The role of mobility in tax and subsidy competition

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Abstract

In this paper, we analyse the role of mobility in tax and subsidy competition. Our primary result is that increasing ‘relocation’ mobility of firms leads to increasing ‘net’ tax revenues under fairly weak conditions. While enhanced relocation mobility intensifies tax competition, it weakens subsidy competition. The resulting fall in the governments’ subsidy payments overcompensates the decline in tax revenues, leading to a rise in net tax revenues. We derive this conclusion in a model in which two governments are first engaged in subsidy competition and thereafter in tax competition, and firms locate and potentially relocate in response to the two political choices.

JEL classifications: H87; H71; F21; H25
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1 Motivation

In this paper, we analyse the role of mobility in international tax and subsidy competition for firms. More specifically, we distinguish between two different concepts of mobility – ‘location’ and ‘relocation’ mobility. The first concept, location mobility, refers to the additional costs that accrue to investors when they set up a new firm or plant in a foreign country rather than in their home country. The second concept, relocation mobility, refers to the costs that arise when an already established firm or plant moves to another jurisdiction. These two types of mobility jointly shape the countries’ subsidy and tax competition. They thus affect each country’s ‘net’ tax revenues, defined as the difference between a government’s tax revenues and its subsidy payments.

Our primary result is that increasing relocation mobility leads to increasing net tax revenues under fairly weak conditions. We derive this conclusion in a four-stage model in which two symmetric jurisdictions compete for firms with subsidies and taxes, each aiming at maximising its net tax revenues. In the first stage, the non-cooperative governments simultaneously set subsidies for attracting investors. In the second stage, the investors decide where they will set up their firms and receive subsidies. After subsidies have been phased out, governments simultaneously choose corporate taxes in the third stage. In the fourth stage, firms decide whether to stay or to relocate, and pay taxes accordingly.

A key feature of the model is that investors face location costs in the second stage, reflecting imperfect location mobility, and relocation costs in the fourth stage, reflecting imperfect relocation mobility. The location costs, i.e., the cost disadvantage from investing abroad, imply that investors are, on average, home biased. This is an empirically well established result (e.g., French and Poterba, 1991; Lewis, 1999; Pinkowitz et al., 2001). The relocation costs imply that firms are, in general, ‘locked in’ once they are operating in a country because, for instance, they develop ties with the regional economy and acquire location-specific knowledge. Reversing the initial location choice is possible but costly. The resulting lock-in effect allows governments to levy higher taxes on firms than is otherwise possible, and it provides incentives to pay subsidies to attract new firms in the first place.

Surprisingly, a decline in relocation costs leads to a rise in net tax revenues in the two countries under ‘reasonable’ assumptions although it weakens the lock-in effect and intensifies tax competition. This outcome occurs because the induced fall in taxes weakens the preceding subsidy competition and is more than offset by the resulting decline in subsidy payments. By contrast, a decline in location costs tends to negatively affect each country’s net tax revenues, since it rather intensifies subsidy competition without weakening tax competition. It thus tends to increase
government payments without enhancing revenues.

Distinguishing between location and relocation costs allows us to disentangle the different channels through which the different types of mobility affect net tax revenues. This is particularly important, because we cannot expect the two types of mobility costs to decrease in line with one another, since the decline in location costs is at least partly driven by forces other than those which determine the decline in relocation costs. We now briefly illustrate this point.

Let us first look at the initial location choice. Investors are, on average, home biased. For a variety of reasons, they prefer to set up new firms or plants in their home region. There are, for instance, international information asymmetries which mean that even large investors are simply better informed about the economic and legal conditions at home than abroad, and this leads to higher transaction costs and greater uncertainties for foreign direct investments (FDIs). This feature is captured by our location costs.

These costs, however, have been decreasing in recent years. International legal and economic harmonisation, the progress of communication and information technologies, and the liberalisation of the world capital markets are the main reasons for this decline. All these measures make the international movement of financial capital less costly and less risky, thereby facilitating foreign investments.

Next, let us consider briefly the relocation choice. Relocation is an option, but it causes substantial opportunity costs. A firm often forges strong links with local business networks and suppliers and acquires location-specific knowledge once it has become established in a region. Local links and knowledge are both worthless in the case of relocation. Also, relocation requires not only the transfer of financial capital, but also the movement of real capital goods and human capital, which is particularly costly.

Nevertheless, we argue that the relocation costs have also been declining over time. Consider the case of a smaller high-tech or services firm initially located in, say, the Netherlands. This firm might be an academic or corporate spin-off, or a ‘regular’ start-up.\(^\text{1}\) The main assets of such smaller firms in the high-tech and services sectors are often their highly skilled employees with a very product-specific know-how, who cannot easily be replaced. In this case, the introduction of the common European labour market substantially reduced the costs of relocating such a firm, including its key employees, to adjacent Belgium. Additionally, the development of modern

\(^{1}\)In the late 1990s, almost 1.8 million start-ups were established in eight European OECD countries in one year, compared to approximately 1.1 million closures. About 230,000 of the new start-ups were corporate spin-offs. In the high-tech industry, in particular, corporate spin-offs are a common way of establishing new firms. See Moncada-Paternò-Castello et al. (2000) and, for further discussion on spin-offs from public sector research institutions, Callan (2001).
communication and transportation technologies and the internationalisation of the former national economies have been diminishing the role of the established local networks.

Alternatively, consider the case of large chip manufacturer in the semiconductor industry. Here the pace of the technological progress has, in some sense, substantially reduced relocation costs. In this industry, the development has been so dynamic that product life cycles are nowadays extremely short. They are, in fact, now measured in months (cf. Henisz and Macher, 2004). Consequently, new production lines are set up very frequently, for example, in order to produce a new generation of microprocessors. Once production facilities have to anyway be replaced, it is only a small step to relocate, or rather replace, the entire factory. In this sense, the relocation costs have been declining as a result of the accelerating speed of technological innovations. These costs are, in general, still positive, given the partial loss of a skilled workforce and the other downsides of relocation. However, the crucial point here is the general downward trend.

The decline in relocation costs is certainly not confined to small high-tech start-ups and large semiconductor firms, but occurs in many industries for various reasons. Irrespective of the underlying reasons, the implications, from the perspective of regional politicians, can be dire. For instance, consider Nokia’s engagement in Bochum, Germany. The Finnish maker of mobile phones had received public investment subsidies of about €90 million to secure the future of its manufacturing plant in Bochum. This substantial financial support could not prevent Nokia from relocating production from Bochum to Cluj, Romania, in 2008 (Financial Times, 2008). The politicians’ hope of having ‘locked-in’ Nokia proved to be an illusion. Having noticed that the lock-in effects are often much weaker than initially thought, politicians have consequently become more and more critical of such subsidies. This is in line with our model, which shows that, in terms of tax revenues, increasing relocation mobility can be a blessing in disguise.

Our paper is related to the ‘tax holiday’ literature. In this strand of literature, governments initially grant tax holidays, or upfront subsidies, to attract foreign

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2 Anticipating this problem, the German-based semiconductor memory producer Qimonda, which filed for insolvency in 2009, had explicitly mentioned a few years earlier in its 2006 IPO prospectus that “[r]eductions in the amount of government subsidies we receive or demands for repayment could increase our reported expenses. (...) The availability of government subsidies is largely outside our control. (...) As a general rule, we believe that government subsidies are becoming less available in each of the countries in which we have received funding in the past” (Qimonda, 2006, pp. 26-27). The semiconductor industry is a prime example of a sector that receives headline catching public financial support. For instance, in 2003, the AMD Fab 36 project in Dresden was officially subsidised by almost €550 million (cf. Grundig et al., 2008).
direct investments and to compensate firms for high time-consistent taxes in the future (e.g., Bond and Samuelson, 1986; Doyle and van Wijnbergen, 1994; Janeba, 2002; Marjit et al., 1999; Thomas and Worrall, 1994). The resulting policy outcome in these papers, i.e., subsidies or low taxes initially followed by high taxes, is similar to our subsidy and tax structure. But, unlike these papers, we analyse the impact of changes in location mobility and relocation mobility on net tax revenues. We also examine how the mobility of firms affects the strategic interactions between the governments in the subsidy and tax stages. By contrast, the articles referred to cannot explore this issue, as they either consider the unilateral policies of a single host country or assume a large number of potential host countries, thus excluding strategic interactions from the outset.

Lee’s (1997) model is more in line with our approach. He analyses a two-period model in which capital is perfectly mobile in the first period and imperfectly mobile in the second period. Governments non-cooperatively levy a tax on capital and use each period’s revenues to provide a public good in the very same period. Lee’s (1997) model excludes initial subsidies. It predicts only one-way capital flows, whereas our model allows for two-way capital flows. Also, Lee (1997) focuses on the question whether the public good is oversupplied or undersupplied in the second period. The answer depends on the strength of two opposing externalities caused by an increase in taxes, a positive fiscal externality and a negative capital income externality (due to international ownership of capital). By contrast, we focus on the impact of gradual changes in location or relocation mobility on the net tax revenues in the two periods together, allowing for subsidies in the first period. Our approach enables us to draw qualitative conclusions on how gradual changes in mobility affect the interaction between tax and subsidy competition and net tax revenues.

Like our paper, the literature on tax competition in models of the ‘new economic geography’ raises some doubts about whether increasing economic integration necessarily erodes government revenues (for instance, Baldwin and Krugman, 2004; Borck and Pflüger, 2006; Kind et al., 2000). In this strand of literature, the arguments hinge on the presence of significant agglomeration economies, which are totally ab-

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3 In an alternative and complementary approach to the tax holiday literature, Chisik and Davies (2004) analyse a bilateral treaty on the taxation of FDIs. They explain the gradual reduction of tax rates over time. Initially, only a treaty that specifies a small tax cut is self-enforcing. This treaty, however, generates an economic environment in which treaties with further, more substantial tax reductions become self-enforcing.

4 Haufler and Wooton (2006) analyse regional tax and subsidy coordination within an economic union when the two members of this union compete with a third country. In their model, however, each government has only one policy instrument at its disposal, which can be either a subsidy or a corporate tax. Their paper thus differs considerably from the tax holiday literature and from our contribution.
sent in our framework. By contrast, our conclusion that rising relocation mobility does not harm the governments’ budgets follows from the interaction between tax and subsidy competition, which is not considered in the ‘new economic geography’ literature.\footnote{Wilson (2005) provides another argument that explains why tax competition can be welfare-enhancing. In his model, the presence of tax competition implies that selfish government officials intensify their efforts in expenditure competition in order to attract mobile capital, and this second type of competition makes residents better off by reducing government ‘waste’. Following a different line of reasoning, Becker and Fuest (2010) show that deeper economic integration, in terms of lower trade costs, can mitigate tax competition. In their model, countries compete for firms with infrastructure investment, which reduces trade costs, and taxes. A coordinated increase in infrastructure investments upon the non-cooperative equilibrium weakens tax competition, since the induced cut in trade costs narrows the price gap between domestically produced and imported goods and thus reduces the benefits to domestic consumers from attracting firms.}

Konrad and Kovenock (2009) is related to both the tax holiday and the new economic geography literature. They analyse tax competition for ‘overlapping FDIs’ in a dynamic model with agglomeration advantages. The vintage property of the FDI prevents a ruinous race to the bottom as long as governments only have non-discriminatory taxes at their disposal. But if governments can also offer subsidies to new FDI, international competition will again be “cut-throat in nature.” Konrad and Kovenock (2009), however, are not interested in the implications of increasing mobility. By contrast, we analyse how rising location and relocation mobility re-shapes tax and subsidy competition, and how it ultimately affects net tax revenues.

Our paper proceeds as follows. In Section 2, the model is presented. Section 3 investigates the outcome of the subsidy and tax competition stages. We analyse the effects of increasing location and relocation mobility on net tax revenues in Section 4. Section 5 concludes with a brief discussion of some policy implications.

