The Evolution of Public Spending on Higher Education in a Democracy

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Abstract

This paper analyses a political force that can cause an initial expansion of public spending on higher education and an ensuing decline in subsidies per student: the increase in the number, and thus voting power, of skilled parents. The rise of the skilled class leads to a majority for an initial expansion of public education spending. This expansion further boosts the number of skilled parents and thus future demand for higher education. The induced shift in demand implies that the initial subsidy per student becomes too expensive to be politically sustainable. The initial educational ‘take-off’ provokes a backlash at the polls. A majority now successfully calls for higher private contributions to the costs of university education. Nevertheless, overall enrolment continues to rise. But equality of opportunity, that went up in the expansion period, declines afterwards.

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1 Motivation

Many European countries traditionally have generous public systems of higher education. Tuition at universities has usually been free, and a combination of grants and subsidised loans has provided a fairly attractive economic environment for students even from lower income families. But the mood has changed in many of these countries, and the call for students to contribute more to the costs of their education has become louder.

This shift in attitude has led to a number of significant policy changes. These changes have been particularly pronounced in the United Kingdom. Initially, tuition fees of £ 1000 per annum for students at UK universities were introduced in 1998. Since 2006, English universities have been allowed to charge up to about £ 3,000 per annum in tuition fees instead of the former flat rate, which had risen to £ 1,115. The current government has further raised the cap on tuition fees sharply, to £ 9,000 (Economist, 2010). Importantly, these changes in funding higher education had already emerged before the current financial crisis unfolded. While the strain on public budgets might have accelerated the existing trend, it did not cause the beginning of this development.

This paper provides a political-economic explanation for the evolution of public spending on higher education. It explores how the changes in political preferences and political power over time first cause the initial expansion of public subsidies, and how the very same forces later lead to a cut in the share of public spending and a corresponding rise in private contributions to education costs.

The basic argument runs as follows: Skilled parents are the main driving force behind subsidising higher education, since most of the college and university students are their children. But, at least at an early stage, their numbers are too small to be pivotal. Only when the size of the skilled class exceeds a threshold value, does a ‘coalition’ in favour of government intervention, consisting of a majority of the skilled citizens and of lower income households with very gifted children, push through extensive public education spending. Higher subsidies foster the number of students and increase the number of the skilled people in the future.

This educational ‘take-off’ later provokes a backlash. Since a bigger skilled class boosts the demand for higher education, public commitments become more expensive. Even the supporters of government intervention then favour a smaller subsidy per student and call for a larger private contribution. After an expansion phase, the ratio between public and private spending on higher education declines. Despite the cut in per-student subsidies, enrolment increases further, but this continuing rise in the number of students is the result of the growth of the skilled class and the ensuing shift in demand for higher education. While equality of opportunity increases in the
expansion phase, it declines afterwards.

This line of reasoning is analysed in a model with two overlapping generations in which young people with different ability attend universities or receive only basic schooling. The education choice takes place in each period’s second stage, after the electorate has voted on public education spending in the first stage. The present student body forms the higher income class in the next period. Thus, two succeeding periods are linked, since today’s choices shape tomorrow’s social stratification, which in turn affects future decisions.

The explicit dynamic modelling distinguishes the current paper from the strand of literature that deals with democratic choices for financing higher education in a static framework. Fernandez and Rogerson (1995) show that redistribution from lower income to better off families occurs in a political equilibrium. The majority favours only partial subsidisation of education as a device to exclude poorer citizens from attending colleges and taking advantage of the transfers. In contrast to Fernandez and Rogerson (1995), households differ in income and ability of their children in the present paper. As in De Fraja (2001), the battle lines are, thus, within income groups and not only between different classes. De Fraja (2001) contrasts an admission test and a subsidy in a framework in which future earnings are uncertain and people are risk averse. A subsidy, for instance, makes lower income households whose children do not attend universities worse off. This issue is taken up in the present paper too. But unlike both Fernandez and Rogerson (1995) and De Fraja (2001), the current analysis explicitly focuses on how education policy evolves over time, reflecting shifts in political preferences and power.

A dynamic political-economic analysis of education and income distribution has been the subject of research in the second strand of literature closely related to the present paper. Cardak (2004), Glomm and Ravikumar (1992), and Gradstein and Justman (1997) compare public and private education in an endogenous growth framework. The applied approaches enable them to gain substantial insights into the relationship between growth, income inequality, and education, but their frameworks are not suitable for exploring the issues of the present paper. In their contributions, the specification of the utility function and the learning technology, combined with a proportional tax on income, implies that all individuals at all times prefer the same tax rate in the public education regime. So any conflict of interests is excluded from the outset. By contrast, in the current paper, citizens differ profoundly in their favoured policies in each period, and political preferences change over time. Voters also disagree on the optimal policy in Saint-Paul and Verdier (1993), where the interplay between democracy, income distribution, and growth is examined.

1 Alternative motives for government intervention are discussed in section 5.
Nevertheless, the political conflicts are again very different from those in the present paper, since in their model all individuals receive publicly funded education in the same way.\(^2\)

To sum up, unlike the static models mentioned above, the current paper explores the evolution of higher education policy. In contrast to the dynamic approaches referred to above, it does this in a framework where citizens differ in their preferred policies and where only a subset of the population benefits from education policy. In this respect, the present contribution extends the two branches of literature mentioned and fills a gap between them.

Thereby, the paper explores both the initial expansion of public spending on higher education and the ensuing ‘reprivatisation’ of higher education. In this sense, the current paper provides a more comprehensive analysis than other approaches that tend to focus either on the expansion of, or on the cuts in, public spending. For instance, several authors link the fall in public spending to globalisation. They stress that labour market integration and the resulting tax competition can restrict the ability of, or the incentives for, governments to subsidise higher education (see Andersson and Konrad, 2003, Justman and Thisse, 1997, and Poutvaara and Kanninen, 2000, for some earlier contributions).\(^3\)

Poutvaara (2011) analyses the circumstances under which public education initially emerges in a democracy, identifying the time-consistency problem of redistribution as a key factor. In his static model, he pays less attention to the potential causes of ensuing cuts in public spending (but he briefly discusses international tax competition as a reason, like some of the papers referred to above). Interestingly, the ageing of the society cannot necessarily explain the fall in public education spending. Kemnitz (1999) argues that, in an ageing society, providing public education becomes cheaper, and that relatively smaller young generations find it easier to lobby effectively. Both effects tend to drive up spending per student.

In contrast to these contributions, the current paper’s mechanisms do not rely on globalisation, ageing or time-consistency problems. Instead, the evolution of public policy is caused by the changes in the size of socioeconomic classes and the induced impact on political preferences and power over time, which in turn is partly brought about by the changes in education finance itself.

\(^2\)There are a number of papers which analyse intergenerational earning mobility but consider education policy as exogenously given or not at all (for instance, Bénabou, 2002, Glomm and Ravikumar, 2003, and Iyigun, 1999). These contributions can be regarded as complementary to the political-economic approaches.

\(^3\)More recent papers include Demange, Fenge and Uebelmesser (2008) and Haupt and Janeba (2009). In response to the international competition for talent, governments might also redirect resources towards country-specific education, as discussed in Poutvaara (2008).
The remainder of this paper is organised as follows: in Section 2, the basic elements of the model are described. Section 3 explores the education choices and the opposing political preferences. The evolution of public education spending, the number of students and the participation rates is analysed in Section 4. In Section 5, I provide some evidence for the key results, and discuss several extensions of the current model. The paper ends with some concluding remarks.

2 The Model

This section presents a simple dynamic model for studying the evolution of public education spending and social stratification in a democracy. I consider a society with two overlapping generations and heterogeneous households. The electorate votes on a tax that finances a subsidy for higher education, and the families decide on the education of their children.

Households and Heterogeneity In each period, the economy consists of a continuum of families, each comprising one parent and one child, with total mass of each generation equal to one. Every parent inelastically supplies one unit of labour, whereas every child either attends an institution of higher education, for brevity referred to as university, or receives only some basic education. For analytical convenience, the following analysis abstracts from the fact that the durations of the two levels of education are different. Instead, the differences are summed up by the single parameter ‘costs of education’, as described below.

Households differ in two respects. First, they can be divided into two groups according to the income and educational achievement of the old generation. Parents who attended university have a skilled occupation and earn gross wage \( w_H \). By contrast, adults who received only basic education have an unskilled occupation and earn only gross wage \( w_L \), with \( w_L < w_H \). The former group is referred to as the higher income, or skilled, class, and the latter one is referred to as the lower income, or unskilled, class.

Second, children differ in their innate ability to acquire human capital. This feature is captured by the costs of education. The lower the talent, the higher are the costs of receiving a university degree.\(^4\) For simplicity, assume that costs of basic schooling equal zero for all individuals while higher education costs are uniformly distributed on the support \([z_l, z_u]\). The density function is identical for all young

\(^4\) Alternatively, different ability levels can, for instance, be modelled by different probabilities of receiving a university degree or by different study durations. These approaches lead to the same qualitative results, but they make the model less tractable.
people of the two classes. It is invariant over time and common knowledge.

**Household Utility and Budget**  The parent-child household is regarded as the basic socioeconomic unit ‘ruled’ by the parent. The parent determines the spending on (family) consumption and education reflecting her valuation of these two components.\(^5\) Her preferences are represented by the utility function

\[
U_{ij} = U(x_{ij}, \theta_{ij}), \quad j = H, L, \tag{1}
\]

where \(x_{ij}\) denotes consumption of the \(i\)th household of income group \(j\) in the current period. The variable \(\theta_{ij}\) captures the utility the household attaches to the child’s education. If the child goes to university, \(\theta_{ij}\) equals \(\theta\); otherwise \(\theta_{ij}\) equals \(\theta < \theta\). Hence, a parent is altruistic in the sense that she values her descendant’s quality of education, which in turn determines the earning capacity she bequeaths to her offspring.\(^6\)

The twice-continuously differentiable utility function is assumed to fulfil three properties: (i) utility is strictly increasing in both consumption and education quality, (ii) \(U(x_{ij}, \theta) > U(0, \theta)\) holds for all \(x_{ij} > 0\), and (iii) if \(U(x', \theta) = U(x'', \theta)\), then \(U(\lambda x', \theta) = U(\lambda x'', \theta)\) for any \(\lambda > 0\). The first property is obvious. The second property excludes implausible boundary solutions without any consumption. As explored below, the third property means that the willingness to pay for higher education is proportional to net income. This property is fulfilled, for instance, by the Cobb-Douglas function \(U(x_{ij}, \theta_{ij}) = x_{ij}^\alpha \theta_{ij}^\beta\), \(\alpha, \beta > 0\), and the quasilinear function \(U(x_{ij}, \theta_{ij}) = \ln x_{ij} + \theta_{ij}\). Since property (iii) is loosely related to the concept of homotheticity, it is referred to as ‘quasi’-homotheticity.

