domestic interest rate reverts to the world level of interest rates with a slower speed, keeping more sizeable interest rate differentials for a longer time. According to equation (6) and uncovered IRP:

$$E_{0^+}(e_t) - e_{0^+} = \int_{\tau=0^+}^{t} (r_\tau - r^*) d\tau = \int_{\tau=0^+}^{t} -\frac{dm}{\lambda} e^{-\varphi\tau} d\tau = \frac{dm}{\lambda\varphi}\left(e^{-\varphi t} - 1\right), \forall t \geq t_{0^+} \quad (15)$$

The IRP effect is greater and lasts longer with a smaller φ. Meanwhile, the domestic price would increase more slowly with a smaller φ. It takes a longer time for the exchange rate to increase by the full amount effected by the monetary expansion. In other words, it takes a longer time for the exchange rate to reach its new equilibrium rate – increase by the same percentage as the increase in money supply. Otherwise with a larger φ, the domestic currency would appreciate to a smaller extent and the IRP effect disappears more quickly. The domestic price would rise more rapidly and it takes a shorter time for the exchange rate to reach its new equilibrium rate. According to equation (12) and equation (7), the departure of the exchange rate from its pre shock equilibrium or PPP rate is:

$$q_t = e_t - p_t + p^f$$

$$= e_0 + \left(1 - e^{-\frac{\beta}{\delta}t}\right)dm + \left(e_{0^+} - e_0\right)e^{-\frac{\beta}{\delta}t} + \frac{\frac{\beta}{\delta} - \varphi + \frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi} dm\left(e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}\right)$$

$$- \left[p_0 + \left(1 - e^{-\varphi t}\right)dm\right] + p^f \quad (16)$$

$$= \left(e_0 - p_0 + p^f\right) + \left(e_{0^+} - e_0\right)e^{-\frac{\beta}{\delta}t} + \frac{\frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi} dm\left(e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}\right)$$

$$= q_0 + \left(e_{0^+} - e_0\right)e^{-\frac{\beta}{\delta}t} + \frac{\frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta} - \varphi} dm\left(e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}\right), \forall t \geq t_{0^+}$$

The first term on the right hand side is the pre shock equilibrium or PPP rate, which the exchange rate would revert to or reach. The second term is the initial shock effect that dies away when time goes by. The third term is concave. It starts at zero, decreases and becomes negative, and then increases to zero again[2]. Therefore, the larger the parameter $\frac{\beta}{\delta}$ and the closer the parameters $\frac{\beta}{\delta}$ and φ, the faster the exchange rate would reach its PPP rate. Also the departure of the exchange rate from its PPP rate would be smaller during the adjustment course.

5. A manifest example

The last financial crisis has provided us with the rare opportunities to inspect the patterns in exchange rate movements following an expansionary monetary policy. The intention to expand the monetary base in most economies during the crisis period was almost solely to prevent the economy from sliding into recession or, at best, to keep the economy as it was, with the outcome of virtually every intervention being just that. The scales of monetary policy intervention have been enormous and unprecedented. The policy tool adopted by most monetary authorities around the developed world is the most direct amongst the three major policy tools – large scale open market purchases of bonds and gilts or quantitative easing (QE). Unlike "conventional" monetary expansions where changes in a few other economic variables may influence exchange rates as much as money supply does, the effect of QE on exchange rates and exchange rate movements greatly dwarfs that of any other economic variables. For this reason, QE effectively isolates the impact of other economic variables on exchange rate movements from that of monetary expansions, offering an immaculate environment in which the effect of monetary expansions on exchange rate adjustment and movement is studied.