\section{Governments and firms}

We start by presenting our two-period, four-stage, model of tax and subsidy competition for imperfectly mobile firms. In the first period (consisting of the first and second stages; see below), the governments of two jurisdictions grant subsidies to attract investors non-cooperatively. Given these subsidies, investors then decide which country they will set up their firms in. In the second period (consisting of the third and fourth stages), the two governments levy corporate taxes. Since the firms are now established in a country, they are locked-in, but only imperfectly, as we will explain in more detail below. Firms can still relocate in response to the tax policies of the jurisdictions. So there is competition for mobile firms in both periods, albeit
to a different degree.

Our framework draws on Haupt and Peters (2005). They, however, deal with tax competition only. But since their model is very tractable, we can enrich the tax competition stages and, more importantly, incorporate the new subsidy competition stages. Let us now look at the model in more detail.

**Firms** Consider two symmetric countries, A and B. In each of these jurisdictions, there is a continuum of home investors, normalised to 1. Here, the term ‘home’ refers to the fact that there are already some links between investors and a country. For instance, the investors might simply reside in this country.

Each of the investors sets up a single firm. Despite these existing links, firms can initially be located either in the investors’ home country or abroad. A firm’s set up costs that occur in the first period are \( c \) if it stays in its home country, and \( c + m_1 \) if it moves abroad. While all firms face identical cost components \( c \), they differ with respect to their \( m_1 \). (For notational convenience, firm indices are not used.) We label the location costs \( m_1 \) and interpret them as the mobility costs or the cost disadvantage of investing abroad in the first period. This characteristic is distributed according to the distribution function \( F_1(m_1) \), whose properties are described below.

In the second period, each firm realises the (gross) return \( \pi \) if it continues to stay in the country where it was established in the first period. Its return is \( \pi - m_2 \) if it relocates in the second period. Again, \( \pi \) is the same for all firms, while the component \( m_2 \) differs across firms. We label the relocation costs \( m_2 \) and interpret them as the mobility costs or the cost disadvantage of relocating in the second period. Denote the ‘number’ or, more correctly, mass of firms which locate in jurisdiction \( i \) in period 1 by \( N_i \). Then, the characteristic \( m_2 \) is distributed across these \( N_i \) firms according to the new distribution function \( F_2(m_2) \).

The distribution functions \( F_1(m_1) \) and \( F_2(m_2) \) are twice continuously differentiable and strictly increasing functions over the intervals \([m_1, \overline{m}_1]\) and \([m_2, \overline{m}_2]\), respectively. They fulfil

**Assumption 1:**

(i) \( F_k(m_k) = 0 \) and \( F_k(\overline{m}_k) = 1 \), \( k = 1, 2 \), (ii) \( m_k < 0 < \overline{m}_k \), (iii) \( F_k(0) < 0.5 \),

(iv) \( m_1 < m_2 \) and \( \overline{m}_1 < \overline{m}_2 \), (v) \( F_1(m) > F_2(m) \) for all \( m \in (m_1, \overline{m}_2) \),

(vi) \( F''_k(m_k) \in (-2(F'_k(m_k))^2 / [1 - F_k(m_k)], 2(F'_k(m_k))^2 / F_k(m_k)) \).

Properties (i) and (ii) restrict the relevant domains of the distribution functions, allowing for both positive and negative values of \( m_1 \) and \( m_2 \). In most cases, set up costs are lower in an investor’s home region, since investors are more familiar with their domestic business environment than with the foreign one. This situation
corresponds with a positive $m_1$. But for some firms, set up costs are lower abroad. They might be able to take advantage of a particularly specialised foreign labour force. Or entrepreneurs might be able to make profitable use of their business ideas only in very specific places. For instance, a fashion label might be successful only in cities such as New York or Paris. These cases are captured by a negative $m_1$. Property (iii), however, implies that the set up costs of the majority of firms indeed favour their home country. Similarly, relocation costs $m_2$ are positive for the majority of firms. For instance, relocation after the start up phase causes the loss of immobile input factors and regional networks built up in the first period. This relocation costs, however, need not be prohibitive. Firms are thus only imperfectly locked in. Moreover, some firms might even benefit from relocating and thus increase their returns. They might, for instance, be closer to clients or suppliers.

Properties (iv) and (v) are most important for our analysis. They capture the feature that second period mobility costs $m_2$ exceed first period mobility costs $m_1$, meaning that distribution function $F_2$ lies to the right of $F_1$, as illustrated in Figure 1. In other words, firms become decreasingly mobile over their life span. This ‘natural’ assumption reflects the imperfect lock-in effect once a firm is located in a country. It drives our results. By contrast, the properties $m_1 < 0$ and $m_2 < 0$ are not important for our economic mechanisms. In fact, our results would go through with $m_1 = m_2 = 0$.\footnote{The ‘technical’ advantage of allowing negative mobility costs is that the distribution functions, and thus the governments’ objective functions below, are ‘smooth’ for a wider range of tax and subsidy differentials. This simplifies our proofs.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Distribution of location and relocation costs}
\end{figure}
Finally, property (vi) is a ‘purely technical’ restriction on the density functions’ slopes that guarantees well-behaved objective functions. This property is satisfied by, among others, a uniform distribution and various specifications of the Beta distribution, which are routinely used in the case of a finite domain.

The functions $F_1$ and $F_2$ are common knowledge. Each firm learns about the realisation of its specific location costs $m_1$ and relocation costs $m_2$ before it makes its location decision in the first period and its relocation decision in the second period, respectively. For simplicity, we assume that a firm’s first period and second period mobility costs are not correlated. This assumption enables us to put forward our arguments as simply as possible.\(^7\)

**Governments** When competing for mobile firms, the non-cooperative governments have subsidies and corporate taxes at their disposal. Subsidies are used in period 1, while taxes are levied in period 2. Governments can implement preferential subsidy and tax regimes. That is, in each country subsidies would then be different for firms of home investors that receive subsidy $s^n_i$, and ‘incoming’ firms of foreign investors that receive subsidy $s^m_i$, where $i = A, B$.\(^8\) Similarly, governments might set differentiated taxes. Firms that have already had their subsidised start up phase in country $i$ then pay tax $t^n_i$, while those firms that relocate ‘newly’ to country $i$ in the second period pay tax $t^m_i$.\(^9\)

**Objectives and timing** Each country maximises its ‘net’ revenues $NR_i$, i.e., the difference between tax revenues $R_i$ and subsidy payments $P_i$, given the decisions of its opponent. As usual, investors maximise the net profits of their firms, taking into account (gross) return $\pi$, set up costs $c$, firm specific mobility costs $m_1$ and $m_2$, subsidies $s^n_i$ and $s^m_j$, and taxes $t^n_i$ and $t^m_j$.

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\(^7\)In fact, it is far from clear whether location and relocation costs are correlated. Take the example of a large, internationally experienced, investor. The location costs of this investor can be minor. But if it sets up a steel factory, the relocation costs will be substantial - if not prohibitive. Low location costs do not imply low relocation costs, and vice versa.

\(^8\)As governments do not know the mobility characteristics of the investors, they cannot offer type-specific subsidies. This assumption is most appropriate for small and medium-sized firms which operate in nascent or rapidly changing high-tech markets. However, even in the case of large firms, governments often find it difficult to predict how mobile investors are before, as well as after, the initial investment, as the example of Nokia in Section 1 illustrates.

\(^9\)A firm is ‘domestic’ in the country where it is set up, and it is taxed accordingly in the second period. At this stage, a government discriminates between domestic and foreign firms, i.e., according to the firms’ initial location, but it treats all domestically set up firms equally. Importantly, in our setting, there are no incentives for governments to discriminate between domestic firms - as defined above - according to the home base of their investors.
The precise timing of the subsidy and tax competition game between the two governments is as follows. In the first stage, the non-cooperative governments simultaneously set subsidies $s^n_A$, $s^m_A$, $s^n_B$ and $s^m_B$. Given these subsidies, investors decide in the second stage whether their firms locate and receive subsidies in either country $A$ or country $B$. In the third stage, the governments simultaneously set their taxes $t^n_A$, $t^m_A$, $t^n_B$, and $t^m_B$, again non-cooperatively. In the fourth stage, firms decide whether they stay or relocate, and pay their taxes accordingly.

This decision structure is illustrated in Figure 2. In terms of time periods, the first two stages can be interpreted as constituting period 1, the third and fourth stages as constituting period 2. As mentioned above, the specific location costs for each firm are revealed prior to the location decision at the beginning of the second stage. Similarly, the relocation costs are revealed to each firm prior to the relocation decision at the beginning of the fourth stage. The distribution of these costs is common knowledge.

### 3 Subsidy and tax competition

As usual, we solve our model by backward induction, starting with the tax competition stages and then going on to the subsidy competition stages.

#### 3.1 Tax competition

The firms’ decisions in the fourth stage are straightforward. A firm that was set up in region $i$ in the first period can stay in this region and receive net return $\pi - t^n_i$ (first period costs and subsidies are sunk at this stage). Alternatively it can move to region $j$ and gain the net return $\pi - m_2 - t^m_j$. A profit maximising firm thus stays
in region \( i \) (relocates to region \( j \)) if, and only if,
\[
m_2 \geq t^n_i - t^m_j \quad (m_2 < t^n_i - t^m_j),
\]
i.e., if, and only if, the tax differential between the countries is smaller (strictly larger) than the firm specific relocation costs.\(^{10}\) Consequently, the share of firms relocating from region \( i \) to \( j \) is \( F_2(t^n_i - t^m_j) \).

Then the tax revenues of government \( i \) are
\[
R_i(t^n_i, t^m_i) = t^n_i \left[ 1 - F_2(t^n_i - t^m_j) \right] N_i + t^m_i F_2(t^n_j - t^m_i) N_j,
\]
where \( N_i \) and \( N_j \) result from the firms’ decisions in the second stage. The first term on the right-hand side captures the tax revenues from all firms that were already located in country \( i \) in the first period (indicated by \( N_i \)) and stay there in the second period.\(^{11}\) By contrast, the second term refers to the revenues from those firms that were initially located in country \( j \) (indicated by \( N_j \)) and only enter country \( i \) in the second period.

In the third stage, government \( i \) chooses taxes \( t^n_i \) and \( t^m_i \) that maximise revenues \( R_i \), given the choices of its competitor (previous subsidy payments \( P_i \) are sunk at this stage). The optimal taxes are characterised by the first-order conditions
\[
\frac{\partial R_i}{\partial t^n_i} = 0 \Leftrightarrow \varepsilon^n_i := \frac{F'_2(t^n_i - t^m_j) t^n_i}{1 - F_2(t^n_i - t^m_j)} = 1,
\]
\[
\frac{\partial R_i}{\partial t^m_i} = 0 \Leftrightarrow \varepsilon^m_i := \frac{F'(t^n_j - t^m_i) t^m_i}{F_2(t^n_j - t^m_i)} = 1,
\]
where \( \varepsilon^n_i \) and \( \varepsilon^m_i \) denote the elasticities of the tax bases with respect to the taxes \( t^n_i \) and \( t^m_i \), respectively. These elasticity rules reflect the traditional trade-off: a higher tax rate increases the revenues from the firms ultimately located in country \( i \), but reduces the number of those firms.

The first-order conditions (3) and (4) give the governments’ reaction functions implicitly. The resultant equilibrium taxes are symmetric, i.e., \( t^n_A = t^n_B =: t^n \) and \( t^m_A = t^m_B =: t^m \), and implicitly given by
\[
t^n = \frac{1 - F_2(t^n - t^m)}{F'_2(t^n - t^m)} \quad \text{and} \quad t^m = \frac{F_2(t^n - t^m)}{F'_2(t^n - t^m)},
\]
\(^{10}\)In principle, subsidies could be contingent on performance. In reality, incomplete contracts and other problems will make it difficult for governments to reclaim subsidies even if firms fail to comply with performance requirements and relocate their production facilities. At most, a firm will be forced to pay back a part of its subsidy in the case of plant closure and relocation. This would obviously increase the relocation costs of the firm and modify condition (1), but it would not change our conclusions qualitatively.