\(^{5}\)The simplification that the parent decides on behalf of her child is widespread in the literature (see, for instance, De Fraja, 2001, and Beviá and Iturbe-Ormaetxe, 2002). This assumption reflects the strong personal and financial ties between parents and their offspring. Parents make, for instance, predetermining schooling decisions at a stage at which the children are more or less passive players without any precise ideas about the implications of these choices for their future. Moreover, young adults face severe credit constraints largely due to information problems and the inability to collateralise human capital. Thus, they can only finance their education if they are supported by their parents or the public. Putting these arguments together, the ‘parentocracy’ serves as a reasonable proxy of educational decisions. Interestingly, many education subsidies are indeed directly targeted at the parents and not at the students. In many countries, parents are eligible to tax deduction or child related transfers as long as their children attend universities.

\(^{6}\)As one of the referees thankfully pointed out, the preferences are reminiscent of impure altruism (see Andreoni, 1989, for a seminal paper). Parents receive a ‘warm glow’ from their contributions to their children’s future consumption. These contributions take the form of giving them education, which can generate an immediate warm glow in the present. Conditioning a parent’s choice on the child’s education quality or similar ‘myopic’ variables instead of the child’s utility is common in dynamic political-economic analyses (see, for instance, Cardak, 2004, Glomm and Ravikumar, 1992, and Gradstein and Justman, 1997). Otherwise, the models are intractable.
A household whose child attends a university receives a uniform subsidy $s$, $s \geq 0$. This subsidy is financed by a lump-sum tax $t$ on the working generation. A household’s budget constraint is thus given by

$$x_{ij} = \begin{cases} w_j - t & \text{if the child receives basic education}, \\ w_j - t + s - z_{ij} & \text{if the child receives higher education}, \end{cases}$$

where $z_{ij}$ denotes the child’s higher education costs.

**Government Budget** The tax is non-negative and, to avoid the unrealistic case of de facto expropriation, is assumed to be limited to $\bar{t} < w_L$. Additionally, the education policy is constrained by the requirement of a balanced government budget in each period:

$$B = t - sE = 0,$$

where $E$ denotes the ‘number’ or, more precisely, the mass of university students. Tax revenues $t$ (recall that the size of each generation is normalized to unity) have to cover public education spending $sE$.

**Timing of Decisions** Given the households’ and government’s budget constraints, the utility-maximising parents make two decisions. In the first stage, they democratically adopt a tax $t$ that finances the uniform education subsidy $s$. A proposal $t$ is collectively chosen if it wins every pairwise comparison against all other candidates. The assumption that only the parents constitute the electorate captures the fact that when students enter university they have barely voted once. The education system can thus be considered as exogenous for the young adults, and depends on the choice of the parents.

In the second stage, the parents decide whether they will provide the financial means for a higher education of their children, taking full account of the tax and its implications on the subsidy level. The households’ consumption then results simply as the difference between net income on the one hand and education expenditures net of subsidy on the other hand.

### 3 Education Choice and Political Preferences

Before I explain the political evolution of public education spending in a democracy and the social stratification implied, the ‘static’ equilibrium in a single period has to be analysed. To this end, the households’ education choices for a given policy $(s, t)$ and the relationship between the tax $t$ and the per-student subsidy $s$ are explored first. Based on the insights gained from this exercise, the political preferences and the emergence of two opposing stances can be outlined.
3.1 Enrolments and Government Budget

When a household decides on the child’s education level, the family’s trade-off is straightforward. A young individual goes to university if the household’s utility gain resulting from a highly educated offspring outweighs the utility loss induced by the private education spending. The inequality \( U(w_j - t + s - z_{ij}, \theta) \geq U(w_j - t, \theta) \), which follows from inserting the budget constraint (2) into the utility function (1), provides the necessary and sufficient condition for this to be the case.

This condition can be characterised more precisely by making use of Lemma 1, which is an immediate implication of the assumption that the utility function is ‘quasi’-homothetic (see property (iii) of this function).

**Lemma 1 Willingness to Pay for Higher Education.**

For a given tax \( t \), a parent’s willingness to pay for the higher education of her child equals the fraction \( m \) of her income \( w_j - t \), i.e., \( m(w_j - t) \). The fraction \( m \) is independent of the net income and lies in the open interval \((0, 1)\).

**Proof.** All proofs are relegated to the Appendix.

Lemma 1 implies that the offspring is sent to university if, and only if, the willingness to pay for higher education \( m(w_j - t) \) and the subsidy \( s \) are together sufficient to cover the costs \( z_{ij} \). That is, a child attends university if, and only if,

\[
z_{ij} \leq m(w_j - t) + s =: \tilde{z}_j(s, t).
\]

Condition (4) has two obvious implications. First, facing a trade off between the education level and consumption, parents are more inclined to invest in the human capital of ‘low cost’ (i.e., highly intelligent) children than in that of costly (i.e., less able) ones. Second, since the willingness to pay for education increases with income, the higher income family of the marginal student \( \tilde{z}_H \) spends more on education than its poorer counterpart. Consequently, children with the same abilities, i.e., same costs \( z \), might acquire different levels of human capital. In this sense, there is no equality of opportunity. The fraction of children from lower income families attending universities falls below that of wealthier children receiving higher education, i.e.,

\[
\gamma_L(s, t) = \frac{\tilde{z}_L - z}{z - \tilde{z}} < \frac{\tilde{z}_H - z}{z - \tilde{z}} = \gamma_H(s, t),
\]

where \( \gamma_L \) and \( \gamma_H \) denote the respective fractions, referred to as participation rates, and \( \tilde{z}_j \) is defined by (4). This gap in participation rates is backed by broad empirical evidence (see, for instance, Blanden and Machin, 2004).

Using (5), the number of university students is given by

\[
E(s, t) = \gamma_L(s, t) L + \gamma_H(s, t) H = \gamma_L(s, t) + [\gamma_H(s, t) - \gamma_L(s, t)] H,
\]
where \(L\) and \(H\) denote the number of lower income and higher income families, respectively. (Recall that the size of the population is normalized to unity, and thus \(L = 1 - H\) holds.) Since the participation rate \(\gamma_H(s, t)\) is greater than \(\gamma_L(s, t)\), enrolment \(E\) increases in the size \(H\) for a given policy \((s, t)\), a relationship which will be important in the following analysis.

For a given tax, the number of students determines the subsidy level, as the government budget constraint (3) shows. This subsidy level, in turn, affects the number of students, as enrolment (6) shows. Taking these interactions between these two figures into account, a unique subsidy \(s\) results for each tax \(t\) such that (i) the government budget is balanced and (ii) each household’s education choice is consistent with utility maximisation for the respective bundle \((s, t)\). (Note that since a single parent has no significant influence on the number of students, she regards the subsidy as independent of her education choice.)

The resulting functional relationship between the tax \(t\), which is determined in the first stage, and the per-student subsidy \(s\) has two important properties. They are described in

**Lemma 2 Relationship between Subsidy and Tax.**

Subsidy \(s\) is a strictly increasing and concave function of tax \(t\), i.e., \(ds(t)/dt > 0\) and \(d^2s(t)/dt^2 < 0\).

The basic intuition for these two properties is straightforward. A higher tax yields larger revenues. Ceteris paribus, it curbs the households’ net incomes and thus the demand for university education. Both larger revenues and lower demand work in favour of a higher subsidy per student.

Moreover, if the tax and, thus, the subsidy and the number of students are rather low, additional revenues stemming from a higher tax are only divided among a small group. The resulting subsidy increase is quite substantial. But the larger the transfer level, the more people attend universities and the larger is the group of recipients demanding their share of additional revenues. The marginal rise in the subsidy then becomes smaller if the tax goes up, yielding a strictly concave relationship between the subsidy and the tax.

Finally, note that the participation rates are only given by (5) if \(\tilde{c}_j \in [\underline{z}, \bar{z}]\) holds. To avoid tedious discussions of rather unrealistic boundary solutions, the focus is on the cases in which the threshold value \(\tilde{c}_j(s, t)\) is indeed between \(\underline{z}\) and \(\bar{z}\). This outcome can be guaranteed by restricting the parameter space:

**Assumption 1:**

(a) \(\underline{z} < mw_L\) and (b) \(\bar{z} > m(w_H - t) + s\) for \(\{(s, t) | t - sE(s, t) = 0\}\).
These conditions are easy to interpret. On the one hand, the brightest children of each income group attend university even if there is no government intervention (cf. inequality (4)). On the other hand, no financially feasible bundle \((s, t)\) makes a higher education degree achievable for the least able child in his social class. (Since the restriction \(t \in [0, T]\) limits the feasible subsidy \(s\), there exists a non-empty set of parameters that fulfil this assumption. As will become evident below, a policy such that all people of a generation go to universities would anyhow never achieve a majority even if it were possible. However, \(\gamma_H = 1\) could, in principle, result if the cost level \(z\) was too small. Considering such boundary solutions, however, would not generate fruitful insights.)

3.2 To be or not to be in Favour of a Subsidy

Having discussed the relationship between the tax \(t\) and the subsidy \(s\), we can now turn to the individuals' political preferences. This analysis is fairly straightforward in the current framework, since households can be clearly divided into two opposing groups: On the one side, there are all those voters who reject any tax and education subsidy. On the other side, there are the supporters of government intervention. Within the latter group, no conflicts of interests arise, since all parents in favour of public education spending agree on their preferred tax level. Hence, a clear-cut dichotomy between the citizens for and those against public education spending emerges. Let us explore the reasons for this outcome before it is precisely stated in Proposition 1 at the end of this section.

Consider first what a parent’s optimal policy in the first stage would be if the parent’s child received higher education in the second stage. In this case, the best policy for the family would be the tax \(t\) that maximises its utility \(U(w_j - t + s(t) - z_{ij}, \theta)\) and thus the differential \(s(t) - t\), referred to as net subsidy. Since the government budget constraint yields a strictly concave functional relationship between the variables \(s\) and \(t\), i.e., \(d^2s/dt^2 < 0\) (see Lemma 2), there exists a unique solution \(t^o\) to this maximisation problem, implying a unique subsidy \(s^o\).