The first round of QE in the US, QE1, is used for case analysis. QE1 started in December 2008 when the Federal Reserve announced it would purchase up to $100 billion in agency debt and up to $500 billion in agency mortgage-backed securities on November 25, 2008. Although the purchases spread over a period, that period was fairly short. The announcement effect would be also considerable, which Gagnon *et al.* (2010) scrutinize for QE1 in detail. The exchange rate used in the study is the US dollar effective exchange rate provided by the US Federal Reserve. The effective exchange rate is re-arranged so that an increase in it corresponds to the depreciation of the US dollar vis-à-vis the currencies of its trading partners, the same way as directly quoted bilateral exchange rates. Figure 2 exhibits US dollar effective exchange rate movements since the start of QE1 in a one-year frame, by which time the rest of the developed economies had also begun implementing their own QE programs and their asset purchases became sizeable. For example, the MPC of the UK announced a £75b asset purchase plan over a three-month period in March 2009; by the November MPC meeting asset purchases were extended to £200b (*cf.* Joyce *et al.* 2011 for the design and operation of QE in the UK). During this period, the ECB also adopted some kind of QE, albeit on a much smaller scale, including a €60b corporate bond purchase program made known in May 2009. Observing Figure 3, US dollar effective exchange rate movements in QE1 fit the theoretical curve delicately well. The US dollar effective exchange rate increased from an index number of around 118 at the beginning of December in 2008 to 129 by the middle of the month, causing 9 percent depreciation. Then the US dollar embarked on a reverse movement course and on March 9, 2009, the index decreased to less than 116, amounting to more than 11 percent of accumulated appreciation in nearly a quarter time period. Afterwards, the US dollar kept depreciating and by December 2009, the US dollar effective exchange rate reached 140. The US dollar depreciated by nearly 19 percent relative to its position a year ago, measured by its effective exchange rate. There are two periods when the exchange rate deviates from the evolution path on the theoretical curve. One is around the end of March 2009 when the Bank of England, following the announcement on March 5, purchased its first large chunk of corporate bonds, lasting for three weeks. This seems to be a counter effect of QE by other economies; the US dollar exchange rate stopped monotonic rising but fluctuated, for about three weeks. The second period is the end of May and June 2009, which coincided the move of the ECB and reflected another counter effect of QE in other economies. These deviations, contributed by other economies' QE, are fairly modest nonetheless.

{Figure 3}

It has been observed that the US dollar depreciated upon the expansion of money supply, but there were no signs of overshooting of exchange rates. The initial depreciation was pulsating for actual asset purchases as well as for the announcement. This kind of initial depreciation following an expansionary monetary policy is prevalent in the empirical literature, featured largely by undershooting of exchange rates, but alluded implicitly by the palpable failure to endorse exchange rate overshooting as well. Reverse movements of the exchange rate in the short-term, however, were manifest in QE. The US dollar depreciated inevitably afterwards in the long-run and the exchange rate moved inevitably in a depreciating manner. The displayed pattern in US dollar effective exchange rate adjustments and movements mirror the theoretical analysis of this paper remarkably agreeably.

6. Summary

A model of exchange rate adjustments in an extended IS-LM analytical framework has been proposed in this paper, to analyze the adjustment and evolution path of the exchange rate following a change in money supply. The proposed model incorporates the IS component that is extended to deal with an external sector in an open economy on the exchange rate-interest rate plane, in addition to the traditional IS-LM analysis on the income-interest rate plane. Assuming neither flexible nor sticky prices, effects of interest rate parity and purchasing power parity can be and are scrutinized with this analytical framework.

The dominant pattern in exchange rate evolutions following an expansionary monetary policy is found to be that the domestic currency appreciates initially after the shock, prior to gradual depreciation towards its new long-run equilibrium level that is the pre monetary expansion equilibrium rate plus the percentage increase in money supply. This pattern in exchange rate adjustments and movements is resulted from the joint and sequential effects of IRP and PPP. This pattern in the evolution path of exchange rate adjustments and movements has manifested by the recent example of the US QE in one of the most rattling epochs of the last financial crisis. The actual exchange rate movements fit the theoretical curve delicately well.

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Endnotes

[1] In relativity, other economies' interest rates converge/revert to the domestic interest rate for a large open economy.

[2] Except for $\varphi = \frac{\beta}{\delta}$, in which case $\frac{e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}}{\frac{\beta}{\delta} - \varphi} = -te^{-\varphi t}$, since $\underset{\varphi \to \frac{\beta}{\delta}}{Lim}\left(\frac{e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}}{\frac{\beta}{\delta} - \varphi}\right) = \underset{\varphi \to \frac{\beta}{\delta}}{Lim}\left(\frac{te^{-\varphi t}}{-1}\right) = te^{-\varphi t} = te^{-\frac{\beta}{\delta}}$

Appendix A

Set:

$$\frac{d\left(e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}\right)}{dt} = \varphi e^{-\varphi t} - \frac{\beta}{\delta} e^{-\frac{\beta}{\delta}t} = 0 \qquad \text{(A1)}$$

It reaches a minimum or maximum at time t_m:

$$t_m = \frac{Ln\left(\frac{\beta}{\delta\varphi}\right)}{\frac{\beta}{\delta} - \varphi} \qquad \text{(A2)}$$