\(^{11}\)Recall that function \( F_2 \) characterises the distribution of relocation costs of all firms whose start-up phase was in the same country, independent of their original home region.
yielding a positive tax differential\footnote{We can exclude $t^n - t^m < 0$, since this implies $F_2(t^n - t^m) < 0.5$ and thus $[1 - 2F_2(t^n - t^m)]/F_2'(t^n - t^m) > 0$, which is obviously a contradiction. Therefore, $t^n - t^m > 0$ results (see Haupt and Peters, 2005).}

$$
t^n - t^m = \frac{1 - 2F_2(t^n - t^m)}{F_2'(t^n - t^m)} =: \Delta t > 0.
$$

These solutions contain two important conclusions. First, government $i$’s tax on firms already established in country $i$ in the first period exceeds the tax on firms that move to region $i$ only in the second period, i.e., $t^n > t^m$. This tax differential arises because firms are locked in, at least imperfectly, once they have settled in a country. Since firms respond less elastically to an increase in the ‘domestic’ tax $t^n$ than to one in the ‘foreign’ tax $t^m$, they end up with higher tax payments if they stick to their initial location choice.

Second, taxes are independent of the number of firms $N_i$ and thus independent of subsidies. By contrast, the optimal subsidies in the first stage are shaped by the future taxes, as will soon become evident. In this sense, there is a one-way link between tax and subsidy competition.

The equilibrium values (5) and (6) are analogous to the results in Haupt and Peters (2005). We derive these results in a more general setting than Haupt and Peters (2005) with respect to mobility. More importantly, they only consider tax competition and completely ignore subsidy competition while we are interested precisely in the relationship between tax and subsidy competition, and we analyse the resulting net tax revenues. Let us therefore turn next to the subsidy competition between the governments.

### 3.2 Subsidy competition

Since the tax $t^n_A$ ($t^n_B$) is equal to $t^n_B$ ($t^n_B$), and since the distributions of migration costs $m_2$ are the same in the two countries, a firm’s expected performance in the second period is independent of its location in the first period. The location choice in the second stage, however, affects a firm’s overall net profit through its location costs and received subsidy. A home investor of country $i$ has net costs of $c - s^n_i$ ($c + m_1 - s^m_j$) in the first period if its firm is set up in country $i$ (country $j$). This firm is thus located in country $i$ (country $j$) in the second stage if, and only if,

$$
m_1 \geq s^m_j - s^n_i \quad (m_1 < s^m_j - s^n_i),
$$

i.e., if, and only if, the subsidy differential between the countries is smaller (strictly larger) than the firm specific location costs. The resultant share of $i$’s investors who
locate their firms in country $j$ is $F_1(s^m_j - s^n_i)$. Consequently, the number of firms established in country $i$ is

$$N_i = \left[1 - F_1(s^m_j - s^n_i)\right] + F_1(s^m_i - s^n_j),$$

where $H_i$ is the number of $i$’s investors setting up their firms in country $i$ and $(1 - H_j)$ is the number of $j$’s investors locating their firms in country $i$.

In the first stage, each government chooses its subsidies $s^n_i$ and $s^m_i$, given the subsidies of its opponent. Government $i$ maximises its net tax revenues

$$NR_i = \Omega^n[H_i + (1 - H_j)] + \Omega^m[(1 - H_i) + H_j] - s^n_iH_i - s^m_i(1 - H_j),$$

where $\Omega^n := t^n[1 - F_2(t^n - t^m)]$ and $\Omega^m := t^mF_2(t^n - t^m)$. The first two terms on the right-hand side capture future tax revenues while the third and the fourth term give the subsidy payments to home and foreign investors.

The optimal subsidies are implicitly given by the first-order conditions

$$\frac{dNR_i}{ds^n_i} = -\left[1 - F_1(s^m_j - s^n_i)\right] + [(\Omega^n - \Omega^m) - s^n_i]F'_1(s^m_j - s^n_i) = 0,$$

$$\frac{dNR_i}{ds^m_i} = -F_1(s^m_i - s^n_j) + [(\Omega^n - \Omega^m) - s^m_i]F'_1(s^m_i - s^n_j) = 0.$$ (10)

A marginal rise in the subsidies $s^n_i$ and $s^m_i$ increases government spending by the number of recipients $H_i$ and $1 - H_j$, respectively. This negative effect of today’s subsidies on net tax revenues is captured by the first term of each of the two derivatives.

By contrast, the second terms show the positive impact of today’s subsidies on future revenues. Note that government $i$’s expected future tax revenue from a firm is $\Omega^n$ if this firm is set up in country $i$, but only $\Omega^m$ if the firm is set up in country $j$. Using (5) and (6), the expected revenue differential is

$$\Omega^n - \Omega^m = t^n - t^m > 0.$$ (12)

That is, country $i$’s revenue increase caused by attracting an additional investor in the first period is exactly equal to the positive tax differential. Taking into account the subsidy payments, the net benefit of attracting an additional home and foreign investor is $(t^n - t^m) - s^n_i$ and $(t^n - t^m) - s^m_i$, respectively. Finally, the derivatives $F'_1(s^m_j - s^n_i)$ and $F'_1(s^m_i - s^n_j)$ tell us how the number of firms established in country $i$ changes in response to a marginal rise in subsidies $s^n_i$ and $s^m_i$.

There is also an alternative interpretation of the optimality conditions. Defining hypothetical taxes $\tau^n_i := (t^n - t^m) - s^n_i$ and $\tau^m_i := (t^n - t^m) - s^m_i$, we can reformulate
the first-order conditions (10) and (11):

\[ \eta_i^n := \frac{F'_1(\tau^n - \tau^m) \tau^n}{1 - F_1(\tau^n - \tau^m)} = 1 \quad \text{and} \quad \eta_i^m := \frac{F'_1(\tau^n - \tau^m) \tau^m}{F_1(\tau^n - \tau^m)} = 1. \]  

(13)

The similarity between the elasticity rules (3) and (4) on the one hand and (13) on the other hand is striking and proves to be convenient later on.

From the first-order conditions, the equilibrium subsidies and hypothetical taxes follow immediately. Not surprisingly, the solution is symmetric, i.e., \( s^n_A = s^n_B =: s^n \), \( s^n_A = s^n_B =: s^n \), etc.:

\[ s^n = \Delta t - \tau^n, \quad \tau^n = \frac{1 - F_1(\tau^n - \tau^m)}{F'_1(\tau^n - \tau^m)}, \]  

(14)

\[ s^m = \Delta t - \tau^m, \quad \tau^m = \frac{F_1(\tau^n - \tau^m)}{F'_1(\tau^n - \tau^m)}. \]  

(15)

These equilibrium values have a straightforward interpretation. If there were no tax differential \( \Delta t \), firms would have had to pay the hypothetical taxes \( \tau^n \) and \( \tau^m \) in the first period (cf. equilibrium taxes (5)). This tax is ‘cut’ by the expected revenue differential (12). In this sense, governments give up current revenues for the benefit of having future ones. But only if the future gain \( t^n - t^m \) strictly exceeds the hypothetical tax \( \tau^n \) or \( \tau^m \), will the subsidy indeed be positive. This outcome, in turn, requires a sufficiently strong lock-in effect.

In any case, the equilibrium levels (14) and (15) directly imply a positive subsidy and hypothetical tax differential

\[ s^m - s^n = \tau^n - \tau^m = \frac{1 - 2F_1(\tau^n - \tau^m)}{F'_1(\tau^n - \tau^m)} =: \Delta \tau > 0. \]  

(16)

Each government grants a higher subsidy to foreign investor than to domestic ones. This preferential treatment reflects the initial home bias and corresponds to our previous result (cf. tax differential (6)). Since investors respond less elastically to subsidy changes at home than to those abroad, they receive less public support for setting up their firms in their home country than for doing the same thing in the other country.\(^{13}\)

We have so far side-stepped the more technical topics of existence and uniqueness of the equilibrium. These issues are taken up in Lemma 1.

**Lemma 1** Tax and subsidy competition.

A subgame perfect equilibrium exists and is unique. Equilibrium taxes and subsidies satisfy conditions (5), (6), (14), (15), and (16). Moreover, \( N_i = N_j = 1 \) results.

**Proof:** See Appendix.  

\(^{13}\)Alternatively, the differential (16) can be explained in terms of hypothetical taxes.
4 Net tax revenues and mobility

We now turn to our key issue, the relationship between mobility and net tax revenues. To analyse the emerging links, we first consider in more detail the net tax revenues in equilibrium.

4.1 Net tax revenues

Using the equilibrium values (5), (12), (14) and (15), each country’s net tax revenues can be expressed as

\[ NR_i = \text{revenues } R_i \left[ \Delta t + 2t^m_i F_2(\Delta t) - \left[ \Delta t - \left[ \tau^n_i (1 - F_1(\Delta \tau)) + \tau^m_i F_1(\Delta \tau) \right] \right] \right]. \]

The revenues can be decomposed into two elements. First, the basic revenues give the tax revenues that would occur in a country if no firm had been located there in the first period. In this case, all firms would be set up in the other country, but the share \( F_2(\Delta t) \) would relocate in the second period, generating revenue \( t^m_i 2F_2(\Delta t) \). Second, the revenue differential (\( \text{rev diff} \)) captures the additional revenues that arise because some firms are initially set up in the respective country and thus pay higher taxes due to the lock-in effect.

The subsidy payments can also be split up into two components: First, the hypothetical tax payments reflect the tax revenues that would result in the first period in the absence of any lock-in effects. In the case of \( \Delta t = 0 \), countries would tax firms similarly in the two periods, as the optimality conditions (3) and (4) on the one hand and (13) on the other hand show. The similarity becomes even more evident if we express the hypothetical tax payments as \( \Delta \tau + 2\tau^m F_1(\Delta \tau) \) and compare these formulation with revenues \( R_i \).

Second, there are hypothetical subsidy payments (\( \text{hyp sub} \)) that reduce these hypothetical tax payments in order to attract firms. This second element – which eventually gives rise to positive real subsidies – constitutes each government’s opportunity costs of attracting firms and generating the revenue differential. These opportunity costs are, in equilibrium, equal to the revenue differential. That is, the costs and benefits of attracting firms exactly cancel out. We refer to this outcome as the What-You-Give-Is-What-You-Get (WYGIWYG) principle. Taking WYGIWYG into account, net tax revenues are

\[ NR_i = 2t^m_i F_2(\Delta t) + \tau^n_i (1 - F_1(\Delta \tau)) + \tau^m_i F_1(\Delta \tau). \]

\(^{14}\)Using Eqs. (14), (15) and (16), we can rearrange the hypothetical tax payments: \( \tau^n (1 - F_1) + \tau^m F_1 = (1 - F_1)^2 / F_1 - F_1^2 / F_1 + 2\tau^m F_1 = \Delta \tau + 2\tau^m F_1. \)
With this simple expression, investigating the impact of mobility on net tax revenues is straightforward. We distinguish between increasing location mobility and increasing relocation mobility. This distinction proves to be crucial.

4.2 Net tax revenues and relocation mobility

In this section, we look at the implications of increasing relocation mobility for net tax revenues. As already argued above, even firms that are well established in a country are for various reasons becoming more and more mobile. In our model, the increase of mobility comes as a reduction in the firms’ relocation costs. More specifically, we capture the rise in mobility as a change in the value of the distribution function \( F_2(\Delta t; z_2) \) in equilibrium \((t^n, t^m)\) which is formally caused by a marginal increase in a parameter \( z_2 \). In particular, we start by considering

**Scenario 1:** \( \partial F_2(\Delta t; z_2)/\partial z_2 > 0 \) and \( \partial F_2(\Delta t; z_2)/\partial z_2 = 0 \)

at the ‘old’ equilibrium level \( \Delta t \). We stick, for convenience, to our notation \( F' = \partial F/\partial \Delta t, F'' = \partial^2 F/\partial \Delta t^2 \), etc. All derivatives with respect to the parameter \( z_2 \) are explicitly expressed as \( \partial F/\partial z_2 \), etc.