Formally, this best policy follows from

\[
\max_t s(t) - t, \quad (7)
\]

which yields the first-order condition

\[
\frac{ds(t)}{dt} = 1. \quad (8)
\]

This condition simply says that, in the optimum, a marginal tax increase equals
the induced rise of the subsidy (given a balanced government budget).\footnote{As \( ds/dt \mid_{t=0} > 1 \) holds, both optimal tax and subsidy have to be strictly positive (see the proof of Proposition 1 in the Appendix).} Since taxes are lump sum, the first-order condition is identical for the two income groups. Thus, the solution \( t^o \) does not depend on whether a higher income or a lower income household is considered. It is also not affected by the \( z_{ij} \)-type. To sum up, if a child attended a university, the tax \( t^o \) and the corresponding subsidy \( s^o \) would be the best that could happen from the perspective of her parent.

By contrast, if a household’s child did not receive higher education in the second stage, the parent would obviously oppose any tax in the first stage, since the resulting transfers would only benefit other families. In this case, the best policy for the family would be \( t = 0 \) (= \( s \)). The remaining question is whether a parent prefers the tax \( t^o \) and sending her child to a university to the policy \( t = 0 \). The former alternative is the family’s best choice if, and only if, \( U (w_j - t^o + s^o (t^o) - z_{ij}, \theta) \geq U (w_j, \theta) \) is fulfilled.

‘Quasi’-homotheticity of the utility function implies that a parent’s willingness to pay for her child’s higher education including tax payments is equal to the fraction \( m \) of the gross income \( w_j \), i.e., \( mw_j \).\footnote{This follows immediately from considering Lemma 1 for \( t = 0 \).} Thus, a parent favours the tax \( t^o \) if, and only if, the resulting net subsidy \( s^o - t^o \) and her gross willingness to pay for education \( mw_j \) together are sufficient to cover the costs \( z_{ij} \). That is, a parent votes for the tax \( t^o \) if, and only if,

\[
z_{ij} \leq mw_j - t^o + s^o (t^o) =: \tilde{z}_j (s^o (t^o), t^o). \tag{9}
\]

If this is not the case, the citizen prefers to pay no taxes at all and to relinquish higher education for her child.

Condition (9) has two important implications. First, since the gross willingness to pay increases with income, a smaller fraction of the lower income class than of the higher income class advocates a subsidy, i.e.,

\[
\eta_L (s^o (t^o), t^o) = \frac{\tilde{z}_L - \tilde{z}}{\tilde{z} - \tilde{z}} < \frac{\tilde{z}_H - \tilde{z}}{\tilde{z} - \tilde{z}} = \eta_H (s^o (t^o), t^o), \tag{10}
\]

where \( \eta_L \) and \( \eta_H \) denote the respective fractions and \( \tilde{z}_j \) is defined by (9). The stronger political support of skilled parents simply stems from the stronger representation of their children at universities.

Second, in each income group the parents whose children receive higher education outnumber the parents who support the proposal \( t^o \), i.e., \( \gamma_j > \eta_j \) for \( t = t^o \) and \( s = s^o \). This implication can be easily explained. If the proposal \( t^o \) is already implemented, the tax payment has to be made anyway and is sunk. In this case,
some families are willing to bear the remaining private education costs \( z_{ij} - s^o \) and send their children to universities, although they prefer no tax at all at the political stage and just a basic education for their children.

More importantly, from the perspective of those parents who are in favour of government intervention, the preferred subsidy \( s^o \) strictly decreases with the number of skilled parents \( H \). This result stems from a negative spending effect. A larger class size \( H \) increases the demand for higher education. To balance the government budget, the per-student subsidy has to fall for a given tax \( t \). Moreover, the additional revenues caused by a marginal tax increase are now distributed among a larger group of recipients. Thus, the rise of the subsidy in response to a marginally higher tax turns out to be smaller, the larger the class size \( H \). Since the per-student subsidy for a given tax and the benefit of a tax increase decline, public funding of higher education becomes less attractive even to those parents who favour public education spending in principle. Hence, their preferred subsidy \( s^o \) definitely decreases.

The main conclusions are summarised in

**Proposition 1 Political Preferences.**

i) A parent prefers the tax \( t^o \) that denotes the solution to maximisation problem (7) to all other alternatives if, and only if, \( z_{ij} \leq \bar{z}_j(s^o(t^o), t^o) \) holds. Conversely, a household favours the policy \( t = 0 \) over all other candidates if, and only if, \( z_{ij} > \bar{z}_j(s^o(t^o), t^o) \) results.

ii) The subsidy \( s^o \) strictly decreases with the size of the skilled class \( H \).

Finally, note that the policy \((s^o, t^o)\) maximises the net subsidy by excluding a fraction of the young generation from higher education. Hence, enrolments definitely fall below unity for this policy.

4 Education Policy and Social Stratification

Having explored the political preferences, let us now turn to the final step and analyse the equilibrium and the dynamic properties of the socioeconomic system. Today’s political choice determines current enrolments, which in turn fix the size of the socioeconomic classes and thus the society’s starting position in the future. Two succeeding periods are linked via the ‘inherited’ number of skilled and unskilled workers. Through this connection and the impact of social stratification on political majorities, public education spending is the result of both current votes and past democratic choices.
4.1 Election Results and the Size of Income Groups

Since higher income and lower income supporters of a government intervention agree on their favoured policy, there is a clear-cut dichotomy of interests. As stated in Proposition 1, the most preferred tax of a household is either $t^\kappa_0$ or 0, where the subscript $\kappa$ stands for the time period. (When analysing the system dynamics, time period subscripts are added to the policy variables $s^\kappa_0$ and $t^\kappa_0$ to make clear that they vary over time.) In any case, one of these two proposals is favoured over all other alternatives. The electorate can thus be divided into two opposing parties, and the favourite proposal of the larger party is the Condorcet winner, the alternative that wins against any other policy at the polls in a pairwise comparison.\(^9\)

Given that each stance is supported by households in both income groups, the political dispute is certainly not a traditional class conflict. The main issue of the current paper is not redistribution between income groups, but redistribution between those who send their children to universities and those who do not. Despite this fact, the question of which of the two proposals gains the upper hand crucially depends on the society’s social composition. As inequality (10) shows, public education spending is more popular among skilled parents than among unskilled. Since the children of skilled parents predominantly go to university, a larger fraction of higher income families benefits from a subsidy. This suggests that the votes in favour of the alternative $t^\kappa_0$ increase in class size $H_\kappa$. And indeed, this notion is confirmed in Proposition 2.

**Proposition 2** ‘Static’ Equilibrium.

The proposal $t^\kappa_0$ wins the election if the size of the skilled class $H_\kappa$ exceeds a threshold value $\overline{H}$. Otherwise, the proposal $t = 0$ gains a majority against any other alternative.

Let us explore the economic intuition behind this result in more detail. First, note that the number of citizens who vote for the policy $t^\kappa_0$, and thus against the proposal $t = 0$, in the decisive election is given by

$$V (s^\kappa_0 (t^\kappa_0), t^\kappa_0) = \eta_LL_\kappa + \eta_HH_\kappa = \eta_LL + [\eta_H - \eta_L] H_\kappa,$$

(11)

where (10) is used. Since $\eta_H$ is greater than $\eta_L$, $V$ increases with $H_\kappa$, ceteris paribus. This direct electoral composition effect means that more skilled parents first of all reinforce the support for public education spending. After all, skilled parents are more inclined to send their children to university, and thus tend to gain more from

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\(^9\)The clear-cut dichotomy guarantees an election winner despite the fact that not all voters have single-peaked political preferences.
a subsidy, than unskilled parents. In addition, a larger skilled class also affects the policy \((s_n^*, t_n^*)\). In particular, it reduces the appetite for a very generous per-student subsidy, as argued above. This policy effect depresses per-student spending and indirectly curbs the rise in support for government intervention. However, while an increase in the number of skilled parents might reduce the benefit per household whose child attends a university, it still raises the overall number of households who benefit from a subsidy. Thus, a larger skilled class strengthens the support for some public education spending, and government intervention wins a majority at the polls only if the number of skilled parents is sufficiently high.

That is, the proposal \(t_n^*\) wins at the polls if \(H_n\) exceeds a critical value \(\overline{H}\), i.e., \(V \geq 0.5\) if \(H_n \geq \overline{H}\). In this case, the majority of skilled parents supported by lower income families with talented (i.e., low cost) children decides the election in favour of the policy \(t_n^*\). Otherwise, i.e., if \(H_n < \overline{H}\) and thus \(V < 0.5\), a majority prefers the alternative \(t = 0\) and rejects any government intervention, as stated in Proposition 2.

Given this ‘static’ election result, the precise evolution of education policy and its impact on class size can finally be analysed. Thereby, a particular focus is on the question of whether a subsidy, once introduced, is politically sustainable.

### 4.2 Educational Take-off and Decline

The dynamics of the system are already partly indicated in Propositions 1 and 2. As shown above, active government involvement can only gain a majority at the polls if the size of the skilled class is sufficiently large. The threshold value \(\overline{H}\) might be smaller than zero or greater than one, meaning that a majority for education subsidies either always exists or can never be achieved.\(^{10}\) More interesting, however, is the case in which \(\overline{H}\) lies in the open interval \((0, 1)\) and will be reached over time. The resultant dynamics of the system is summarised in Proposition 3.

**Proposition 3 Evolution of Education Policy and Social Stratification.**

Assume that the system described above starts with no skilled citizens, i.e., \(H_0 = 0\). Then, it shows the following dynamic properties:

i) The size of the skilled class \(H_n\) and the number of university students \(E_n\) strictly increase over time and converge towards positive values in the stable steady state. On this path, the policy \(t = 0\) wins the elections as long as the number of skilled parents is below the threshold value \(\overline{H}\). If the size of the skilled class exceeds the

\[^{10}\text{A simple sufficient condition for } \overline{H} < 1 \text{ to hold is } mw_H > (1/2)(\overline{\tau} + \underline{\tau}). \text{ In this case, more than 50% of the higher income households send their children to universities even if the government does not intervene, and thus necessarily support the policy } t^*\]
critical level $\bar{H}$ in period $\pi$, the tax $t^\pi$ will gain a majority at the polls in this and all succeeding periods, leading to the corresponding per-student subsidy $s^\pi$.

ii) After introducing a positive tax and subsidy in period $\pi$, the per-student subsidy $s^\pi$ and the net subsidy $s^\pi - t^\pi$ strictly decrease over time and converge towards positive steady state levels.

iii) The ratio of the participation rates $\gamma_L/\gamma_H$ strictly increases in period $\pi$. It strictly decreases afterwards and converge towards its positive steady state value.