Note $Ln\left(\frac{\beta}{\delta\varphi}\right) > 0$ when $\frac{\beta}{\delta} > \varphi$ and $Ln\left(\frac{\beta}{\delta\varphi}\right) < 0$ when $\frac{\beta}{\delta} < \varphi$, so $t_m \in \mathbb{R}_{++}$ and is always viable. At $t = t_m$, the second derivative is:

$$\left.\frac{d^2\left(e^{-\frac{\beta}{\delta}t} - e^{-\varphi t}\right)}{dt^2}\right|_{t=t_m} = \left(\frac{\beta}{\delta}\right)^2 e^{-\frac{\beta}{\delta}\frac{Ln\left(\frac{\beta}{\delta\varphi}\right)}{\frac{\beta}{\delta}-\varphi}} - \varphi^2 e^{-\varphi\frac{Ln\left(\frac{\beta}{\delta\varphi}\right)}{\frac{\beta}{\delta}-\varphi}}$$

$$= e^{-\frac{\beta}{\delta}\frac{Ln\left(\frac{\beta}{\delta\varphi}\right)}{\frac{\beta}{\delta}-\varphi}}\left[\left(\frac{\beta}{\delta}\right)^2 - \varphi^2 e^{\left(\frac{\beta}{\delta}-\varphi\right)\frac{Ln\left(\frac{\beta}{\delta\varphi}\right)}{\frac{\beta}{\delta}-\varphi}}\right] \qquad \text{(A3)}$$

$$= e^{-\frac{\beta}{\delta}\frac{Ln\left(\frac{\beta}{\delta\varphi}\right)}{\frac{\beta}{\delta}-\varphi}}\left[\left(\frac{\beta}{\delta}\right)^2 - \varphi^2 e^{Ln\left(\frac{\beta}{\delta\varphi}\right)}\right] = e^{-\frac{\beta}{\delta}\frac{Ln\left(\frac{\beta}{\delta\varphi}\right)}{\frac{\beta}{\delta}-\varphi}}\varphi^2\left[\left(\frac{\beta}{\delta\varphi}\right)^2 - e^{Ln\left(\frac{\beta}{\delta\varphi}\right)}\right]$$

Therefore, whether the second derivative is positive or negative is decided by the sign of $\left(\frac{\beta}{\delta\varphi}\right)^2 - e^{Ln\left(\frac{\beta}{\delta\varphi}\right)}$. Since:

$$2Ln\left(\frac{\beta}{\delta\varphi}\right)\begin{cases} > e^{Ln\left(\frac{\beta}{\delta\varphi}\right)}\Big|_{\frac{\beta}{\delta}>\varphi} \\ < e^{Ln\left(\frac{\beta}{\delta\varphi}\right)}\Big|_{\frac{\beta}{\delta}>\varphi} \end{cases} \qquad \text{(A4)}$$

Therefore:

$$\left.\frac{d^2\left(e^{-\frac{\beta}{\delta}t}-e^{-\varphi t}\right)}{dt^2}\right|_{t=t_m}\begin{cases}>0\big|_{\frac{\beta}{\delta}>\varphi}\\ \\ <0\big|_{\frac{\beta}{\delta}<\varphi}\end{cases} \tag{A5}$$

So, $e^{-\frac{\beta}{\delta}t}-e^{-\varphi t}$ is concave for $\frac{\beta}{\delta}>\varphi$ and $e^{-\frac{\beta}{\delta}t}-e^{-\varphi t}$ is convex for $\frac{\beta}{\delta}<\varphi$.

Appendix B

Express equation (12) in a condensed way:

$$e_t = e_0 + dm + \left[\left(e_{0^+}-e_0\right)+\left(\Psi-1\right)dm\right]e^{-\frac{\beta}{\delta}t} - \Psi\cdot dm\cdot e^{-\varphi t}, \forall t \geq t_{0^+} \tag{B1}$$

where $\Psi = \frac{\frac{\beta}{\delta}-\varphi+\frac{\gamma}{\delta\lambda}}{\frac{\beta}{\delta}-\varphi}\begin{cases}>1\big|_{\frac{\beta}{\delta}>\varphi}\\ <0\big|_{\frac{\beta}{\delta}<\varphi}\end{cases}$. Set:

$$\frac{de_t}{dt} = \varphi\cdot\Psi\cdot dm\cdot e^{-\varphi t} - \frac{\beta}{\delta}\left[\left(e_{0^+}-e_0\right)+\left(\Psi-1\right)dm\right]e^{-\frac{\beta}{\delta}t} = 0 \tag{B2}$$