Scenario 1 means that we consider an upward shift of the distribution curve that leaves its slope, i.e., the density \( F'_2 \), at the ‘old’ equilibrium level \( \Delta t \) unaltered, as illustrated in Figure 3. The corresponding rise in mobility weakens the lock-in effect. Since established firms are more inclined to relocate and to respond more elastically to international tax differentials, the old tax differential \( \Delta t \) cannot be maintained. In this sense, tax competition is intensified and erodes the revenue differential in equation (17).

Nevertheless, this revenue differential is always identical in magnitude to the hypothetical subsidy, as the WYGIWYG principle stresses. That is, any decline in the revenue differential does not matter, since it is matched by an equal fall in subsidy payments. Attracting firms in the first period is simply less beneficial if these firms are more mobile and pay fewer taxes in the second period. Consequently, subsidy competition is reduced. All that ultimately matters is the impact of relocation mobility on basic revenues, as reflected in the derivative

\[
\frac{dNR_i}{dz_2} = 2t_i^n \frac{\partial F_2 (\Delta t; z_2)}{\partial z_2} + 2t_i^m F'_2 (\Delta t; z_2) \frac{dt_i^n}{dz_2},
\]

where we made use of the envelope theorem, i.e., \( \partial NR_i/\partial t_i^n = 2 \partial [t_i^n F_2(\Delta t)]/\partial t_i^m = 2 (\partial R_i/\partial t_i^m) = 0 \).

The first term on the right-hand side captures the direct effect of increasing mobility in the second period. For given taxes \( t^n \) and \( t^m \), the number of relocating
firms $F_2(\Delta t; z_2)$ rises, since the lock-in effect is weakened. This positive effect on country $i$’s ‘basic’ tax base drives up net tax revenues.

The second term shows the indirect effect of increasing relocation mobility through the tax change in equilibrium. If the tax $t^n_j$ decreases (increases) with mobility parameter $z_2$, country $i$’s tax base erodes (grows). This negative (positive) effect reduces (raises) net revenues. As long as this indirect effect is not too negative, the direct effect dominates, and net revenues increase with mobility parameter $z_2$.

Proposition 1 relates the overall outcome to a simple elasticity rule.

**Proposition 1** Net tax revenues and relocation mobility.

In Scenario 1, the net tax revenues $NR_i$ increase (decrease) with the firms’ mobility parameter $z_2$ if, and only if, the elasticity of the elasticity $\varepsilon^n_j$ with respect to $t^n_j$ is greater (smaller) than unity in equilibrium. That is,

$$\frac{dNR_i}{dz_2} \geq 0 \iff \frac{\partial \varepsilon^n_j}{\partial t^n_j} \frac{t^n_j}{\varepsilon^n_j} \geq 1.$$ (20)

*Proof: See Appendix.*

The intuition for this relationship is as follows. The rise in mobility, which is captured by $\partial F_2(\Delta t; z_2)/\partial z_2 > 0$, increases the elasticity $\varepsilon^n_j$ for given taxes, and thus distorts the initial equilibrium, as the first-order condition (3) reveals. The tax $t^n_j$ faces downward pressure. To restore the equilibrium, the tax $t^n_j$ has to adjust more, the less elastically the elasticity $\varepsilon^n_j$ responds to changes in $t^n_j$. Only if the elasticity of $\varepsilon^n_j$ is sufficiently small (i.e., below one), will the tax $t^n_j$ decline so drastically that
the negative indirect effect dominates (see second term of (19)). Then, country i’s tax base will erode substantially, and net tax revenues \( NR_i \) will fall. By contrast, if the elasticity of \( \varepsilon^n_j \) is above one, net revenues will increase.

The rise in net tax revenues is not an odd abnormality in this framework, but a very likely outcome. To see this, we reinterpret the relationship (20) in Proposition 2, where \( g^n_j(t^m_i) = t^n_j \) stands for country j’s reaction function in the tax competition game for j’s domestic firms.

**Proposition 2** **Net tax revenues and relocation mobility (continued).**

In Scenario 1, the net tax revenues \( NR_i \) increase (decrease) with the firms’ mobility parameter \( z_2 \) if, and only if, country j’s optimal tax \( t^n_j \) increases (decreases) with country i’s tax \( t^m_i \). That is,

\[
\frac{dNR_i}{dz_2} \gtrless 0 \quad \Leftrightarrow \quad \frac{dg^n_j(t^m_i)}{dt^m_i} \gtrless 0. \tag{21}
\]

**Proof:** See Appendix.

Proposition 2 unambiguously relates the impact of relocation mobility on net revenues to the nature of the countries’ strategic interaction. In particular, net revenues increase with relocation mobility if, and only if, the policy choices \( t^n_j \) and \( t^m_i \) are strategic complements, as expressed in condition (21). Taxes are strategic complements in traditional tax competition models under standard assumptions, and exactly under these ‘conventional’ circumstances, our ‘unconventional’ conclusion holds: an increase in relocation mobility of firms raises net revenues. The government revenues would be negatively affected only if the taxes \( t^n_j \) and \( t^m_i \) were strategic substitutes, as expressed in condition (21).\(^{15}\)

Since the two countries are symmetric, both of the them will experience the same equilibrium effects of a rise in mobility. That is, net tax revenues will increase in both countries, or will fall in both countries.

To illustrate our conclusion, we consider the case of a uniform distribution of relocation costs as an example.

**Example:** Consider the case of uniformly distributed mobility costs, i.e., \( F_k(m_k) = \frac{m_k - m_{k-1}}{m_k - m_1} \) and \( F'_k(m_k) = -\frac{1}{m_{k-1} - m_k} \), where \( k = 1, 2 \). Then, property (vi) of Assumption 1 is fulfilled, and we continue to assume that all other properties of Assumption 1

\(^{15}\)We have to interpret the strategic interactions between the two countries carefully. In our framework, the fact that country j’s optimal tax \( t^n_j \) increases (decreases) with country i’s tax \( t^m_i \) does not imply that country i’s optimal tax \( t^m_i \) increases (decreases) with country j’s tax \( t^n_j \). While the countries are symmetric, the taxes \( t^n_j \) and \( t^m_i \) are not.
are satisfied. (Notice that, with a uniform distribution, property (iii) $F_k(0) < 0.5$ implies $m_k > |m_k|$. In this example, the two distribution curves in Figures 2 and 3 are straight lines. As in the general case, the relocation costs exceed location costs, investors are home biased, and firms are locked in.

Following our previous line of reasoning, the first-order conditions of the governments lead to the reaction functions

$$t^n_j = g_j^n(t^n_i) = \frac{m_i}{2} + \frac{1}{2} t^n_i \quad \text{and} \quad t^n_i = g_i^n(t^n_j) = -\frac{m_j}{2} + \frac{1}{2} t^n_j; \quad (22)$$

$$s^n_j = \frac{\Delta t - \bar{m}_1}{2} + \frac{1}{2} s^n_i \quad \text{and} \quad s^n_i = \frac{\Delta t + \bar{m}_1}{2} + \frac{1}{2} s^n_j, \quad (23)$$

where (23) is equivalent to the hypothetical reaction functions

$$r^n_j = h_j^n(t^n_i) = \frac{\bar{m}_1}{2} + \frac{1}{2} r^n_i \quad \text{and} \quad r^n_i = h_i^n(r^n_j) = -\frac{m_j}{2} + \frac{1}{2} r^n_j. \quad (24)$$

Thus, taxes and subsidies (or, alternatively, hypothetical taxes) are strategic complements.

Then, the equilibrium taxes and subsidies are

$$t^n = \frac{2\bar{m}_2 - m_2}{3} > \frac{\bar{m}_2 - 2m_2}{3} = t^m, \quad (25)$$

$$s^n = \frac{\bar{m}_2 + m_2}{3} - \frac{2\bar{m}_1 - m_1}{3} < \frac{\bar{m}_2 + m_2}{3} - \frac{\bar{m}_1 - 2m_1}{3} = s^m. \quad (26)$$

The home bias of investors and the lock-in effect that established firms experience lead to preferential tax and subsidy regimes in favour of foreign investors and firms, i.e., $t^n > t^m$ and $s^n > s^m$.\(^{16}\) Using equilibrium taxes and subsidies and the equilibrium outcome $N_i = N_j = 1$, the resulting net tax revenues can be determined:

$$NR_i = \left(\frac{2(\bar{m}_2 - 2m_2)}{9(\bar{m}_2 - m_2)} + \frac{2m_1 - m_1}{9(\bar{m}_1 - m_1)}\right) + \left(\frac{\bar{m}_1 - 2m_1}{9(\bar{m}_1 - m_1)}\right). \quad (27)$$

Let us define $\bar{m}_k = \omega_k - z_k$ and $\bar{m}_k = \bar{w}_k - z_k$. In line with Scenario 1, we can then formally capture an increase in relocation mobility, i.e., a decline in relocation costs, by an increase in the parameter $z_2$, shifting the distribution $F_2(m_2)$ to the left without changing its slope. Differentiating (27) then yields

$$\frac{dNR_i}{dz_2} = \frac{4(\bar{m}_2 - 2m_2)}{9(\bar{m}_2 - m_2)} > 0. \quad (28)$$

\(^{16}\)Equilibrium values (25) and (26) yield $t^n - t^m = (\bar{m}_2 + m_2)/3 > 0$ and $s^n - s^m = (\bar{m}_1 + m_1)/3 > 0$, since $F_k(0) < 0.5$ implies $m_1 > |m_1|$ and $m_2 > |m_2|$ under a uniform distribution of (re-)location costs. Both subsidies, $s^n$ and $s^m$, are positive if the condition $\bar{m}_2 > 2m_1 - m_2 - m_3$ is satisfied. By contrast, if this condition is not fulfilled, at least domestic firms already face a tax in period 1. Even in this case, however, this tax will be lower than the tax on domestic firms in period 2.
Hence, a decrease in relocation costs, resulting in a higher relocation mobility, unambiguously increases net revenues. We sum up the outcome in this example in Corollary 1.

**Corollary 1** Net tax revenues and relocation mobility: uniform distribution.
Assume that relocation costs are uniformly distributed as specified above. Then, the taxes $t_j^n$ and $t_i^m$ are strategic complements, and the net tax revenues $NR_i$ increase with the firms’ mobility parameter $z_2$.  

Returning to our general discussion, we now take into account the fact that changes in relocation mobility might also affect the slope of the distribution function. The additional effects that arise if $\partial F'_2(\Delta t; z_2)/\partial z \neq 0$ holds at the ‘old’ equilibrium level $\Delta t$ are stated in Proposition 3.

**Proposition 3** Net tax revenues and relocation mobility (further continued). The revenue increasing effect of a marginal change in relocation mobility is reinforced (counteracted) if $\partial F'_2(\Delta t; z_2)/\partial z_2 < 0$ ($\partial F'_2(\Delta t; z_2)/\partial z_2 > 0$) holds.

Proof: See Appendix.  

The economic explanation for this conclusion is straightforward. If the density $F'_2$ decreases (increases) with the mobility parameter $z_2$, the firms’ response to tax increases becomes less (more) elastic, as the first-order conditions (3) shows. This causes a rise (decline) in tax $t_j^n$. Such a tax change, however, increases (erodes) the basic revenues of country $i$. This additional channel would be captured by a change in the second term of derivative (19).

### 4.3 Net tax revenues and location mobility

Next, we investigate the implications of rising location mobility. That is, we analyse the case in which investors are more mobile and less home biased when they decide where their firms are set up in the first period.

Analogously to Scenario 1, we now consider

**Scenario 2:** $\partial F_1(\Delta \tau; z_1)/\partial z_1 > 0$ and $\partial F'_1(\Delta \tau; z_1)/\partial z_1 = 0$.