Figure 1 illustrates these conclusions. For convenience, continuous lines capture the dynamics of the system, despite the fact that time is taken to be a discrete variable in the current model. Also, this figure illustrates the interesting case in which an educational take-off occurs in period $\bar{\pi} > 0$.\(^{11}\)

Let us explore the intuition for the evolution of public education spending and social stratification step by step. Assume that the size of the skilled class in the starting period $H_0$ is very small and falls below $\bar{H}$ (subscripts of variables changing

\(^{11}\)Note that proposition 3 does not rule out $\bar{H} = 0$ and thus $\bar{\pi} = 0$. In this case, the educational take-off occurs at the very beginning, and the section to the left of $\bar{\pi}$ vanishes in figure 1. That is, the subsidy per student is positive from the outset but decreases over all periods. Conversely, if the number of skilled parents never reaches the critical value $\bar{H}$, then the educational take-off will never occur, and the section to the right of $\bar{\pi}$ will disappear in figure 1. That is, the tax and subsidy will stay at zero forever.

Figure 1: Evolution of Education Policy and Social Stratification

Let us explore the intuition for the evolution of public education spending and social stratification step by step. Assume that the size of the skilled class in the starting period $H_0$ is very small and falls below $\bar{H}$ (subscripts of variables changing
over time refer to the respective period). There is, thus, no majority for an active education policy at the beginning of the time horizon. But as, for a sufficiently small size of the skilled class, the number of university students from lower income families outweighs the number of higher income class children who do not attend universities, the group of skilled people will grow over time. This dynamic process is described by the difference equation (6) for \[ s = t = 0 \], which relates the size of the skilled class in the next period \( E_{t+1} \), \( E_{t+1} = H_t \), to its present size \( H_t \).

This development heads towards a stable steady state with no education subsidies unless the figure \( H \) tops the threshold value \( \overline{H} \) in any period. If wages and willingness to pay for education are rather high, or costs are sufficiently low, the class size \( H_t \) will once be above \( \overline{H} \). Suppose this is the case in period \( t \), as shown in Figure 1. An educational take-off then occurs, meaning that a majority of skilled and unskilled citizens with talented children push through a tax financed education subsidy. The distributional effects of this policy are ambiguous. On the one hand, this policy is at the expense of the many unskilled parents who pay taxes but whose children only receive basic schooling. On the other hand, the public education spending indeed opens up access to universities for children from lower income families and improves the equality of opportunity. The ratio of the participation rates \( \frac{L}{H} \) increases, indicating the enhanced social mobility and educational integration. The subsidy, moreover, boosts the number of students in general.

But this phase does not last forever. After a period of extension, the ‘success’ of the education spending causes its decline. The government intervention has fostered the change of the society’s social composition. As a result, a larger group of skilled parents further increases the demand for education and thus makes publicly financed, or at least supported, universities more costly. Maintaining the same subsidy per student would require a significant tax increase. Even more, the positive impact of a marginal tax change on the subsidy is smaller, since the number of recipients has grown. In response to this negative spending effect, even the supporters of government intervention now favour cuts in the resulting per-student subsidy \( s_t \).

There is still a majority for an active government role, but this majority successfully calls for larger private contributions to higher education. Unavoidably, the net subsidy \( s_t - t_t \) falls along with the per-student subsidy \( s_t \).

Despite the backlash at the polls, the overall number of students further increases. This rise is due to the change in the society’s social composition. An increase in class size \( H_t \) leads to a greater demand for university places and drives the number of students up. Importantly, the shift in the political preferences shapes the social openness of the university system and thus social stratification. Since lower income families are particularly affected by the cuts in the subsidy per student, the fraction
of their children attending university $\gamma_L$ drops in relation to that of children from skilled parents $\gamma_H$.

The decline in per-student subsidy in the aftermath of the educational take-off continues in the ensuing periods. Rising enrolments today further increase future education demand via the class-size effect. Although the subsidy system is put under even more strain, a majority continues to support some government intervention. But this majority imposes further cuts in the per-student subsidy. While the fraction of unskilled parents drops, the remaining children from lower income families fall more and more into an education trap. The whole process finally converges towards a steady state with a smaller but still positive subsidy.

Let me conclude this section with two remarks. First, this model explains an initial educational take-off and the ensuing decline in public spending per student as an almost ‘smooth’ development. The size of the skilled class increases over the whole time horizon, monotonically converging towards its steady state value. Also, the subsidy per student declines ‘smoothly’ after the initial rise, and converges towards a positive steady state value. Despite the decline in public spending per student, there is no return to a situation in which a majority rejects any government intervention (i.e., there is no return to $t = s = 0$). Obviously, the almost ‘smooth’ evolution is a stylised representation of a long-term trend that neglects short-term disruptions. The model cannot explain temporary or permanent voting cycles, which are ruled out in this simple framework (see the proof of Proposition 3). Some issues that complicate political life and may lead to a less ‘smooth’ development are discussed in Section 5.

Second, changes in private spending on higher education counteract the changes in public spending. In the periods prior to the educational take-off, average private spending per student remains constant over time within each income class. However, since the share of skilled parents increases, and since skilled parents spend more on education on average than unskilled parents, private spending per student grows when considering the society as a whole. Then, the introduction of public subsidies in the course of the educational take-off leads to a fall in private spending per student within each income group. But once the subsidy per student declines, private spending picks up again. That is, private spending per student increases over time in each income group as well as in the society as a whole.$^{12}$

12 Household $i$ of group $j$ will privately spend $z_{ij} - s$ on education if the child attends university. Thus, private spending per student of parents in income group $j$ (i.e., average private spending of $j$’s households who send their children to university) is given by $\int_{\frac{\bar{z}}{2}}^{\frac{\bar{z}}{2}} (z_{ij} - s) / (\bar{z} - \bar{z}) \, dz_{ij} = 0.5 (\bar{z} - s) - s = 0.5 \left[ \bar{m} (\bar{w} - t) - s + \bar{z} \right]$. The evolution of this term over time implies the results mentioned above. At the aggregate level, the growth in private spending on higher education can outpace the rise in income. Further details can be obtained upon request.
5 Discussion

A thorough econometric underpinning of the theoretical results is beyond the scope of this paper, but I now provide some evidence for the key conclusions. Also, several potential extensions of the current analysis and some qualifications are discussed in this section.

5.1 Evidence

The major implication of the current political-economic analysis is straightforward. After an initial expansion period, the recipients of higher education have to cover an increasing share of their education costs privately, as the majority pushes through cuts in per-students subsidies over time. A further result is that, again after an initial expansion period, equality of opportunity deteriorates over time. Finally, the number of students continues to increase even after private education costs begin to rise.

There is some evidence that backs these claims. In the United Kingdom, for instance, participation in higher education has significantly risen. The Age Participation Index (API) increased from about 6% in the early 1960s to more than 30% in 2001 (Blanden and Machin, 2004). The Higher Education Initial Participation Rate (HEIPR), a new government measure, has also risen over recent years and reached 46% for 2008–2009 (BIS, 2010).

Public support for students reached its highest levels from 1977 to 1984, with fairly generous maintenance grants and access to housing benefits and, during vacation, unemployment benefits. In the second half of the 1980s, the real value of the maintenance grant deteriorated and access to social transfers was cut off. Loans increasingly replaced grants in the 1990s (Blanden and Machin, 2004).

In the same vein, Greenaway and Haynes (2003) describe the drastic drop in public university funding in the United Kingdom. The index of funding per student plummeted from 100 in the base year 1980/1981 to just about 50 in the year 1999/2000. In the same period, student numbers doubled, clearly indicating that public spending on higher education was far from matching the rising intake of the British universities. Thus, not only public support for students but also per-student funding of universities decreased sharply.

13 The API and the HEIPR are not directly comparable. Also, different methodologies have been used to calculate the HEIPR for the years from 1999 to 2006 and for the years from 2006 to 2009. There was no clear trend in the former period, with a HEIPR of 39.2% for 1999–2000 and 39.8% for 2006–2007 (old methodology). In the latter period, the HEIPR has gone up from 42.0 for 2006–2007 to 45.5% for 2008–2009 (new methodology). See BIS (2010) for details.
Major reforms of the tuition fee system have further raised the private costs of higher education. In 1998, the government implemented a tuition fee of £ 1,000 per annum (which was partly waived for poor students and did not apply to Scottish students who studied at Scottish universities). Just a few years later, the government again overhauled the fee system in the Higher Education Act 2004, leading to even higher private contribution to the costs of university education. Under the new system, which was introduced in 2006, universities in England and Northern Ireland can currently charge domestic students up to £ 3,290 a year. Students receive loans to cover the tuition fees, but these loans have to be paid back after graduation. The cap on tuition fees will rise to £ 9,000 in 2012, accompanied by a corresponding drop in public funding for teaching (Economist, 2010). Together these measures effectively ‘privatise’ many higher education courses which will then no longer receive any direct public money. These changes constitute a clear shift from public to private funding of higher education.

There is some empirical evidence that such a shift is indeed at the expense of lower income households. For instance, Blanden and Machin (2004) argue that the participation gap in higher education increased between 1981 and 1999. Making use of two British cohort studies and the British Household Panel Survey, they provide data about the proportions of young people who have acquired a degree by the age of 23 for three different parental income groups. Denote the degree acquisition rates for children from parents of the top quintile (middle 60%, bottom quintile) by $\mu_{H1}$, $\mu_{H2}$, $\mu_L$. Then the ratio of the degree acquisition rates $\mu_L/\mu_{H1}$ fell from 0.75 in 1981 to 0.39 in 1999. Similarly, the ratio $\mu_L/\mu_{H2}$ dropped from 0.30 in 1981 to 0.20 in 1999. Applying probit models and the Nadaraya-Watson kernel regression, Blanden and Machin (2004) show that the positive relationship between parental income and degree acquisition became stronger between 1981 and 1999.