The exchange rate reaches a minimum or maximum value at t_m, which can be solved by:

$$t_m = \frac{Ln\left[\frac{\left(e_{0^+}-e_0\right)+\left(\Psi-1\right)dm}{\frac{\delta\varphi}{\beta}\Psi\cdot dm}\right]}{\frac{\beta}{\delta}-\varphi} \tag{B3}$$

At $t = t_m$, the second derivative of the exchange rate with respect to t is:

$$\left.\frac{d^2 e_t}{dt^2}\right|_{t=t_m} = \left(\frac{\beta}{\delta}\right)^2 \left[\left(e_{0^+} - e_0\right) + \left(\Psi - 1\right)dm\right] e^{-\frac{\beta}{\delta}\frac{Ln\left[\frac{\left(e_{0^+}-e_0\right)+\left(\Psi-1\right)dm}{\frac{\delta\varphi}{\beta}\Psi\cdot dm}\right]}{\frac{\beta}{\delta}-\varphi}}$$

$$- \varphi^2 \cdot \Psi \cdot dm \cdot e^{-\varphi\frac{Ln\left[\frac{\left(e_{0^+}-e_0\right)+\left(\Psi-1\right)dm}{\frac{\delta\varphi}{\beta}\Psi\cdot dm}\right]}{\frac{\beta}{\delta}-\varphi}} \tag{B5}$$

$$= e^{-\frac{\beta}{\delta}\frac{Ln\left[\frac{\left(e_{0^+}-e_0\right)+\left(\Psi-1\right)dm}{\frac{\delta\varphi}{\beta}\Psi\cdot dm}\right]}{\frac{\beta}{\delta}-\varphi}} \varphi^2 \left\{ \left(\frac{\beta}{\delta\varphi}\right)^2 \left[\left(e_{0^+} - e_0\right) + \left(\Psi - 1\right)dm\right] - \Psi \cdot dm \cdot e^{Ln\left[\frac{\left(e_{0^+}-e_0\right)+\left(\Psi-1\right)dm}{\frac{\delta\varphi}{\beta}\Psi\cdot dm}\right]} \right\}$$

The sign of the second derivative is the same as the sign of $\left(\frac{\beta}{\delta\varphi}\right)^2 \left[\left(e_{0^+} - e_0\right) + \left(\Psi - 1\right)dm\right] - \Psi \cdot dm \cdot e^{Ln\left[\frac{\left(e_{0^+}-e_0\right)+\left(\Psi-1\right)dm}{\frac{\delta\varphi}{\beta}\Psi\cdot dm}\right]}$, which in turn has the same sign as:

$$2Ln\left(\frac{\beta}{\delta\varphi}\right) + Ln\left[\frac{\left(e_{0^+} - e_0\right) + \left(\Psi - 1\right)dm}{\Psi \cdot dm}\right] - Ln\left[\frac{\left(e_{0^+} - e_0\right) + \left(\Psi - 1\right)dm}{\frac{\delta\varphi}{\beta}\Psi \cdot dm}\right] \tag{B6}$$

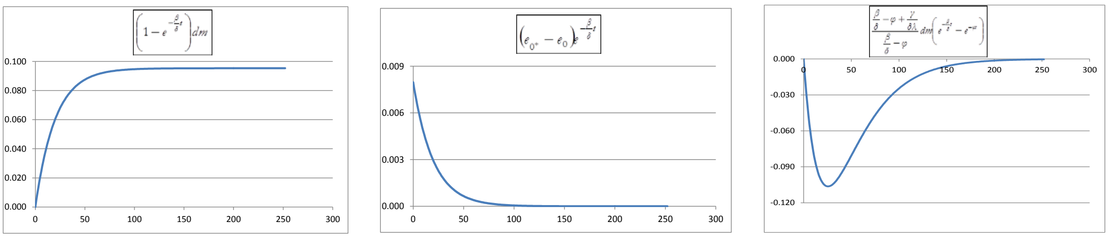
$$= 3Ln\left(\frac{\beta}{\delta\varphi}\right) \begin{cases} > 0 \big|_{\frac{\beta}{\delta} > \varphi} \\ < 0 \big|_{\frac{\beta}{\delta} < \varphi} \end{cases}$$