We formally express Scenario 2 in terms of hypothetical taxes instead of subsidies. The two interpretations are equivalent, since a rise in hypothetical taxes $\tau^n$ and $\tau^m$ corresponds with a decline in subsidies $s^n$ and $s^m$ of the same magnitude. Referring to taxes, however, proves to be more convenient and allows us to compare the differences between rising location and relocation mobilities more explicitly.
Increasing location mobility does not affect future real taxes, but only current hypothetical tax revenues or, equivalently, real subsidy payments:

\[
\frac{dNR_i}{dz_1} = - (\tau^n_i - \tau^m_i) \frac{\partial F_1(\Delta \tau; z_1)}{\partial z_1} + \tau^m_i F'_1(\Delta \tau; z_1) \frac{d\tau^n_j}{dz_1} + \tau^m_i F'_1(\Delta \tau; z_1) \frac{d\tau^m_j}{dz_1},
\]

where we take again advantage of the envelope theorem, i.e., of the fact that \(\partial NR_i/\partial \tau^n_i = \partial NR_i/\partial s^n_i = 0\) and \(\partial NR_i/\partial \tau^m_i = \partial NR_i/\partial s^m_i = 0\) is satisfied in equilibrium.

The first term on the right-hand side again reflects the direct impact of mobility on the tax bases. In contrast to its counterpart in derivative (19), this effect is now negative. For given hypothetical taxes, and thus subsidies, increasing mobility reduces the number of home firms located in each country \(1 - F_1(\Delta \tau; z_1)\), but it increases the number of foreign firms \(F_1(\Delta \tau; z_1)\) by the same amount. The impact of these changes on net revenues is negative, since the former firms pay more hypothetical taxes than the later ones. To put it differently, increasing mobility implies that highly subsidised foreign investors who take advantage of the subsidy differential replace less subsidised home investors who set up their firms abroad, thereby increasing each country’s overall subsidy payments.

The second and third term capture the indirect effects of location mobility via its influence on equilibrium taxes \(\tau^n\) and \(\tau^m\). The indirect effect erodes (raises) country \(i\)’s tax bases, if country \(j\)’s hypothetical taxes \(\tau^n_j\) and \(\tau^m_j\) decrease (increase), and thus real subsidies \(s^n_j\) and \(s^m_j\) rise (decline).\(^{17}\) This negative (positive) effect depresses (raises) net revenues \(NR_i\). However, as long as the indirect effect is not too positive, the direct effect dominates, and net revenues of country \(i\) fall.

As this discussion shows, there are two major differences between the effects of increasing location mobility and relocation mobility. First, the direct impact is now negative because, for given hypothetical taxes, hypothetical revenues from home firms decline with location mobility. This negative effect has no counterpart in the case of changes in relocation mobility. Then, any decline in the additional tax revenues generated by domestic firms (i.e., by firms that were already set up in the country considered) in the second period is exactly offset by a decrease in subsidy payments, as already stated by the WYGIWYG principle. The remaining direct effect of an increase in relocation mobility is positive (see Section 4.2).

Second, the induced changes in both hypothetical taxes of country \(j\), \(\tau^n_j\) and \(\tau^m_j\), matter for the net tax revenues of country \(i\) in the case of increasing location mobility.

\(^{17}\)We know that the hypothetical tax, or subsidy, differential (16) decreases with location mobility. The previous discrimination against home investors is simply no longer viable once they become less attached to their home country. However, both taxes \(\tau^n\) and \(\tau^m\) might rise or fall, or \(\tau^n\) falls and \(\tau^m\) rises in response to a larger location mobility.
In the previous case of increasing relocation mobility, only the induced changes in tax $t^m_j$ ultimately had an impact on country $i$, since only the basic revenues count in the second period. These basic revenues of country $i$ are only affected by the opponent’s tax $t^m_j$, and not by tax $t^n_j$.

As a consequence of the differences between the two scenarios, the elasticity rule that determines the overall impact of an increasing mobility on net tax revenues is now more complicated.

**Proposition 4 Net tax revenues and location mobility.**

In Scenario 2, the net tax revenues $NR_i$ increase (decrease) with the investors’ mobility parameter $z_1$ if, and only if, the weighted and corrected elasticities of the elasticities $\eta^n_j$ and $\eta^m_j$ with respect to $t^n_j$ and $t^m_j$, respectively, are positive (negative) in equilibrium. More precisely,

$$\frac{dNR_i}{dz_1} > 0 \iff \frac{\tau^m_i}{\tau^n_i - \tau^m_i} \left( \frac{\partial \eta^n_j}{\partial \tau^n_j} \frac{\tau^n_j}{\eta^n_j} - 1 \right) - \frac{\tau^n_i}{\tau^n_i - \tau^m_i} \left( \frac{\partial \eta^m_j}{\partial \tau^m_j} \frac{\tau^m_j}{\eta^m_j} - 1 \right) \geq 0. \quad (30)$$

**Proof: See Appendix.**

The intuition behind Proposition 4 resembles that of Proposition 2. A rise in mobility, which is captured by $\partial F_1(\Delta \tau; z_1)/\partial z_1 > 0$, increases the elasticity $\eta^n_j$ for given hypothetical taxes. Consequently, there is downward pressure on the hypothetical tax $\tau^n_j$, equivalent to upward pressure on the subsidy $s^n_j$ (see the first-order conditions (10) and (13)). The hypothetical tax $\tau^n_j$ has to adjust more, the less elastically the elasticity $\eta^n_j$ responds to changes in $\tau^n_j$. The more the tax $\tau^n_j$ declines, and thus the subsidy $s^n_j$ increases, the more the tax base of country $i$ erodes, and thus the net revenues decrease.

This reasoning is very much in line with our discussion of Scenario 1, whereas the following conclusion is specific to Scenario 2. An increase in location mobility decreases the elasticity $\eta^m_j$ for given hypothetical taxes. This exerts upward pressure on the hypothetical tax $\tau^m_j$, equivalent to downward pressure on the subsidy $s^m_j$ (see condition (13)). The hypothetical tax $\tau^m_j$ has to adjust more, the less elastically the elasticity $\eta^m_j$ responds to changes in $\tau^m_j$. The more the tax $\tau^m_j$ increases, and thus the subsidy $s^m_j$ declines, the more the tax base of country $i$ grows, and thus the net revenues rise.

Overall, net tax revenues decline with mobility if the elasticity $\left( \frac{\partial \eta^m_j}{\partial \tau^m_j} \right) \left( \frac{\tau^m_j}{\eta^m_j} \right)$ is not too inelastic compared to the elasticity $\left( \frac{\partial \eta^n_j}{\partial \tau^n_j} \right) \left( \frac{\tau^n_j}{\eta^n_j} \right)$. Otherwise, net revenues increase. Importantly, a greater weight is assigned to the former elasticity than to the latter one, since $\tau^n_i$ is greater than $\tau^m_i$ in equilibrium.
The induced changes in revenues can be again related to the nature of the strategic interactions between the two countries, as Proposition 5 shows. Here, \( h^j_n(\tau^m_i) = \tau^j_n \) and \( h^m_n(\tau^p_i) = \tau^m_j \) stand for country \( j \)'s hypothetical reaction functions in the first stage. The rearranged first-order conditions (13) implicitly define these functions.

**Proposition 5** Net tax revenues and location mobility (continued).

In Scenario 2, the hypothetical tax payments from foreign firms \( \tau^m_i F_1(\Delta \tau) \) increase (decrease) with the firms’ mobility parameter \( z_1 \) if, and only if, country \( j \)'s optimal tax \( \tau^p_j \) increases (decreases) with country \( i \)'s tax \( \tau^m_i \). That is,

\[
\frac{d\tau^m_i F_1(\Delta \tau)}{dz_1} \geq 0 \iff \frac{dh^j_n(\tau^m_i)}{d\tau^m_i} \geq 0. 
\]

(31)

The hypothetical tax payments from domestic firms \( \tau^p_i [1 - F_1(\Delta \tau)] \) increase (decrease) with the firms’ mobility parameter \( z_1 \) if, and only if, country \( j \)'s optimal tax \( \tau^m_j \) decreases (increases) with country \( i \)'s tax \( \tau^p_i \). That is,

\[
\frac{d\tau^p_i [1 - F_1(\Delta \tau)]}{dz_1} \geq 0 \iff \frac{dh^m_n(\tau^p_i)}{d\tau^p_i} \leq 0.
\]

(32)

Proof: See Appendix.

The competition for domestic investors is completely disentangled from the competition for foreign investors. The countries are engaged in two separated ‘markets’. As a consequence, the type of strategic interaction between the countries in one market is unambiguously related to the revenues generated in this market only. In particular, the hypothetical tax payments of foreign investors \( \tau^m_i F_1(\Delta \tau) \) increase, and thus the subsidy payments they receive decline, if \( \tau^p_j \) and \( \tau^m_i \) are strategic complements, as expressed in condition (31). By contrast, the hypothetical tax payments of domestic investors \( \tau^p_i [1 - F_1(\Delta \tau)] \) decline, and thus the subsidy payments they receive increase, if \( \tau^m_j \) and \( \tau^p_i \) are strategic complements, as expressed in condition (32).

Overall, in the ‘conventional’ case, in which all tax choices are strategic complements, the impact of increasing location mobility on net revenues is inconclusive. Clear-cut implications for net tax revenues \( NR_i \) result under two circumstances. First, net revenues definitely decline if at the same time (i) the optimal tax \( \tau^p_j \) decreases with \( \tau^m_i \) and (ii) the optimal tax \( \tau^m_j \) increases with \( \tau^p_i \). Second, net revenues unambiguously increase if (i) the optimal tax \( \tau^p_j \) increases with \( \tau^m_i \) and (ii) the optimal tax \( \tau^m_j \) decreases with \( \tau^p_i \).
Example (continued): Let us return briefly to our example of uniformly distributed mobility costs. In line with our reasoning in Section 4.2, we now consider the impact of an increase in the location mobility, i.e., a decline in location costs, captured by a marginal shift of the distribution \( F_1(m_1) \) to the left. Formally, we analyse a marginal change in \( z_1 \). Differentiating the net-revenue function (27) yields

\[
\frac{dNR_i}{dz_1} = -\frac{2(m_1 + m_1)}{9(m_1 - m_1)} < 0.
\]

(33)

Hence, we can sum up the second part of our example as follows.

**Corollary 2** Net tax revenues and location mobility: uniform distribution.

Assume that relocation costs are uniformly distributed as specified above. Then, the net tax revenues \( NR_i \) decrease with the firms’ mobility parameter \( z_1 \).

In the case of uniformly distributed location and relocation costs, we derive a clear-cut result. Net revenues increase with relocation mobility, but decline with location mobility.

Again returning to the general discussion, we extend our analysis to the case \( dF'_0(\Delta t; z_2)/dz_2 \neq 0 \).

**Proposition 6** Net tax revenues and location mobility (continued).

The revenue decreasing effect of a marginal change in location mobility is strengthened (weakened) if \( \partial F'_2(\Delta t; z_2)/\partial z_2 > 0 \) \( (\partial F'_2(\Delta t; z_2)/\partial z_2 < 0) \) holds.

*Proof: See Appendix.*

Proposition 6 is completely in line with Proposition 3. The conclusion of Proposition 6 reflects again the fact that the tax base becomes less (more) elastic if \( dF'_2(\Delta t; z_2)/dz_2 < 0 \) \( (dF'_2(\Delta t; z_2)/dz_2 > 0) \) holds. This change pushes the hypothetical taxes \( \tau^n_j \) and \( \tau^m_j \) up (down), and the corresponding subsidies \( s_j^n \) and \( s_j^m \) fall (rise). Consequently, country \( i \)'s tax base and net revenues increase (decrease). Formally, these additional effects would be captured by changes in the second and third term of derivative (29).