Political shifts in funding higher education can be observed in many countries, and international data underline the changes in education finance. OECD (2001, 2010) provides data about the relative proportions of public and private expenditures on tertiary educational institutions. Comparing these data for 1995 and 2007, the year before the current financial crisis began to unfold, provides some interesting descriptive insights. For these two years, the data on spending shares is available for 20 OECD countries. It shows that the public share of total (public and private) spending on higher education declined, and that the private share increased correspondingly, in 16 out of these 20 countries. The public share of total expenditures on tertiary education plummeted by more than 10 percentage points in Australia (from 64.6% to 44.3%), Austria (from 96.1% to 85.4%), Italy (from 82.9% to 69.9%), Portugal (from 96.5% to 70%), and three further OECD countries. The decline was
less sharp in other countries, such as Germany (from 89.2% to 84.7%) and Sweden (from 93.6% to 89.3%).

The OECD data certainly has to be interpreted with caution. For instance, private spending includes expenditures that are publicly subsidised, and several countries do not provide information about the magnitude of this indirect public spending. In the case of the countries for which this information is available, however, calculating total public spending including public subsidies and using these numbers does not affect the overall picture. In the United Kingdom, for instance, the share of public spending including subsidies to households and private entities fell from 79.9% in 1995 to 52.9% in 2007.

5.2 Extensions and Qualifications

Next, let us discuss the assumptions about the utility function and the tax system in more detail.

Preferences and Taxes  First, preferences are assumed to be ‘quasi’-homothetic. As argued above, this assumption guarantees that, for a given tax, the households’ willingness to pay for higher education, i.e., \( m(w_j - t) \), is proportional to net income. This property is fulfilled, for instance, by the Cobb-Douglas utility function and its monotonic transforms, which are routinely used in dynamic analyses (Cardak, 2004, and Gradstein and Justman, 1997, among others). As a result, the demand for higher education is linear in the tax and the number of skilled parents, which in turn eliminates ‘second-order’ effects. That is, the magnitude of the negative impact of a higher tax, and the positive impact of a larger skilled class, on the number of students, i.e., \( \partial E/\partial t < 0 \) and \( \partial E/\partial H > 0 \), does not depend on the tax level and the size of the skilled class, i.e., \( \partial^2 E/\partial t^2 = 0 , \partial^2 E/\partial t \partial H = 0 \) and \( \partial^2 E/\partial H^2 = 0 \).14

To get a taste of how alternative preferences affect the conclusions of the paper, the willingness to pay for education, denoted by \( g_j = g(w_j - t) \), is now assumed to be an increasing and strictly concave function of income, i.e., \( g' \in (0, 1) \) and \( g'' < 0 \) (instead of \( g' = m \) and \( g'' = 0 \), as above). First of all, this modification will not change the evolution of the size of the skilled class and the number of students up to point in time when the educational take-off occurs. Both the size of the skilled class and the number of students will grow over time if the higher income households exhibit a greater willingness to pay for education than the lower income households exhibit.

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14To be precise, the ‘second-order’ effects \( \partial^2 E/\partial t^2 , \partial^2 E/\partial t \partial H \) and \( \partial^2 E/\partial H^2 \) vanish because of the quasi-homotheticity of the preferences and the assumption that innate abilities are uniformly distributed.
ones. Similarly, there still exists a threshold level $\overline{H}$ so that a majority favours government intervention once the size of the skilled class exceeds this threshold.\footnote{For instance, the sufficient condition $g(w_H) > (1/2)(\overline{\tau} + \overline{\xi})$ guarantees the existence of such a threshold $\overline{H}$ with $\overline{H} < 1$ (cf. footnote 10).}

While the alternative preferences leave the dynamics of the system up to the educational take-off and the educational take-off itself unchanged, they influence to some extent the evolution of the education policy thereafter. To see this, consider the relationship between the subsidy level preferred by those who favour government intervention and the size of the skilled class. Let us focus on the additional, second-order effects that arise under the modified assumption.

Now, a tax increase reduces the number of students from lower income families more drastically than that from higher income families, since lower income parents cut their private education spending more significantly in response to a tax rise than the higher income parents. As a result, the negative effect of a tax increase on the overall number of students will diminish if the skilled class becomes larger (i.e., $\partial^2 E / (\partial t \partial H) > 0$). Consequently, a rise in the size of the skilled class will put additional strain on public finances, and will further reduce the feasible subsidy per student. Then, those who support public education spending in principle favour an even more drastic cut in subsidies as the size of the skilled class increases.

While this additional, second-order effect reinforces the previous conclusion about the relationship between the subsidy and the size of the skilled class, there is another one that weakens this conclusion. This further effect stems from the fact that, as the willingness to pay is strictly concave in income, the negative impact of a tax increase on the number of students becomes stronger with the tax level (i.e., $\partial^2 E / \partial t^2 < 0$). This second-order effect stabilises public finances. It thus counteracts the call for drastic cuts in response to a larger skilled class. As a consequence, the overall direction of the ‘new’ effects on the relationship between the per-student subsidy and the size of the skilled class is inconclusive.

By contrast, other conclusions definitely remain untouched qualitatively. The net subsidy which results if the supporters of government intervention constitute a majority declines with the size of the skilled class, irrespective of which of the alternative assumptions about the willingness to pay for education is made. However, the identified ambiguities of the additional, second-order effects carry over to the analysis of the overall dynamics of the system after the initial educational take-off. For instance, if the decline in the subsidy favoured by the majority is less drastic, then the rise in the number of students will tend to be more pronounced. Accordingly, the further change in the social stratification and the strains on public finances will tend to be more drastic, which will tend to reinforce the backlash at the polls.
in the succeeding periods.

To sum up, the alternative assumption about preferences leaves the evolution of the system up to the educational take-off and this take-off qualitatively unaffected. The net impact of the ‘new’ effects on some variables and their evolution thereafter is ambiguous.\textsuperscript{16} In principle, they could give rise to a more complex dynamics. Consequently, the clear-cut conclusions in the propositions are to be qualified. However, the additional, second-order effects would only reverse the previous results if the net impact of the ‘new’ effects did not only counteract the ‘old’, first-order effects, but also dominated them. To put it differently, the basic premise is that the unambiguous first-order effects drive the overall outcome, and not the ambiguous second-order effects.

Second, preferences are defined over consumption and education only. In particular, leisure is not included, and labour supply is thus exogenously given. However, labour supply can be endogenised without affecting the results of the paper. Even with endogenous labour supply, the individual willingness to pay for a child’s higher education increases with income, as long as education is a normal good, and it can still be a constant share \( m \) of household income, as in the paper’s model.

To illustrate this assertion, consider the utility function

\[
U_{ij}(x_{ij}, y_{ij}, \theta_{ij}) = [\alpha \ln x_{ij} + (1 - \alpha) \ln y_{ij}] + \theta_{ij},
\]

where \( y_{ij} \) stands for the time the parent allocates to leisure. Normalising an individual’s total time to unity, family consumption is then given by (i) \( x_{ij} = (1 - y_{ij}) w_j - t \) if the child receives basic education and (ii) \( x_{ij} = (1 - y_{ij}) w_j - t + s - z_{ij} \) if the child participates in higher education (cf. budget constraint (2)). Then, it turns out that the threshold levels \( \bar{z}_{ij} \) and \( \tilde{z}_{ij} \) are still defined by conditions (4) and (9), with the income share \( m = (1 - e^{-(\theta - \tilde{\theta})}) \) being independent of household income.\textsuperscript{17} As a result, all further conclusions of the paper are completely unaffected. For more general utility functions, however, the income share \( m \), which a parent is willing to sacrifice for her child’s education, cannot be expected to be constant. But as the previous discussion indicated, the relationship between income and the willingness to pay for higher education depends on the general properties of the utility functions, and not necessarily on the particular question

\textsuperscript{16}In the same vein, ambiguities will also emerge if the willingness to pay is strongly increasing with income, i.e., if \( g' > 0 \) and \( g'' > 0 \).

\textsuperscript{17}This solution can be easily checked. The optimal levels of consumption and leisure are (i) \( x^*_{ij} = \alpha (w_j - t) \) and \( y^*_{ij} = (1 - \alpha) (w_j - t) / w_j \) if the child receives basic education and (ii) \( x^+_{ij} = \alpha (w_j - t + s - z_{ij}) \) and \( y^+_{ij} = (1 - \alpha) (w_j - t + s - z_{ij}) / w_j \) if the child participates in higher education. Inserting these values into the utility function and comparing the utility levels (i) \( U_{ij}(x^*_{ij}, y^*_{ij}, \theta) \) and (ii) \( U_{ij}(x^+_{ij}, y^+_{ij}, \theta) \) along the lines explored in Section 3 gives the threshold levels \( \bar{z}_{ij} \) and \( \tilde{z}_{ij} \) and the corresponding income share \( m = (1 - e^{-(\theta - \tilde{\theta})}) \).
of whether leisure, or any other additional good, is taken into account.

Finally, subsidising higher education raises the size of the future skilled class, which in turn has two implications for public finance. First, the number of students increases not only in the current period but even more in the future, and so does government spending. The current model captures this spending effect. Second, aggregate income rises in the future, and so would tax revenues if tax payments increased with income. Assuming lump-sum taxation, this tax revenue effect, which counteracts the spending effect, is ignored in the current analysis.

For general tax systems, the spending effect will dominate the tax revenue effect if the share of public education spending that the lower income households receive is greater than their share of the total tax payments. More specifically, consider the case where taxes are proportional to income. Then the spending effect will be dominant as long as the subsidy falls short of the minimum education costs $\frac{z}{z}$. That is, if all students still face some private education costs, the surge in public education spending will outpace the rise in revenues, as the size of the skilled class increases.\(^{18}\) In any case, the tax revenue effect eases the strain on public finances. Thus it should certainly weaken the political pressure to cut education subsidies after an educational take-off.