Therefore:

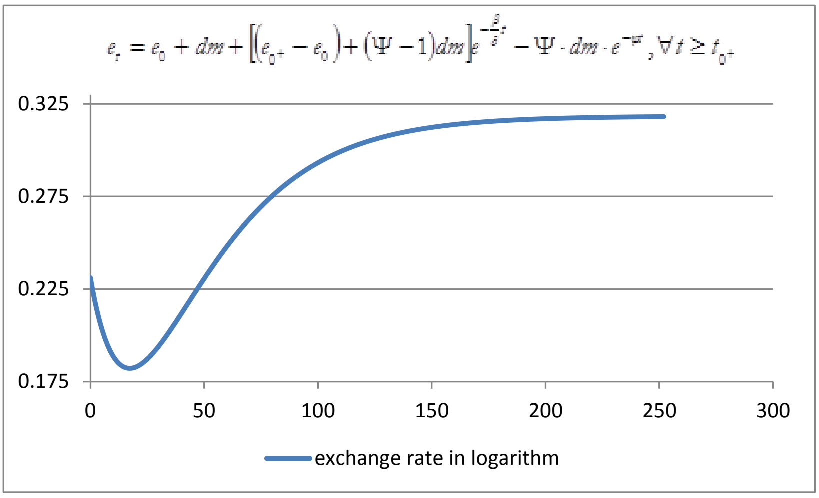
$$\left.\frac{d^2 e_t}{dt^2}\right|_{t=t_m} \begin{cases} > 0 \big|_{\frac{\beta}{\delta} > \varphi} \\ < 0 \big|_{\frac{\beta}{\delta} < \varphi} \end{cases} \tag{B7}$$

The exchange rate reaches its minimum e_{min} at t_m if $\frac{\beta}{\delta} > \varphi$. That is, the domestic currency would appreciate initially after the shock, and then depreciate towards its new long-run equilibrium rate. The exchange rate reaches its maximum e_{max} at t_m if $\frac{\beta}{\delta} < \varphi$. However, $e_{max} > e_{0^+}$. The domestic currency would further depreciate from e_{0^+} after the initial shock, and then appreciate towards its new long-run equilibrium rate. This is almost the overshooting case of Dornbusch (1976), except that the domestic currency does not depreciate beyond the new equilibrium rate immediately, it takes some time. This is the cases in pro-overshooting empirical studies where the time horizons range from one quarter to four quarters.