### 4.4 Repeated relocation choice

Up to this point, firms can relocate only once after the initial set-up period. Obviously, this assumption is a crude simplification, since firms can repeatedly reconsider
their location choice in their usual life spans.\textsuperscript{18} The question arises whether our results about the implications of increasing relocation and location mobility are still valid when we allow for repeated relocations over time.

To get an idea of how robust our results are, let us return to our previous example of a uniform distribution. We introduce an additional, intermediate period lying between the two periods considered so far; that is, we now have an initial set-up period (period 1), an intermediate period (period 2), and the final period (period 3). Firms can relocate in both the second and third period. Assumption 1 now describes not only the declining mobility between the first and the second period, but also between the second and the third period. For the latter relationship, just replace the subscripts 1 and 2 with 2 and 3 in Assumption 1. Firms experience the lowest mobility costs (i.e., the highest mobility) in the initial set-up period, higher mobility costs in the intermediate period, and even higher mobility costs in the final period. In this sense, the lock-in effect becomes gradually stronger over time.

As before, firms cash in subsidy payments $s^n$ or $s^m$ in the initial set-up period and face a tax $t^n$ or $t^m$ in the final period. In the intermediate period, firms receive a subsidy, or pay a tax, $v^n$ or $v^m$, where $v > 0$ ($v < 0$) indicates a subsidy (tax). In keeping with the previous convention, a firm will be considered a domestic firm in country $i$ in the second (third) period if it stayed in, or relocated to, country $i$ in the first (second) period.

Following precisely the line of reasoning in Section 4.2, the extended system arrives at the following equilibrium tax and subsidy payments:

\begin{align}
  t^n &= \frac{2\bar{m}_3 - m_3}{3} > \frac{\bar{m}_3 - 2m_3}{3} = t^m, \\
v^n &= \frac{\bar{m}_3 + m_3 - 2\bar{m}_2 - m_2}{3} < \frac{\bar{m}_3 + m_3}{3} = v^m, \\
s^n &= \frac{\bar{m}_2 + m_2 - 2\bar{m}_1 - m_1}{3} < \frac{\bar{m}_2 + m_2}{3} = s^m.
\end{align}

The resemblance between the equilibrium values (34) to (36) on the one hand and (25) to (26) is remarkable. The relationships between the taxes in the final period and the parameters of the uniform distribution of the mobility costs in the very same period are exactly identical in the two-period and three-period scenario (see Eqs. (25) and (34)). Similarly, the formulas for the subsidies, or taxes, in the first and intermediate period of the current scenario are completely in line with the corresponding expressions for the first-period subsidies in Section 4.2 (see Eqs. (26), (35), and (36)). Again, the home bias of investors and firms give rise to preferential tax and subsidy regimes in all periods.

\textsuperscript{18}For instance, consider the example of Nokia, which moved its production from Bochum, Germany, to Cluj, Romania, in 2008, as discussed in Section 1. The Finnish firm closed the Cluj factory, which was replaced by Asian plants, only three years later (Financial Times, 2011).
Using equilibrium taxes and subsidies (34) to (36), we can calculate the net tax revenues:

\[ NR_i = \frac{2(m_3 - 2m_3)^2}{9(m_3 - m_3)} + \frac{2(m_2 - 2m_2)^2}{9(m_2 - m_2)} + \frac{(2m_1 - m_1)^2}{9(m_1 - m_1)} + \frac{(m_1 - 2m_1)^2}{9(m_1 - m_1)}. \]  

(37)

The similarity between net tax revenues (37) and (27) is obvious. The net tax revenues (37) can be decomposed into three components: (i) the hypothetical tax payments in the initial set-up period, (ii) the basic revenues in the final period, and (iii) as the new component in the case of three periods, the basic revenues in the intermediate period. Thus, introducing an intermediate period does not alter the WYGIWYG principle. The opportunity costs of attracting investors in the initial set-up period exactly offset the generated revenue differentials in the succeeding periods. Each country is left with the hypothetical tax payments in the initial period and the basic revenues in the following periods.

Using the previous definition \( m_k = \omega_k - z_k \) and \( \bar{m}_k = \bar{\omega}_k - z_k \), now with \( k = 1, 2, 3 \), we can again formally capture an increase in mobility in period \( k \), i.e., a decline in (re-)location costs, by an increase in the parameter \( z_k \). This is completely in line with Scenarios 1 and 2 in Sections 4.2 and 4.3. Differentiating net tax revenues (37) gives

\[ \frac{dNR_i}{dz_3} = \frac{4(m_3 - 2m_3)}{9(m_3 - m_3)} > 0, \]  

(38)

\[ \frac{dNR_i}{dz_2} = \frac{4(m_2 - 2m_2)}{9(m_2 - m_2)} > 0, \]  

(39)

\[ \frac{dNR_i}{dz_1} = -\frac{2(m_1 + m_1)}{9(m_1 - m_1)} < 0. \]  

(40)

These derivatives confirm and extend our previous results, as a comparison with the derivatives (28) and (33) shows. An increase in the location mobility in the initial set-up period reduces net revenues, since it intensifies subsidy competition. By contrast, a higher relocation mobility in the second or the third period raises net revenues, as it boosts basic revenues. Introducing an intermediate period leaves our fundamental conclusions unaffected. On the contrary, an increase in the relocation mobility in both the second and the third period now positively affects net revenues.

We summarise the results above in the final proposition:

**Proposition 7** *Net tax revenues and mobility in the three-period case.*

Consider the three-period case with uniformly distributed mobility costs as specified above. Then, the net tax revenues \( NR_i \) increase with the firms’ mobility parameters \( z_3 \) (final period) and \( z_2 \) (intermediate period), but decrease with \( z_1 \) (initial set-up period).

\[ \square \]
Finally, note that the marginal effect of a change in the relocation mobility in the second and third period is proportional to the share of country $j$’s firms that relocate to country $i$ in the respective period, i.e., to $F_k = \frac{(\bar{m}_k - 2m_k_2)}{3(\bar{m}_k - \bar{m}_k)}$. Also, this share is greater in the second period than in the third period, which reflects the fact that firms are more mobile in the second period. As a result, the marginal impact of an increase in the relocation mobility in the second period is even stronger than that in the third period.\(^\text{19}\)

5 Concluding remarks

Governments compete for mobile firms with both subsidies and taxes. We have analysed the resulting interplay between tax competition and subsidy competition, leading to the WYGIWYG principle. That is, the additional revenues generated by attracting firms through subsidies are exactly offset by the opportunity costs of these subsidies. This result has helped us to shed some light on the impact of rising mobility on net tax revenues, thereby distinguishing between location mobility and relocation mobility. Our key conclusion is that a rise in relocation mobility increases net tax revenues under fairly weak conditions. A higher relocation mobility reinforces tax competition, but weakens subsidy competition. Overall, the fall in subsidy payments overcompensates for the decline in tax revenues, yielding higher net tax revenues. Considering the example of a uniform distribution of mobility costs, we have shown that our key conclusion remains valid when we allow for repeated relocation choices. An increase in the relocation mobility in the intermediate period boosts net revenues even more than a similar increase in the final period.

These conclusions are in contrast to the common belief that increasing mobility erodes national revenues – a belief that is backed by ‘pure’ tax competition models. Notably, our contrasting conclusions are derived in a ‘conventional’ tax competition framework, but in one that is supplemented by subsidy competition stages. In this setting, we also argue that rising location mobility tends to reduce net tax revenues, somewhat in line with the ‘conventional’ tax competition literature and common beliefs.

Our findings have important policy implications. They directly imply that fiercer tax competition (here, due to rising relocation mobility) might be advantageous to governments because of its feedback effect on subsidy competition. In the public debate, however, the focus is on weakening tax competition, or preventing harmful tax competition, through various measures (cf. OECD, 1998). In our model,

\(^\text{19}\)Comparing Eq. (38) and (39) reveals that $dNR_i/dz_2 > dNR_i/dz_3 \iff -\bar{m}_3m_2 > -\bar{m}_2m_3$, where the latter inequality follows from $\bar{m}_3 > \bar{m}_2 > 0 > m_3 > m_2$ (see Assumption 1 (ii) and (iv)).
weakening tax competition actually implies intensifying subsidy competition, with potentially adverse effects on net tax revenues. So an exclusive concentration on tax harmonisation might be misleading and thus detrimental to future revenues. In this sense, our paper cautions politicians against narrow minded tax harmonisation on grounds different from those previously discussed in the literature.\textsuperscript{20} Our paper also indicates that more attention should be paid to subsidy competition and its interaction with tax competition. Reducing subsidy competition might indeed be a more successful avenue for larger tax revenues than restrictions on tax competition.

Exploring the implication of various forms of harmonisation and cooperation in our framework in detail can be a promising extension of our analysis. Such an extension would also include the discussion of limitations on preferential tax and subsidy regimes – as far as such limitations are enforceable, given that subsidies are frequently granted in the form of somewhat hidden and indirect transfers, and even preferential tax treatments are often hidden.\textsuperscript{21} As a further extension, the impact of correlated location and relocation costs could be checked. Firms might then sort themselves according to their mobility characteristics, and multiple equilibria might arise. Nevertheless, the underlying mechanisms explored in our simplified version should remain the same, and our conclusions should therefore still be valid, perhaps with some modifications.

Going one step further, we could endogenise relocation mobility. As briefly indicated in Section 1, relocation costs are at least partly driven down by political decisions, such as the European labour market integration. Also, firms can reduce relocation costs, for instance, by renting production facilities rather than buying. Many small start-ups use the facilities of application-oriented research institutes, such as the Fraunhofer Institute in Germany.

Another challenging extension would be to combine our approach of repeated decisions of governments and firms on policies and location with models which analyse other motives for attracting firms. For instance, Haufler and Mittermaier (2011) argue that governments face an incentive to attract a foreign firm as a means to curb the wage setting power of unions. In their model, however, governments decide on a tax or subsidy only once at the very beginning. They show that a country with strong unions is particularly prone to grant high subsidies. It would be interesting to see whether this conclusion still holds in the case of repeated decisions

\textsuperscript{20}See, for instance, Zodrow (2003) for a survey on tax competition in the European Union and the standard arguments against tax harmonisation.

on taxes/subsidies, location and wages. Similarly, it would be interesting to see to what extent our conclusions would still hold in an economic framework such as the one in Haufler and Mittermaier (2011).

Appendix

Proof of Lemma 1  We start by analysing the tax competition equilibrium (third and fourth stage). As argued above, this equilibrium is independent of the governments’ subsidies (first stage) and the investors’ initial location choice (second stage). In step 1, we exclude any ‘boundary’ equilibria. Uniqueness and existence of the tax competition equilibrium are proved in step 2. In step 3, we show that our lines of reasoning can easily be repeated to prove existence and uniqueness of the subsidy competition equilibrium, and thus of the subgame perfect equilibrium.

Step 1 (No ‘boundary’ equilibrium)  The first-order conditions

\[
\frac{\partial R_i}{\partial t_i^n} = \left\{ 1 - F_2(t_i^n-t_j^m) \right\} \frac{t_i^n F_2'(t_i^n-t_j^m)}{F_2(\Delta t)} N_i = 0, \quad (41)
\]

\[
\frac{\partial R_j}{\partial t_j^m} = \left[ F_2(t_i^n-t_j^m) - t_j^m F_2'(t_i^n-t_j^m) \right] N_i = 0, \quad (42)
\]

implicitly define the governments’ continuous reaction functions \(g_i^n\) and \(g_j^m\) in the case of an interior solution, since, first, the second-order conditions

\[
\frac{\partial^2 R_i}{\partial (t_i^n)^2} = -2F_2'(\Delta t) + \left[ \frac{1 - F_2(\Delta t)}{F_2(\Delta t)} \right] N_i < 0, \quad (43)
\]

\[
\frac{\partial^2 R_j}{\partial (t_j^m)^2} = -2F_2'(\Delta t) + \left[ \frac{F_2(\Delta t) F_2''(\Delta t)}{F_2(\Delta t)} \right] N_i < 0, \quad (44)
\]

are fulfilled for all taxes that constitute a solution to (41) and (42) according to Assumption 1 (vi) and, second, \(F\) is a twice continuously differentiable function.