Importantly, however, the choice of the tax system affects the general support for government intervention in different income groups. Considering lump-sum taxation, the current analysis focuses on the conflict between those families whose children attend universities and those whose offspring does not. If a different tax schedule is implemented, further conflicts of interests arise. In the case of a proportional or progressive tax on labour income, lower income families whose children attend universities benefit twice from public education spending. They receive a subsidy financed by all households, and they contribute less in absolute terms to the tax revenues than their wealthier counterparts. Because of the additional redistributive impact, overall support for a subsidy among the lower income households is strengthened while that among higher income households is diluted. Since the latter group gains political power over time, a less regressive taxation might rather

\(^{18}\)To see whether the spending effect or the tax revenue effect dominates, let us calculate how the government budget changes with the size of the skilled class, assuming that the budget is initially balanced. Consider the reformulated budget

$$B = T_L(1 - H) + T_H H - s \left[ g_L (1 - H) + g_H H + s - \frac{z}{z} \right] / (\tau - z),$$

where $T_i = T(w_i)$ and $g_i = g(w_i - T_i)$ stands for the tax payments and the willingness to pay for higher education as a function of gross and net income, respectively. Then,

$$\frac{\partial B}{\partial H} = T_H - T_L - s \left( g_H - g_L \right) / (\tau - z) = \left[ -T_L + s \left( g_L + s - \frac{z}{z} \right) / (\tau - z) \right] / H,$$

where $B = 0$ is used. Consequently,

$$\frac{\partial B}{\partial H} \geq 0 \iff s \gamma_L \geq T_L \Rightarrow \gamma_L / [\gamma_L (1 - H) + \gamma_H H] \geq T_L (1 - H) / [T_L (1 - H) + T_H H] \iff T_H / T_L \geq \gamma_H / \gamma_L,$$

where again $B = 0$ is used. For a proportional tax, i.e., $T_i = \tau w_i$, where $\tau > 0$ is the tax rate, these inequalities imply

$$\frac{\partial B}{\partial H} \geq 0 \iff s \geq \frac{z}{z},$$

as suggested.
reinforce the decline in education spending.\textsuperscript{19} Whether a less regressive taxation fosters or impedes the initial take-off is unclear. Also, in the case of more than two income groups, an ‘ends against the middle’ phenomenon similar to that in Epple and Romano (1996) and Anderberg (2006) might appear.

**Further Issues** Finally, a few further extensions of the current analysis and qualifications are briefly explored. First, in the current model there are no subsidies before the educational take-off occurs (see Figure 1). This feature certainly exaggerates the rise in public spending. Even before an expansion of the type described above takes place, some kind of government intervention is usually supported by the electorate. Many citizens who do not directly gain from an education subsidy nevertheless vote for some public spending. These individuals might indirectly benefit from increased aggregate human capital because it positively affects the earnings of the unskilled (Creedy and Francois, 1990), the return on capital (Soares, 2003), public pensions (Kemnitz, 2000) and the tax base and thus the financial means for redistributive measures (Beviá and Iturbe-Ormaetxe, 2002). So the education subsidy should be positive in the pre-expansion periods. However, this does not contradict the current arguments for an educational take-off and an ensuing decline in public spending.

Second, by considering only two person families, the model suggests that a majority for public spending requires an overall participation rate that exceeds 0.5. This feature, which frequently appears in the literature on the political economics of higher education (for instance, Fernandez and Rogerson, 1995), of course overstates the number of immediate beneficiaries necessary for an expansion of the public education system to occur. It could be avoided in several ways without affecting the basic mechanisms analysed in this paper. For instance, once families with two parents and more than one child are considered, already one talented child provides sufficient incentives for the parents to support government intervention. Thus, a majority of citizens can favour an education subsidy, although only a minority of the children attends universities. Moreover, parents are well-informed about the abilities of their children, but not with absolute certainty. In the case of risk-averse

\textsuperscript{19}With proportional or progressive taxation, the lower income parents who support government intervention prefer higher taxes than their higher income counterparts. (Skilled parents will further support government intervention if the tax system is not too progressive and the wage gap not too large.) This additional conflict of interest can give rise to ‘technical’ problems such as the potential non-existence of a voting equilibrium. Also, the resulting dynamics can be more complex than the simple evolution of public education spending under lump-sum taxation. Within each income group, however, the preferred tax level of those who support education subsidies still declines with the size of the skilled class as long as the spending effect dominates. Thus, the upper bound of the politically feasible subsidy decreases.
parents, they might vote in favour of public education spending even if the probability that their children are talented and will indeed attend a university is not particularly high. Thereby, they insure themselves against the ‘costly’ outcome that receiving higher education is indeed worthwhile for their children. Again, the number of parents supporting subsidies can substantially exceed the number of students.

Third, the median voter approach is obviously a short cut in order to analyse political decisions in a democracy. Busemeyer (2009), for instance, explains increasing public spending on higher education as a result of social democratic strategies of forging cross-class alliances between working and middle classes. Reaching out to middle class voters enables social democrats to gain majorities in an era in which their traditional working class constituency dwindles away. This argument is consistent with the explanation in the current paper. The skilled class is particularly keen on the initial expansion of public spending. Once its size is sufficiently large, a majority ‘coalition’ forms in favour of substantial education subsidies, consisting of a majority of skilled parents and of unskilled parents with very talented children. In the context of party politics, this cross-class ‘coalition’ can be brought about by social democratic parties in their quest for a majority at a time when the size of the working class diminishes. Importantly, the current paper explores how the interests of this ‘coalition’, and thus the funding of higher education, evolve over time.

Fourth, only an income-independent education subsidy is considered, although in reality we also observe regressive and progressive financial assistance. However, income-independent subsidies, mainly in the form of low, or no, tuition fees, still constitute a major component of total spending on higher education in many countries. From a political-economic perspective, progressive subsidies like means-tested grants can be seen as an instrument to broaden the support for public education spending among lower income households. For instance, a ‘coalition’ in favour of education subsidies might be short of a majority. In particular, too few lower income households support an expansion of public funding, since the optimal subsidy of the ‘coalition’ is not sufficient for them to send their children to university. Then, means-tested grants can lure them into the ‘coalition’ and guarantee a majority without increasing public funding too much above the coalition’s optimal level. They thus reduce the threshold level $H$, but do not necessarily contradict the present conclusion of a decline in the share of public spending after the initial expansion period.

Fifth, the budget constraints of the households and the government in connection with the parents’ preferences might need some further clarification in addition to the remarks in Section 2. These constraints imply that the costs of education in the current period cannot be shifted to the next generation. A strict interpretation
is that these constraints are exogenously given. A less strict interpretation is that these constraints are self-imposed, as the parents derive utility from their children’s education only if the resulting expenditures do not burden their offspring. This excludes, for instance, a pure student loan scheme to finance higher education. However, even if public student loans cannot then simply replace education subsidies, they might emerge as an instrument that complements these subsidies. A loan scheme might increase participation rates, and hence the number of beneficiaries of subsidies. Like means-tested grants, an accompanying loan scheme could therefore broaden the public support for subsidies. On the other hand, more students, and thus more subsidy recipients, mean that those families who would have sent their children to university even without accompanying public loans benefit less from an education subsidy. This curbs the political enthusiasm for a public loan scheme. In this sense, such a scheme is subject to similar political-economic considerations as the subsidy scheme itself.

Sixth, the present approach ignores the impact of economic growth on social mobility (for instance, Galor and Tsidion, 1997, and Owen and Weil, 1998). If technological inventions and aggregate human capital positively affect general productivity and thus increase gross wages of the skilled and unskilled, they boost the demand for university places of both income groups. This not only makes it more likely that the educational take-off occurs because the threshold value $H$ decreases. It also mitigates, or reverses, the ensuing decline in participation rates. However, the political-economic forces that cause the change in education finance are still at work. The call for a higher private contribution to education costs still gains momentum over time, as the number of students increases. Importantly, it is this rise in private contributions that constitutes the major message of the current paper.

Finally, the present model implies a simple relationship between family income

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20 Interpreted differently, parents get a warm glow from funding their own children’s higher education. If students had to pay back the costs of their education, this warm glow would not arise. However, government subsidies do not diminish the warm glow. Key is that the offspring is not burdened. In this sense, this interpretation of impure altruism somewhat differs from the traditional one (cf. Andreoni, 1989).

21 In endogenous wage models without growth and technological progress, the outcome could be slightly different. Consider the case of the Cobb-Douglas production function $Y = H^a L^{1-a}$. Then, a rise in the number of skilled workers drives up unskilled wages but lowers skilled wages. Hence, participation rates $\gamma_L$ and $\gamma_H$ converge, as $\gamma_L$ increases and $\gamma_H$ decreases. However, overall income and thus willingness to pay for education still rise, pushing up the size of the student population. Again, this leads to a backlash at the polls, and increasing private contributions have to make up for a falling per-student subsidy. (Note that a declining wage gap $w_H - w_L$ will curb the rise in the number of students if the parameter $\theta_{ij}$ is identified with the expected future gross income of the young generation.)
and private education spending. In reality, the link between social background and educational choice is certainly more complex. For instance, Sjögren (2000) argues that an individual’s uncertainty about the ability to be successful in an occupation is greater in the case of occupations distant from the parents’ occupation than in the case of familiar ones. Thus, risk-averse individuals tend to choose careers that are similar to those of their parents. So even in countries with modest income disparity, the family background should affect the educational and occupational choices.

This influence of family characteristics, however, does not mean that economic incentives are unimportant. By contrast, Sjögren (2000) shows that under certain conditions individuals from lower income families are particularly sensitive to economic incentives.

6 Conclusion

The present paper highlights some of the underlying mechanisms that can induce an educational take-off and an ensuing decline in subsidies per student. It analyses how the political outcome interacts with the evolution of class size and the changing political preferences of the groups supporting a tax financed subsidy. As we have seen, the current call for higher private contributions might rather reflect a broad trend in public opinion than a short-lived political mood. One appeal of the current approach is that both the rise and the fall in subsidies per student follows from a single cause: the increase in the size, and thus voting power, of the skilled class.

The line of reasoning in this paper has a simple point of departure: the demand for higher education increases with the number of skilled parents because their children attend universities more than proportionally, and the families of students are those who support the respective tax financed subsidy as a means of redistributing resources to them. Given this starting point, the initial expansion of public spending and the ensuing cuts in per-student subsidies are driven by two opposing forces that are generated by the same source, the increasing number of skilled parents. The rise of the skilled class leads to a majority for an educational take-off. This expansion of public education spending further boosts the number of skilled individuals and thus future demand for higher education. This shift in demand implies that the initial subsidy per student becomes too expensive to be politically sustainable. Although the majority for some public spending is broadened, the preferred levels of per-student support decline over time. Nevertheless, the number of students

\[22\] This notion is supported by numerous papers which stress the role of family characteristics like the human capital of parents for the children’s educational attainment. See, for instance, Haveman and Wolfe (1995) for a review.

26
rises further, both reflecting the changing social composition and reinforcing these changes. But despite growing enrolments, equality of opportunity deteriorates.