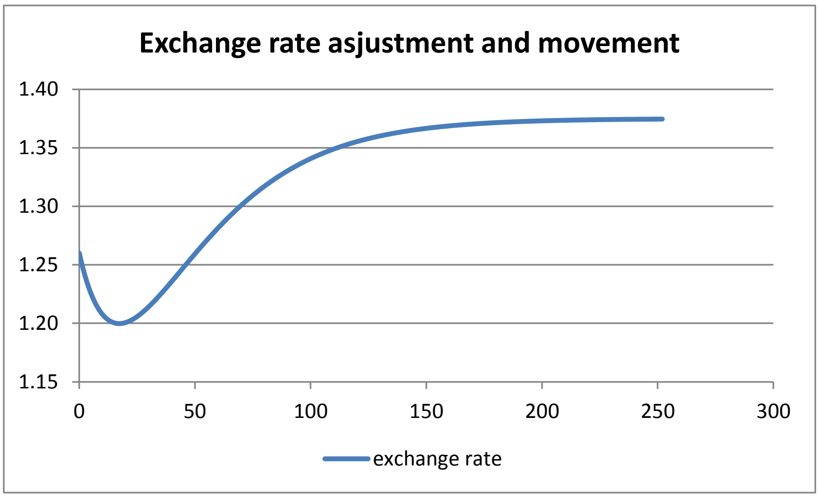
Figures



(a) Patterns in three terms



(b) Patterns in exchange rate adjustment and movement in logarithm



(c) Exchange rates

Figure 1. Exchange rate adjustment and movement

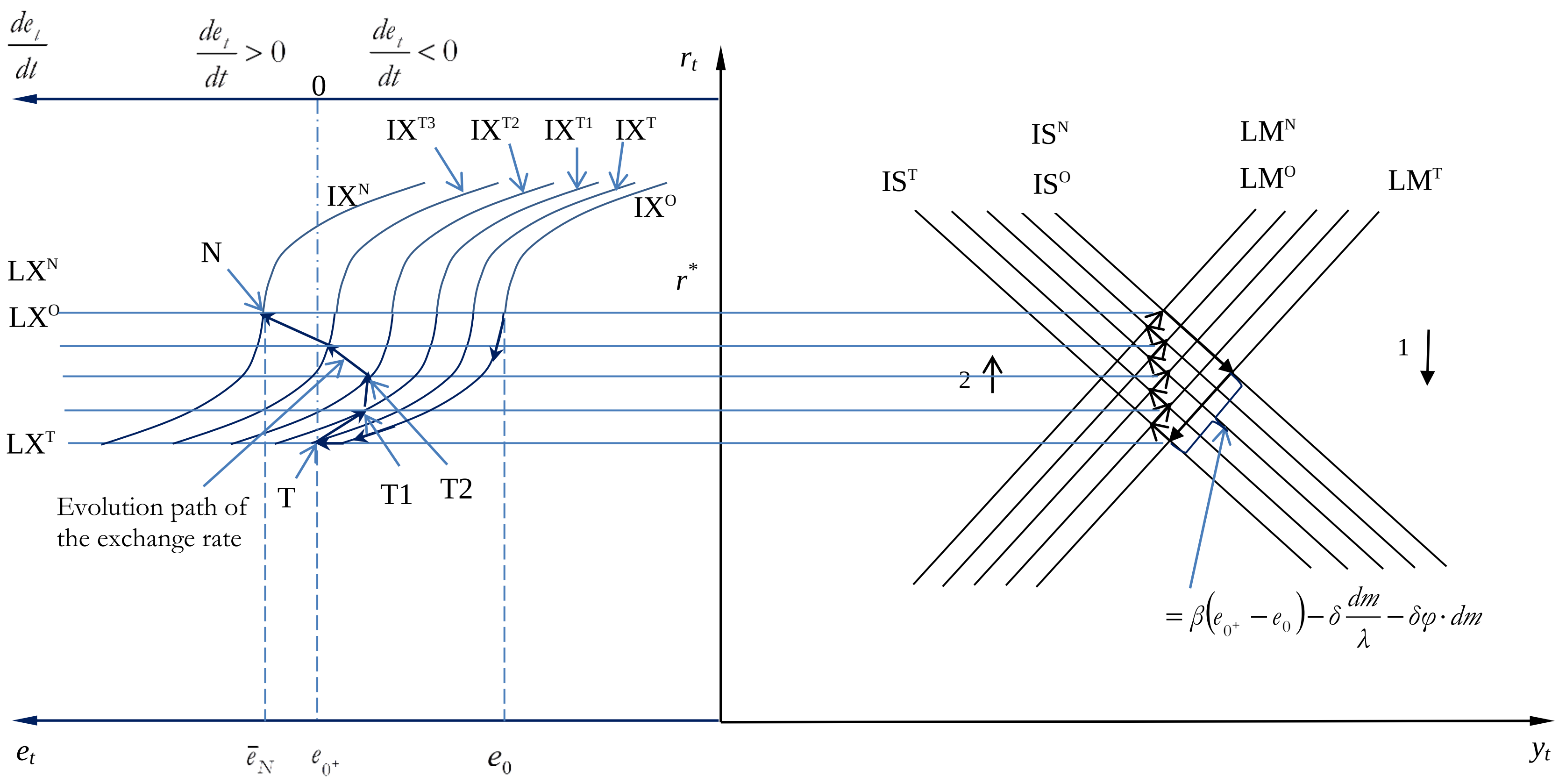
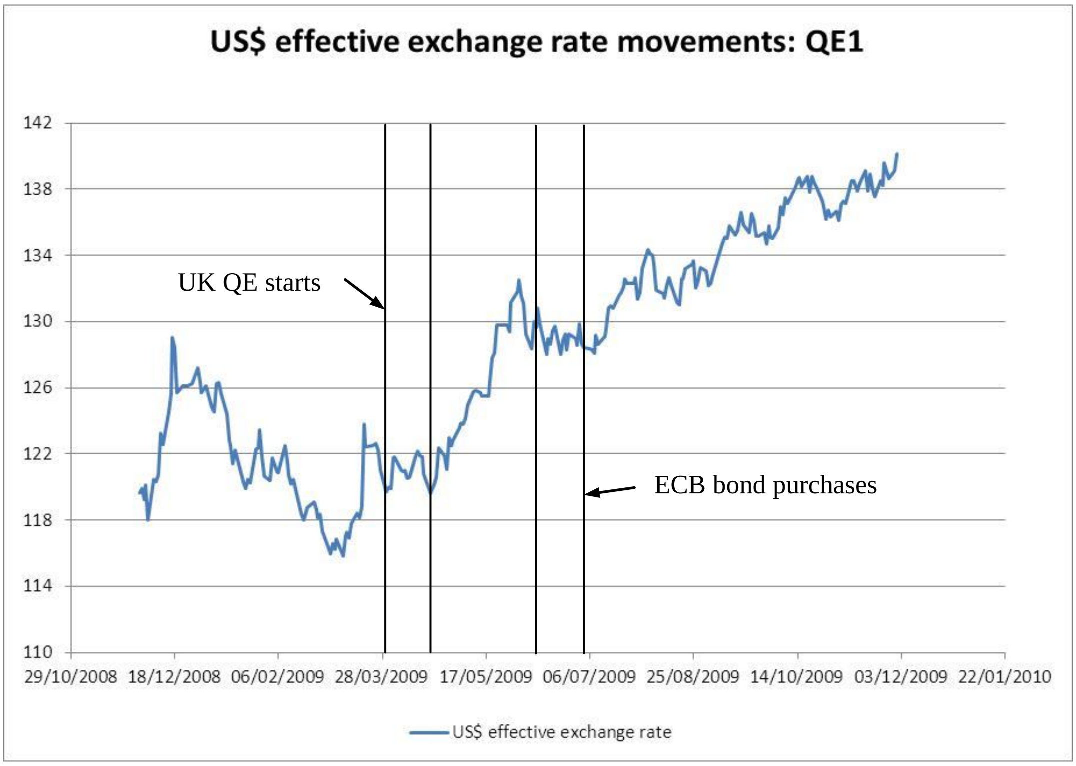