Obviously, negative taxes can never be revenue maximising so that we can focus on non-negative solutions, i.e., \(t_i^n, t_j^m, t_i^* , t_j^* \geq 0\). Moreover, \(\partial R_i/\partial t_i^n \big|_{t_i^n=0} = \left[ 1 - F_2(-t_j^m) \right] N_i > 0\) and \(\partial R_j/\partial t_j^m \big|_{t_j^m=m_2} = -t_i^n F_2'(m_2) < 0\), implying that \(0 < t_i^n = g_i^n(t_j^m) \prec t_j^m + m_2\). Similarly, \(\partial R_j/\partial t_j^m \big|_{t_j^m=m_2} = -t_i^n F_2'(m_2) < 0\), implying that \(0 < t_j^m = g_j^m(t_i^n) \prec t_i^n - m_2\). Thus, taxes are positive and boundary solutions with \(F_2(\Delta t) = F_2(m_2) = 1\) or \(F_2(\Delta t) = F_2(m_2) = 0\) can be excluded. Then, the reaction function \(g_i^n, g_j^m\) gives a unique optimal tax \(t_i^n \ (t_j^m)\) for each tax \(t_j^m \ (t_i^n)\), and any equilibrium is characterised by conditions (5) and (6). (We implicitly assume that the firms’ gross returns \(\pi\) are sufficiently large so that they do not constrain government taxation.)
Step 2 (Existence and uniqueness) We first show that a solution to conditions (6), or equivalently to condition \( \Delta t - [1 - 2F_2(\Delta t)] / F'_2(\Delta t) = 0 \), exists and is unique. To this end, we differentiate the term \( [1 - 2F_2(\Delta t)] / F'_2(\Delta t) =: \Phi(\Delta t) \) with respect to \( \Delta t \), leading to

\[
\frac{\partial \Phi(\Delta t)}{\partial \Delta t} < 0 \iff F''_2(\Delta t) > -2 \frac{[F'_2(\Delta t)]^2}{1 - 2F_2(\Delta t)} \tag{45}
\]

for \( F_2(\Delta t) \in [0, 0.5] \iff \Delta t \in [m_2, m_{\text{crit}}] \), where \( m_{\text{crit}} \) is defined as \( m_{\text{crit}} : F_2(m_{\text{crit}}) = 0.5 \) and \( m_{\text{crit}} > 0 \) holds (see Assumption 1 (iii)). Furthermore, inequality \( F''_2(\Delta t) > -2 [F'_2(\Delta t)]^2 / [1 - F_2(\Delta t)] \) is satisfied (see Assumption 1 (vi)), and, additionally, inequality \( -2 [F'_2(\Delta t)]^2 / [1 - F_2(\Delta t)] \geq -2 [F'_2(\Delta t)]^2 / [1 - 2F_2(\Delta t)] \) is fulfilled for \( \Delta t \in [m_2, m_{\text{crit}}] \). Thus, \( F''_2(\Delta t) > -2 [F'_2(\Delta t)]^2 / [1 - 2F_2(\Delta t)] \) indeed results for \( \Delta t \in [m_2, m_{\text{crit}}] \), and \( \Phi(\Delta t) \) continuously declines with \( \Delta t \) in the interval \([m_2, m_{\text{crit}}]\). Also, we know that \( \Phi(0) = [1 - 2F_2(0)] / F'_2(0) > 0 \) (which follows from Assumption 1 (iii)), \( \Phi(m_{\text{crit}}) = 0 \), and, for \( \Delta t \in (m_{\text{crit}}, m_2] \), \( \Phi(\Delta t) < 0 \) hold. As a result, the term \( \Delta t - \Phi(\Delta t) = 0 \) continuously increases with \( \Delta t \) in the interval \([m_2, m_{\text{crit}}]\), with \([\Delta t - \Phi(\Delta t)]|_{\Delta t=0} < 0 \) and \([\Delta t - \Phi(\Delta t)]|_{\Delta t=m_{\text{crit}}} > 0 \). Given these properties, the intermediate value theorem implies that a solution \( \Delta t \) to the condition \( \Delta t - \Phi(\Delta t) = 0 \) (or, equivalently, to the condition (6)) exists and is unique, with \( \Delta t \in [0, m_{\text{crit}}] \).

Then, equilibrium taxes \( n^A = t^B = n^m \) and \( n^A = t^B = n^m \) exist and are uniquely determined by (5).

Step 3 (Subsidy Competition and Subgame-Perfect Equilibrium) The first-order conditions (10) and (11) are equivalent to

\[
\frac{\partial NR_i}{\partial \tau^n_i} = [1 - F_1(\tau^n_i - \tau^m_j)] - \tau^n_i F'_1(\tau^n_i - \tau^m_j) = 0, \tag{46}
\]

\[
\frac{\partial NR_j}{\partial \tau^m_j} = F_1(\tau^n_i - \tau^m_j) - \tau^m_j F'_1(\tau^n_i - \tau^m_j) = 0, \tag{47}
\]

where Eq. (12), the definitions \( \tau^n_i := (t^n - t^m) - s^n_i \) and \( \tau^m_i := (t^n - t^m) - s^m_i \), and Eq. (16) are used. The similarity between (46) and (47) on the one hand and (41) and (42) on the other hand is obvious. Not surprisingly, the proof of existence and uniqueness of the subsidy competition equilibrium follows the lines of reasoning explored in step 1 and 2, which need not be repeated here. The hypothetical taxes \( \tau^n_i \) and \( \tau^m_i \) are independent of the second period equilibrium. The only impact of the second period equilibrium on the first period equilibrium is that the taxes \( t^n \) and \( t^m \) raise the resulting subsidies \( s^n \) and \( s^m \) by the tax differential \( \Delta t \). The symmetric nature of the framework and the resulting equilibrium imply \( N_i = N_j = 1 \).

Consequently, we can conclude that (i) a subgame-perfect equilibrium exists and is unique, (ii) equilibrium taxes and subsidies are characterised by (5), (6), (14), (15), (16), and (iii) \( N_i = N_j = 1 \) results.
Proof of Propositions 1, 2, and 3

Preliminary Results  Inserting the optimal taxes (5), (14) and (15) into the net tax revenues (18) and rearranging to resulting terms lead to

\[ NR_i = \frac{F'_2(\Delta t; z_2)}{F'_1(\Delta t; z_1)} + \frac{[1 - F_1(\Delta t; z_1)]^2}{F'_1(\Delta t; z_1)} + F_1^2(\Delta t; z_1). \]  

(48)

Differentiating net tax revenues (48) with respect to mobility parameter \( z_2 \) yields

\[ \frac{dNR_i}{dz_2} = \frac{\partial NR_i}{\partial z_2} + \frac{\partial NR_i}{\partial \Delta t} \frac{d\Delta t}{dz_2}. \]  

(49)

The components of this derivative are given by

\[ \frac{\partial NR_i}{\partial z_2} = 2 \frac{F'_2(\Delta t; z_2) F_2(\Delta t; z_2) \frac{\partial F_2(\Delta t; z_2)}{\partial z_2} - [F_2(\Delta t; z_2)]^2 \frac{\partial F'_2(\Delta t; z_2)}{\partial z_2}}{[F'_2(\Delta t; z_2)]^2}, \]  

(50)

\[ \frac{\partial NR_i}{\partial \Delta t} = -2 \frac{[F'_2(\Delta t; z_2)]^2 F_2(\Delta t; z_2) - [F_2(\Delta t; z_2)]^2 F''_2(\Delta t; z_2)}{[F'_2(\Delta t; z_2)]^2}, \]  

(51)

\[ \frac{d\Delta t}{dz_2} = \frac{2 F'_2(\Delta t; z_2) \frac{\partial F_2(\Delta t; z_2)}{\partial z_2} + [1 - 2 F_2(\Delta t; z_2)] \frac{\partial F'_2(\Delta t; z_2)}{\partial z_2}}{[F'_2(\Delta t; z_2)]^2 \left[ 3 + \rho_2 \right]}, \]  

(52)

where

\[ \rho_2 = \frac{\Delta t F''_2(\Delta t; z_2)}{F'_2(\Delta t; z_2)} = \frac{[1 - 2 F_2(\Delta t; z_2)] F''_2(\Delta t; z_2)}{[F'_2(\Delta t; z_2)]^2}. \]  

(53)

is the elasticity of the density function \( F'_2(\Delta t; z_2) \) with respect to changes in the tax differential \( \Delta t \), evaluated at the equilibrium. Note that derivative (52) follows from tax differential (6) and the associated comparative statics: \( d\Delta t/dz_2 = -(\partial \kappa_2 / \partial z_2) / (\partial \kappa_2 / \partial \Delta t) \), where \( \kappa_2(\Delta t; z_2) := \Delta t - [1 - 2 F_2(\Delta t; z_2)] / F'_2(\Delta t; z_2) \) and \( \partial \kappa_2 / \partial \Delta t = 3 + \rho_2 \).

We can prove propositions 1, 2 and 3 in a more convenient and shorter manner by making use of the derivatives (49)-(52) instead of the more intuitive derivative (19) and the tedious comparative statics that leads to \( dt^n/dz_2 \).

Proposition 1  We now consider Scenario 1 with \( \partial F_2(\Delta t; z_2)/\partial z_2 > 0 \) and \( \partial F'_2(\Delta t; z_2)/\partial z_2 = 0 \) at the equilibrium value of \( \Delta t \), which simplifies the derivatives (50) and (52). To prove Proposition 1, we insert (50), (51) and (52) into derivative (49) and rearrange the resulting terms (using Eq. (53)):

\[ \frac{dNR_i}{dz_2} = \frac{4 F_2 \frac{\partial F_2}{\partial z_2}}{F'_2(3 + \rho_2)} \left( \frac{1}{F'_2(3 + \rho_2)} - \frac{\partial \varepsilon_j^n}{\partial \varepsilon_j^n} \right) = \frac{4 F_2 \frac{\partial F_2}{\partial z_2}}{F'_2(3 + \rho_2)} \left( \frac{\partial \varepsilon_j^n t^n}{\partial \varepsilon_j^n t^n} - 1 \right) \Rightarrow 0 \Leftrightarrow \frac{\partial \varepsilon_j^n t^n}{\partial \varepsilon_j^n t^n} \geq 1, \]  

(54)
where the elasticity of the elasticity $\varepsilon_j^n$ with respect to $t_j^n$
\[
\frac{\partial \varepsilon_j^n}{\partial t_j^n} \varepsilon_j^n = \left[\frac{1 - F_2(\Delta t)}{F_2(\Delta t)}\right] + 2 (55)
\]
is evaluated at the equilibrium (see the first-order and equilibrium conditions (3) and (5)). The functions’ argument $\Delta t$ and parameter $z_2$ are suppressed in Eq. (54) for notational convenience. The sign of derivative (54) depends on the terms in the brackets, since all other terms are positive. In particular, $F_2'' > -2(F_2')^2 / (1 - F_2)$ (see Assumption 1 (vi)) implies that the inequality $3 + \rho_2 > 3 - 2[(1 - 2F_2) / (1 - F_2)] > 1$ is fulfilled, where Eq. (53) is used.

**Proposition 2** Comparative statics yields
\[
\frac{dg(t_i^m)}{dt_i^m} > 0 \Leftrightarrow \frac{\partial^2 R_i}{\partial t_j^n \partial t_j^n} = \left[F_2'(\Delta t) + \frac{1 - F_2(\Delta t)}{F_2(\Delta t)} F_2''(\Delta t)\right] N_i \geq 0
\]

\[
\Leftrightarrow \frac{\partial \varepsilon_j^n}{\partial t_j^n} \varepsilon_j^n = \frac{1 - F_2(\Delta t)}{[F_2'(\Delta t)]^2} F_2''(\Delta t) + 2 \geq 1,
\]
where the first-order condition (41) and Eqs. (5) and (55) are used. Then, conditions (54) and (56) imply Proposition 2.