As argued in the previous section, the path shown describes a broad tendency rather than a precise development. The basic argument is that universities will become fenced in so that the increase in demand is curbed. An obvious strategy to achieve this goal is to require higher private contributions. There are, however, other measures that can serve this end. For instance, institutional and financial arrangements at the school level can work in favour of social selection, thus reducing the group of potential recipients of a subsidy for higher education. In addition to analysing the evolution of public spending on higher education, it is thus worthwhile exploring the evolution of the education system as a whole. This demands a closer look at the interplay between the school system and the system of higher education. This issue is beyond the scope of the current paper.23

Appendix

Proof of Lemma 1

Define \( m := [(w_j - t) - x'']/(w_j - t) \). Properties (i) and (ii) of the utility function (1) guarantee that for all levels of net income \( w_j - t > 0 \), a (hypothetical) consumption level \( x'' \) exists such that \( U(w_j - t, \theta) = U(x'', \theta) \), with \( U(w_j - t, \theta) > U(x'', \theta) \) for all \( x'' < x'' \). Hence, for a net income of \( w_j - t \), the private willingness to pay equals \( (w_j - t) - x'' = m(w_j - t) \). Then, property (iii) implies that a parent with the net income of \( \lambda(w_j - t) \) is ready to give up \( \lambda(w_j - t) - \lambda x'' = m[\lambda(w_j - t)] \). Thus, fraction \( m \) does not depend on the level of net income. Property (i) guarantees \( m > 0 \), and property (ii) implies \( m < 1 \).

Proof of Lemma 2

To make the following derivatives more accessible, the government budget is reformulated, using (4), (5) and (6):

\[
B(s, t; H) = t - sE(s, t; H) = t - s \frac{m[w_L + (w_H - w_L)H - t] + s - \tilde{z}}{\tilde{z} - \tilde{z}}, \quad (A1)
\]

where \( E(s, t; H) \) captures the optimal education choices of the utility-maximising households. For later reference in this and the succeeding proofs, providing a list of

23In a recent paper on education policy, for instance, Di Gioacchino and Sabani (2009) give an political-economic explanation for why societies with a high wealth inequality relative to income inequality are inclined to comparatively spend more on higher education than on lower levels of education. However, this paper does not consider that spending on basic education might have an impact on participation in higher education.
the partial derivatives of (A1) proves to be convenient:

\[
\begin{align*}
\frac{\partial B}{\partial s} &= -E - s \frac{\partial E}{\partial s} < 0, \\
\frac{\partial^2 B}{\partial s^2} &= -2 \frac{\partial E}{\partial s} < 0, \\
\frac{\partial B}{\partial t} &= 1 - s \frac{\partial E}{\partial t} > 0, \\
\frac{\partial^2 B}{\partial s \partial t} &= 0, \\
\frac{\partial^2 B}{\partial s^2} &= 0, \\
\frac{\partial^2 B}{\partial t^2} &= 0.
\end{align*}
\]  
(A2)

The budget constraint \( B(s, t; H) = 0 \) implicitly yields \( s \) as a function of \( t \) and \( H \): \( s = s(t; H) \). Thus, comparative statics yields the derivative

\[
\frac{ds}{dt} = \frac{\partial B(s, t; H)}{\partial t} / \frac{\partial B(s, t; H)}{\partial s} > 0.
\]  
(A3)

This expression leads to

\[
\frac{d^2 s}{dt^2} = -\frac{\frac{\partial B}{\partial s} \frac{\partial^2 B}{\partial s \partial t} ds}{\left( \frac{\partial B}{\partial s} \right)^2},
\]  
(A4)

where \( \frac{\partial^2 B}{\partial t^2} = 0 \) is used (see (A2)).

The denominator of (A4) is positive. Denote the numerator of (A4) by \( \Omega \). Inserting (A3) and (A2) into \( \Omega \) and reformulating the resulting term show that

\[
\Omega = \frac{\partial B}{\partial s} \frac{\partial^2 B}{\partial s \partial t} ds - \frac{\partial B}{\partial t} \left( \frac{\partial^2 B}{\partial s^2} ds^2 + \frac{\partial^2 B}{\partial t \partial s} ds \right) = -\frac{\partial B}{\partial t} \left( 2 \frac{\partial B}{\partial s} \frac{\partial^2 B}{\partial s \partial t} - \frac{\partial B}{\partial t} \frac{\partial^2 B}{\partial t \partial s} \right) > 0 \iff
\]

\[
2 \frac{\partial B}{\partial s} \frac{\partial^2 B}{\partial t \partial s} - \frac{\partial B}{\partial t} \frac{\partial^2 B}{\partial s^2} > 0 \iff 2(E + s \frac{\partial E}{\partial s}) \frac{\partial E}{\partial t} + 2(1 - s \frac{\partial E}{\partial t}) \frac{\partial E}{\partial s} > 0 \iff \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} E > 0,
\]

where the last inequality is implied by \( E < 1 \) and

\[
\frac{\partial E}{\partial s} = \frac{1}{z - z} > \frac{m}{z - z} = -\frac{\partial E}{\partial t}.
\]  
(A5)

(Note that \( m \in (0, 1) \) according to Lemma 1.) Thus, \( d^2 s / dt^2 < 0 \) results.

**Proof of Proposition 1**

i) First of all, note that the properties of the budget constraint shown in Lemma 2 and the property \( ds/dt \bigg|_{t=0} = 1/E(0, 0; H) > 1 \) imply a unique solution to (7), where \( s^0(t^o) > t^o > 0 \) holds. Then, the proof follows the lines of reasoning presented in Section 3. As argued above, a child either attends university or receives basic schooling. In the former case, the inequality \( U(s^0(t^o), t^o; \theta) = U(w_j - t^o + s^0(t^o) - z_{ij}, \theta) > U(w_j - t + s(t) - z_{ij}, \theta) = U(s(t), t; \theta) \) for all \( t \neq t^o \) follows from maximisation (7). In the latter case, \( U(0, 0; \theta) = U(w_j, \theta) > U(w_j - t, \theta) = U_{ij}(s(t), t; \theta) \) for all \( t > 0 \). Consequently, if inequality (9) is fulfilled (not fulfilled), \( U_{ij}(s^0(t^o), t^o; \theta) \geq U_{ij}(0, 0; \theta) \) (\( U_{ij}(s^0(t^o), t^o; \theta) < U_{ij}(0, 0; \theta) \)) and therefore \( U_{ij}(s^0(t^o), t^o; \theta) \) for all \( t \). The parent prefers the tax \( t^o \) (the tax \( t = 0 \)).

ii) Usual comparative statics proves the second part of Proposition 1. A glance at the government budget constraint (A1) and (A3) reveals that both \( H \) and \( t \) affect the
first-order condition (8) directly and indirectly via \( s \). Utilising these relationships, comparative statics leads to \( \frac{dt}{dH} = - \frac{[d^2 s(t; H)]}{(dtdH) / (d^2 s(t; H) / dt^2)} \). The numerator can be calculated analogously to (A4): 

\[
\frac{d^2 s}{dt dH} = - \frac{\frac{\partial^2 s \partial B}{\partial s \partial H} \partial s}{\left(\frac{\partial B}{\partial s}\right)^2}
\]

where (A3) and \( \partial^2 B / \partial t^2 = 0 \) (see (A2)) are used. The government budget constraint \( B(s, t; H) = 0 \) implies \( \partial s(t; H) / \partial H = -[\partial B(s, t; H) / \partial H] / [\partial B(s, t; H) / \partial s] \). Inserting this equation, the first-order condition \( ds(t; H) / dt = 1 \Leftrightarrow \partial B / \partial t = -\partial B / \partial s \) (see (A3)), and the partial derivatives (A2) into (A6) and (A4) leads to:

\[
\frac{dt}{dH} = \frac{-\frac{\partial E}{\partial s} \frac{\partial B}{\partial H} + \frac{\partial B}{\partial H} \left( \frac{\partial^2 s}{\partial s^2} + 2 \frac{\partial^2 B}{\partial s \partial H} \right)}{-\frac{\partial B}{\partial s} \left( \frac{\partial^2 s}{\partial s^2} + 2 \frac{\partial^2 B}{\partial s \partial H} \right)} = -\frac{\frac{\partial E}{\partial s}}{2 \left(1 - s \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \right)} < 0.
\]

The denominator of (A7) is positive, since it is equal to the numerator of (A4) for \( ds / dt = -[\partial B(s, t) / \partial t] / [\partial B(s, t) / \partial s] = 1 \). Concerning the numerator, \( \partial E / \partial H = m(w_H - w_L) / (\tau - z) > 0 \) follows from (A1). Moreover, some reformulations show that

\[
E - s \left( \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \right) = m \left[ w_L + (w_H - w_L) H - t^o + s^o \right] / (\tau - z) \in (0, 1)
\]

because of the chain of inequalities \( \tau > m(w_H - t^o) + s^o > m[w_L + (w_H - w_L) H - t^o + s^o] > mw_L > \bar{z} \), which follows from assumption 1 and \( s^o(t^o) - t^o > 0 \) (see first remark in the proof of Proposition 1). Thus, the numerator is negative, and \( dt / dH < 0 \) results.

Finally, \( ds(t; H) / dH = \partial s(t; H) / \partial H + (ds(t; H) / dt)(dt / dH) \), \( ds / dt = 1 \), (A3) and (A7) lead to:

\[
\frac{ds}{dH} = \frac{-\frac{\partial^2 B}{\partial s \partial H} \partial H + \frac{\partial B}{\partial H} \partial s \partial B}{-\frac{\partial B}{\partial s} \left( \frac{\partial^2 s}{\partial s^2} + 2 \frac{\partial^2 B}{\partial s \partial H} \right)} = \frac{-\frac{\partial E}{\partial H}}{2 \left(1 - s \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \right)} < 0,
\]

since the numerator is again negative.

**Proof of Proposition 2**

Using (A1) and (A3) to reformulate first-order condition (8) yields

\[
s_{foc} = \frac{1}{2 - m} \left\{ \bar{z} - m \left[ w_L + (w_H - w_L) H - t \right] \right\},
\]

which is referred to as \( s_{foc} \)-function (\( foc \) stands for first-order condition). Next, using (9), (10) and (11) leads to \( V(s(t), t) \geq 0.5 \Leftrightarrow s(t) \geq s^v \), where

\[
s^v = \frac{\bar{z} + \bar{z}}{2} - m \left[ w_L + (w_H - w_L) H \right] + t.
\]

(11)
(A11) is labelled $s^v$-function ($v$ stands for 50 percent of the vote). It implies that $V(s^v(t'), t') \geq 0.5 \Leftrightarrow s^v(t') \geq \frac{1}{2} s^v$ for $t = t'$ in (A10) and (A11).