Figure 2. Extended IS-LM framework



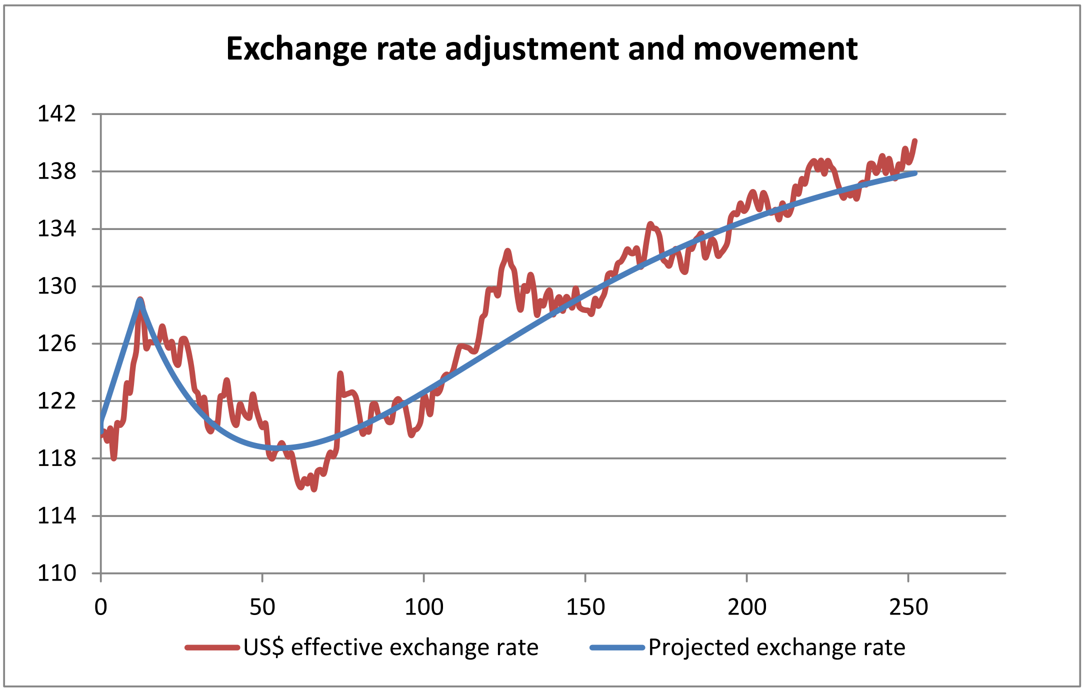


Figure 3. Exchange rate adjustments and movements: actual and projected