**Proposition 3** To calculate the additional impact of a change in the mobility parameter $z_2$ on the net tax revenues $NR_i$ that arises if $\partial F_2'(\Delta t; z_2)/\partial z_2 > 0$, we evaluate the derivatives (50) and (52) for $\partial F_2(\Delta t; z_2)/\partial z_2 = 0$ and $\partial F_2'(\Delta t; z_2)/\partial z_2 > 0$ at the equilibrium value of $\Delta t$. Inserting again the derivatives (50)-(52) into derivative (49) yields, after some rearrangements,
\[
\frac{dNR_i}{dz_2} = -2 \frac{\partial F_2'}{\partial z_2} \left[F_2^2 + \left(\frac{2(F_2')^2 F_2 - (F_2')^2 F_2''}{(F_2')^2}\right) \left(\frac{1 - 2F_2}{3 + \rho_2}\right)\right] \geq 0
\]
where we again suppress the functions’ argument $\Delta t$ and parameter $z_2$. Recall that both second-order conditions (43) and (44) are fulfilled if, and only if, $F_2'' \in (-2(F_2')^2 / (1 - F_2), 2(F_2')^2 / F_2$ holds, which is assumed to be the case by Assumption 1 (vi). Then, $F_2' < 2(F_2')^2 / F_2$ implies that $(F_2')^2 F_2 - (F_2')^2 F_2'' > 0$ holds. Also, $F_2'' > -2(F_2')^2 / (1 - F_2)$ implies that the inequality $3 + \rho_2 > 3 - 2[(1 - 2F_2) / (1 - F_2)] > 1$ is fulfilled, where we use (53). Finally, $F_2 < 0.5$ and thus $1 - 2F_2 > 0$ hold in equilibrium (see tax differential (6) and the explanation in footnote 12). Thus, all terms in the square brackets are positive, resulting in
\[
\frac{dNR_i}{dz_2} \geq 0 \Leftrightarrow \frac{\partial F_2'(\Delta t; z_2)}{\partial z_2} \leq 0,
\]
which proves Proposition 3.
Proof of Propositions 4, 5, and 6

Preliminary results This proof follows along the lines of the previous reasoning. Now, equilibrium net tax revenues (48) are affected by a change in the mobility parameter $z_1$:

$$
\frac{dNR_i}{dz_1} = \frac{\partial NR_i}{\partial z_1} + \frac{\partial NR_i}{\partial \Delta \tau} \frac{d\Delta \tau}{dz_1}.
$$

(59)

The three terms in (59) are given by

$$
\frac{\partial NR_i}{\partial z_1} = -2 \frac{F'_1(\Delta \tau; z_1) [1 - 2F_1(\Delta \tau; z_1)] \frac{\partial F_1(\Delta \tau; z_1)}{\partial z_1}}{[F'_1(\Delta \tau; z_1)]^2} - [1 - 2F_1(\Delta \tau; z_1) + 2F^2_1(\Delta \tau; z_1)] \frac{\partial F_1(\Delta \tau; z_1)}{\partial z_1},
$$

(60)

$$
\frac{\partial NR_i}{\partial \Delta \tau} = -2 \frac{[F'_1(\Delta \tau; z_1)]^2 [1 - 2F_1(\Delta \tau; z_1)]}{[F'_1(\Delta \tau; z_1)]^2} - [1 - 2F_1(\Delta \tau; z_1) + 2F^2_1(\Delta \tau; z_1)] F''_1(\Delta \tau; z_1),
$$

(61)

$$
\frac{d\Delta \tau}{dz_1} = -2 \frac{F'_1(\Delta \tau; z_1) \frac{\partial F_1(\Delta \tau; z_1)}{\partial z_1} + [1 - 2F_1(\Delta \tau; z_1)] \frac{\partial F_1(\Delta \tau; z_1)}{\partial z_1}}{[F'_1(\Delta \tau; z_1)]^2 [3 + \rho_1]},
$$

(62)

where

$$
\rho_1 = \frac{\Delta \tau F''_1(\Delta \tau; z_1)}{F'_1(\Delta \tau; z_1)} = \frac{[1 - 2F_1(\Delta \tau; z_1)] F''_1(\Delta \tau; z_1)}{[F'_1(\Delta \tau; z_1)]^2}.
$$

(63)

is the elasticity of the density function $F'_1(\Delta \tau; z_1)$ with respect to changes in the differential $\Delta \tau$. Analogously to (52), derivative (62) follows from differential (16) and the respective comparative statics: $d\Delta \tau/dz_1 = -(\partial \kappa_1/\partial z_1)/(\partial \kappa_1/\partial \Delta \tau)$, where $\kappa_1(\Delta \tau; z_1) := \Delta \tau - [1 - 2F_1(\Delta \tau; z_1)]/F'_1(\Delta \tau; z_1)$ and $\partial \kappa_1/\partial \Delta \tau = 3 + \rho_1$.

Proposition 4 We consider Scenario 2; that is, $dF_1(\Delta \tau; z_1)/dz_1 > 0$ and $dF'_1(\Delta \tau; z_1)/dz_1 = 0$ hold at the equilibrium differential $\Delta \tau$. To prove Proposition 4, we insert the derivatives (60), (61) and (62) into derivative (59) and rearrange the resulting terms:

$$
\frac{dNR_i}{dz_1} = \frac{2 \frac{\partial F_1}{\partial z_1}}{F'_1(3 + \rho_1)} \left[ \left( 1 + \frac{1 - F_1}{F'_1} \right) F_1 - \left( 1 - \frac{F_1F''_1}{(F'_1)^2} \right) (1 - F_1) \right] = \frac{2 (1 - 2F_1) \frac{\partial F_1}{\partial z_1}}{F'_1(3 + \rho_1)} \left[ \left( \frac{\partial \eta_j^n \tau_j^n \tau_j^m}{\partial \tau_j^m \eta_j^n} - 1 \right) \frac{\tau_j^m}{\tau_j^n - \tau_j^m} - \left( \frac{\partial \eta_j^n \tau_j^m \tau_j^m}{\partial \tau_j^m \eta_j^n} - 1 \right) \frac{\tau_j^n}{\tau_j^n - \tau_j^m} \right],
$$

(64)
where we make use of Eq. (63) and of the elasticities of the elasticities $\eta_j^n$ and $\eta_j^m$ with respect to $\tau_j^n$ and $\tau_j^m$,
\[
\frac{\partial \eta_j^n}{\partial \tau_j^n} \frac{\tau_j^n}{\eta_j^n} = \frac{(1 - F_1) F'_n}{(F'_1)^2} + 2 \quad \text{and} \quad \frac{\partial \eta_j^m}{\partial \tau_j^m} \frac{\tau_j^m}{\eta_j^m} = 2 - \frac{F_1 F''_n}{(F'_1)^2},
\] (65)
again evaluated at the equilibrium (see Eqs. (13), (14) and (15)). For notational convenience, we again omit the functions’ argument $\Delta \tau$ and parameter $z_1$.

Note that $F''_n > -2(F'_1)^2/(1 - F_1)$ holds according to Assumption 1 (vi) – otherwise the second-order condition would not be fulfilled. Thus, the inequality $3 + \rho_1 > 3 - 2[(1 - 2F_1)/(1 - F_1)] > 1$ results, where we use (63). Since $F'_1$, $\partial F_1/\partial z_1$ and, in equilibrium, $1 - 2F_1$ are also positive, the quotient outside the square brackets is definitely positive. The overall sign of (64) then depends on the sign of the terms in the square brackets, which implies condition (30) in Proposition 4.

**Proposition 5** The proof of the first part of Proposition 5 completely follows along the lines of the proof of Proposition 2 and need not be repeated. To prove the second part of Proposition 5, we first establish the relationship between the hypothetical tax payments from domestic firms and the elasticity of the elasticity $\eta_j^m$ with respect to $\tau_j^m$:
\[
\frac{d}[\tau_i^n (1 - F_i)]}{dz_1} = -2(1 - F_1) \frac{\partial F_1}{\partial z_1} \left(1 - \frac{F''_n F_1}{(F'_1)^2}\right) = -2 \frac{(1 - F_1)}{F'_1(3 + \rho_1)} \left(\frac{\partial \eta_j^m}{\partial \tau_j^m} \frac{\tau_j^m}{\eta_j^m} - 1\right) \geq 0 \iff \frac{\partial \eta_j^m}{\partial \tau_j^m} \frac{\tau_j^m}{\eta_j^m} \leq 1, \quad (66)
\]
where Eq. (66) coincides with the second part of Eq. (64), which is weighted by the term $\tau_j^n / (\tau_j^n - \tau_j^m)$.

Comparative statics yields
\[
\frac{d h_i^m (\tau_i^n)}{d \tau_i} \geq 0 \iff \frac{\partial^2 N R_i}{\partial \tau_j^m \partial \tau_i} = \left[F'_1(\Delta \tau) - \frac{F_1(\Delta \tau)}{F'_1(\Delta \tau)} F''_1(\Delta \tau) \right] N_i \geq 0
\]
\[
\iff \frac{\partial \eta_j^m}{\partial \tau_j^m} \frac{\tau_j^m}{\eta_j^m} = 2 - \frac{F_1(\Delta \tau)}{[F'_1(\Delta \tau)]^2} F''_1(\Delta \tau) \geq 1, \quad (67)
\]
where the first-order condition (47) and Eq. (65) are used. Jointly, (66) and (67) imply $d[\tau_i^n (1 - F_i)]/dz_1 \geq 0 \iff \frac{\partial \eta_j^m}{\partial \tau_j^m} \frac{\tau_j^m}{\eta_j^m} \leq 1 \iff \frac{dh_i^m (\tau_i^n)}{d \tau_i} \leq 0$, which proves relationship (32) and thus the second part of Proposition 5.
Proposition 6  We follow the lines of reasoning applied in the proof of Proposition 3. That is, to determine the additional impact of a change in the parameter \( z_1 \) on the net tax revenues \( NR_i \) that arises if \( \partial F'_1(\Delta t; z_1) / \partial z_1 > 0 \), we evaluate the derivatives (60) and (62) for \( \partial F'_1(\Delta t; z_1) / \partial z_1 = 0 \) and \( \partial F'_1(\Delta t; z_1) / \partial z_1 > 0 \) at the equilibrium value of \( \Delta t \). Then, inserting (60)-(62) into derivative (59) yields, after some rearrangements,

\[
\frac{dNR_i}{dz_1} = -\frac{\partial F'_1}{\partial z_1 (F'_1)^2} \left[ (1 - F'_1)^2 + F''_1 - \frac{2(1 - 2F'_1)(F''_1)^2 + (1 - 2F'_1) F''_1}{3(F'_1)^2 + (1 - 2F'_1) F''_1} (1 - 2F'_1) \right],
\]

(68)

where we again omit the functions’ argument \( \Delta t \) and parameter \( z_2 \). Note that \( 3(F'_1)^2 + (1 - 2F'_1) F''_1 = (F''_1)^2 (3 + \rho_1) > 0 \), since \( 3 + \rho_1 > 1 \), which is shown in the proof of Proposition 4. Then, the quotient in the square brackets is positive and smaller than one. This conclusion, jointly with the fact that \( (1 - F'_1)^2 + F''_1 = 1 - 2F'_1 + 2F''_1 > 1 - 2F'_1 > 0 \) (where the last inequality follows from \( F'_1 < 0.5 \) in equilibrium; see again the proof of Proposition 4), implies that the expression in the square bracket is positive. Consequently, \( dNR_i/dz_1 \geq 0 \Leftrightarrow \partial F'_1/\partial z_1 \geq 0 \) results, which proves Proposition 6.

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