Next, let us calculate the intersection $(\tilde{s}, \tilde{t})$ between the two functions. This yields

$$\tilde{t} = \frac{m}{2} \left[ w_L + (w_H - w_L) H + \frac{\tilde{s}}{2(1-m)} \right] - \frac{2-m}{4(1-m^2)} \tilde{s},$$

(A12)

where $\partial \tilde{t} / \partial H > 0$ results. Since $\partial s^v / \partial t = 1 > m/(2-m) = ds^v/\partial t > 0$ holds, $s^v(t') \geq \frac{1}{2} s^v \Leftrightarrow t \leq \tilde{t}$. (Recall that $m \in (0, 1)$ according to Lemma 1.) Therefore, $V(s^v(t'), t') \geq 0.5 \Leftrightarrow t^o \leq \tilde{t}.$

Define $\overline{H}$ as the threshold value such that $V(s^v(\overline{H}), t^o(\overline{H})) = 0.5$. Consider the case where $H = \overline{H}$. In this case the point $(s^v, t^o)$ coincides with the intersection $(\tilde{s}, \tilde{t})$, i.e., $(s^v(\overline{H}), t^o(\overline{H})) = (\tilde{s}(\overline{H}), \tilde{t}(\overline{H}))$. As the optimal values $s^o$ and $t^o$ decrease in $H$ (see (A7) and (A9)), $t^o(H) \leq t^o(\overline{H}) \Leftrightarrow H \geq \overline{H}$. By contrast, (A12) implies that $\tilde{t}(H) \leq \overline{H}(\overline{H}) \Leftrightarrow H \geq \overline{H}$. Thus, $t^o(H) \leq \overline{H}(\overline{H}) \Leftrightarrow H \geq \overline{H}$ results, which leads to $V(s^o(t'), t') \geq 0.5 \Leftrightarrow H \geq \overline{H}$, where $s^o(t')$ stands for 50 percent of the vote. It implies that $V(s^o(t'), t') = 0.5$ in (A10) and (A11). Consequently, if the systems starts with $H = 0$ and $H(0,0) < \overline{H}$ holds, $H_\kappa$ never exceeds $\overline{H}$, the policy $t = 0$ wins every election (see Proposition 2), and $H_\kappa$ strictly increases over time and converges towards the stable steady state $\hat{H}(0,0) = \tilde{H}(0,0)$.

Proof of Proposition 3

i) First, the proof shows that if $(s,t) = (0,0)$ is implemented in each period, $H_\kappa$ strictly increases over time and converges towards a stable steady state $\hat{H}(0,0)$. Second, I argue that if $(s^o, t^o)$ is implemented in each period, $H_\kappa$ strictly increases and converges towards a stable steady state $\hat{H}(s^o, t^o)$. Third, $\hat{H}(s^o, t^o) > \hat{H}(0,0)$ is shown to hold. These three parts together imply Proposition 3 i), as argued below.

**Step 1:** Assume that $(s,t) = (0,0)$ is implemented in each period. In this case, the difference equation (6), which describes the evolution of the number of the skilled, reduces to $H_\kappa = E(H_{\kappa-1}) = \gamma_L(0,0) + [\gamma_H(0,0) - \gamma_L(0,0)] H_{\kappa-1}$. This in turn leads to $H_\kappa = [H_0 - \gamma_L/ (1 - \gamma_H + \gamma_L)] (\gamma_H - \gamma_L)^\kappa + \gamma_L/ (1 - \gamma_H + \gamma_L)$. Note that $[\gamma_H(0,0) - \gamma_L(0,0)] \in (0, 1)$ by Assumption 1. This guarantees both stability and monotonicity; i.e., $H_\kappa$ strictly increases over time and converges monotonically to the asymptotically stable steady state $\hat{H}(0,0) = \gamma_L(0,0)/ [1 - \gamma_H(0,0) + \gamma_L(0,0)]$. $\hat{H}$ is strictly positive but smaller than unity because $\gamma_H < 1$ (again by Assumption 1). Consequently, if the systems starts with $H = 0$ and $\hat{H}(0,0) < \overline{H}$ holds, $H_\kappa$ never exceeds $\overline{H}$, the policy $t = 0$ wins every election (see Proposition 2), and $H_\kappa$ strictly increases over time and converges towards the stable steady state $\hat{H}(0,0)$.

**Step 2:** Assume that $(s^o, t^o)$ is implemented in each period. In this case, (6) implies

$$\frac{dH_\kappa}{dh_{\kappa-1}} = \frac{\partial E}{\partial H_{\kappa-1}} + \frac{\partial E}{\partial s} \frac{ds}{dH_{\kappa-1}} + \frac{\partial E}{\partial t} \frac{dt}{dH_{\kappa-1}}.$$
Inserting (A7) and (A9) into (A13) leads to

$$\frac{dH_\kappa}{dH_{\kappa-1}} = \frac{\partial E}{\partial H} \left[ 1 - \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \left[ E - s \left( \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \right) \right] \right].$$

(A14)

The numerator of the quotient is positive, since $\partial E/\partial s > -\partial E/\partial t > 0$ (see (A5)) and $E - s [(\partial E/\partial s) + (\partial E/\partial t)] \in (0, 1)$ (see (A8)). The denominator is positive too (cf. (A7)). Furthermore, simple reformulations show that the numerator is smaller than the denominator:

$$\frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \left[ E - s \left( \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \right) \right] = \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \left[ 1 - 2s \left( \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \right) \right] < 2 \left( 1 - s \frac{\partial E}{\partial s} \right) \left( \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \right) \iff \frac{\partial E}{\partial s} > -\frac{\partial E}{\partial t},$$

(A15)

where $ds/dt = 1 \iff -dB/\partial s = dB/\partial t \iff E + s \partial E/\partial s = 1 - s \partial E/\partial t$ and (A5) are utilised. Thus, the whole term in the brackets is positive and smaller than one. In addition, $\partial E/\partial H = \gamma_H - \gamma_L \in (0, 1)$ holds. Therefore, $dH_\kappa/dH_{\kappa-1} \in (0, 1)$ results.

The fact that $dH_\kappa/dH_{\kappa-1} \in (0, 1)$ guarantees both stability and monotonicity. That is, $H$ converges monotonically to an asymptotically stable steady state $\hat{H}(s^o, t^o) = \gamma_L(s^o, t^o) / [1 - \gamma_H(s^o, t^o) + \gamma_L(s^o, t^o)]$. (Note that $(s^o, t^o)$ remains constant over time if, and only if, $H$ does not change.) Moreover, $\hat{H}(s^o, t^o) \in (0, 1)$ follows from $\gamma_H < 1$ (see Assumption 1).

Step 3: The relationships $1 - \gamma_H(0, 0) + \gamma_L(0, 0) = 1 - [m(w_H - w_L)] / (\bar{z} - \underline{z}) = 1 - \gamma_H(s^o_\kappa, t^o_\kappa) + \gamma_L(s^o_\kappa, t^o_\kappa)$ (cf. (4) and (5)) and $\gamma_L(0, 0) = (mw_L - \bar{z}) / (\bar{z} - \underline{z}) < [m(w_L - t^o_\kappa) + s^o_\kappa - \bar{z}] / (\bar{z} - \underline{z}) = \gamma_L(s^o_\kappa, t^o_\kappa)$, which follows from $s^o_\kappa > t^o_\kappa$ (see first remark in the proof of Proposition 1), lead to $\hat{H}(0, 0) < \hat{H}(s^o, t^o)$. If the system starts with $H = 0$ and $\hat{H}(0, 0) > \overline{H}$ holds, $H$ exceeds $\overline{H}$ in some period $\pi$. From step 1, we know that $H_\kappa$ increases until period $\pi$. Then, the policy $t^o_\kappa$ wins the election and the dynamics is described by (A14). Since $H_\pi < \hat{H}(0, 0)$ still holds in period $\pi$, $H_\kappa$ is still below $\hat{H}(s^o, t^o)$. Therefore, $H_\kappa$ further increases in the succeeding periods, converging monotonically to the asymptotically stable steady state $\hat{H}(s^o, t^o)$ (see step 2), and the proposal $t^o_\kappa$ wins each election from period $\pi$ onwards (see Proposition 2) because $H_\kappa > H_\pi > \overline{H}$ holds for all $\kappa > \pi$.

ii) As shown above, $H_\kappa$ increases further in the periods following $\pi$ and converges monotonically to its steady state level. Thus, $ds/dH < 0$ (see Proposition 1, part ii) implies that $s^o_\kappa$ decreases over time, converging monotonically to its asymptotically stable steady state level. Showing the relation $\partial(s - t)/\partial H < 0$ is sufficient to prove that $(s^o_\kappa - t^o_\kappa)$ also declines monotonically from period $\pi$ on (towards the steady state level). Using (A7) and (A9) yields

$$\frac{\partial (s - t)}{\partial H} = -\frac{\partial E}{\partial H} \left\{ 1 - \left[ E - s \left( \frac{\partial E}{\partial s} + \frac{\partial E}{\partial t} \right) \right] \right\}.$$
The denominator of (A16) is positive (cf. (A7)). By contrast, the numerator is negative because $\partial E/\partial H = m (w_H - w_L) / (\tau - \bar{z}) > 0$ follows from (A1) and $E - s [(\partial E/\partial s) + (\partial E/\partial t)] \in (0, 1)$ holds (see (A8)). All in all, $\partial (s - t) / \partial H < 0$ results. Finally, the first remark in the proof of Proposition 1 implies that $s^o$ and $s^o - t^o$ are strictly positive.

iii) First, $\gamma_L/\gamma_H = [m (w_L - t^o_s) + s^o - \bar{z}] / [m (w_H - t^o_s) + s^o - \bar{z}]$ is positively correlated with the term $[s^o - mt^o_s]$, i.e.,

$$\frac{\partial (\gamma_L/\gamma_H)}{\partial [s - mt]} = \frac{m (w_H - w_L)}{[m (w_H - t) + s - \bar{z}]^2} > 0.$$  \hfill (A17)

Second, the inequalities $\partial s/\partial H < 0$, $\partial t/\partial H < 0$ and $\partial (s - t)/\partial H < 0$ (see (A7), (A9) and (A16)) imply $(\partial s/\partial H) - m (\partial t/\partial H) < (\partial s/\partial H) - (\partial t/\partial H) < 0$, which in turn leads to $\partial [s - mt] / \partial H < \partial (s - t) / \partial H < 0$. Consequently, the term $[s^o - mt^o_s]$ moves in the same direction as $(s^o - t^o_s)$ does over time. This relationship in connection with (A17) and ii) completes the proof.

References


