Labour Market Integration, Human Capital Formation, and Mobility*

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Abstract

In this paper, we analyse the implications of labour market integration in a two-region model with local human capital externalities and congestion effects. We show that integration can be a double-edged sword. Integration and the ensuing agglomeration of skilled labour can reduce ‘real’ income in both regions. Even if there is a ‘winning’ region, human capital and real income in the two regions together might decline (but need not). However, integration can increase total real income even if it depresses human capital formation. We further explore how the degree of labour mobility and the strength of the congestion effects shape the impact of integration on human capital and income.

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1 Motivation

Labour market integration is often promoted as a means to enhance overall efficiency, as it enables mobile skilled workers to become employed where they are most productive. In the presence of scale economies, skilled workers can agglomerate in specific regions and take advantage of positive externalities, thereby increasing aggregate income. By creating better job opportunities for skilled workers, labour market integration also provides greater incentives to become skilled in the first place.

In this paper, we scrutinise this line of reasoning. We analyse the interactions between labour market integration, agglomeration and human capital formation. More specifically, we explore how these interactions depend on labour mobility. This issue is important because dismantling all legal barriers to labour migration might have a very different impact on different regions and cities, depending on how willing the population is to take advantage of the new liberties. For example, consider the labour market integration in the European Union. In this case, agglomeration might reshape, say, competing German and Austrian regions and cities much more profoundly than German and French regions and cities. In the former case, a common language and culture suggests a highly mobile workforce after integration, whereas in the latter case language and cultural barriers still impose substantial impediments to mobility even once legal obstacles to migration have been abolished. Similarly, agglomeration is expected to affect the integrated European labour market very differently from the US one.

To explore the implication of integration for different degrees of mobility, we develop a model with two regions, each of them consisting of an urban and a rural area. Industrial production takes place in the two urban areas, while the rural areas are home to the agricultural sector. The natives of each region differ in ability and inter-regional mobility, and they decide on their education and location, with the caveat that inter-regional migration is only possible if the labour markets of the two regions are integrated. Workers employed in the urban industrial sector benefit from local human capital externalities, but suffer from negative congestion effects.

In this framework, labour market integration leads to agglomeration of skilled workers. Not surprisingly, one region might benefit from agglomeration at the expense of the other region. More surprisingly, both regions might be worse off after integration in the presence of both human capital externalities and congestion effects, with each region experiencing not only lower ‘real’ income (that is, wage income net of education and congestion costs) but also fewer skilled workers than in the case of non-integrated markets. In general, and when considering the two regions together, both total real income and total human capital might be higher or lower after integration. Interestingly, integration can yield higher total real income, although it depresses human capital formation.
Analysing how the degree of labour mobility exactly affects the impact of integration, we identify different patterns. In one pattern, labour market integration decreases total human capital and total real income for low levels of inter-regional labour mobility, raises total real income but depresses total human capital for intermediate levels of mobility, and increases both total human capital and total real income for high levels of mobility. Relating the emerging patterns to the strength of the urban congestion effects allows us to find further relationships. While integration will have a positive impact on total human capital and real income if mobility is high and congestion effects are weak, it reduces real income in both regions and total human capital if mobility is high and congestion effects are strong. The variety of possible outcomes cautions against simple conclusions and policy recommendations. It also shows that labour market integration can have very different implications for human capital and income, depending on inter-regional labour mobility and congestion effects.

Our paper connects labour market integration to agglomeration, human capital formation and inter-regional mobility. As a result, we contribute to two strands of the literature. First, there is the literature on brain drain (see, for instance, Miyagiwa, 1991, Mountford, 1997, Stark et al., 1998, Beine et al., 2001, Grossmann and Stadelmann, 2011, and Mountford and Rapoport, 2011). These contributions analyse the impact of migration of skilled workers on human capital formation, usually from the perspective of the poor country, assuming exogenous productivity differentials between rich and poor countries. In contrast, we consider two ex-ante identical regions, and analyse the emergence of endogenous asymmetries through agglomeration. We are interested in the implications of agglomeration for human capital and real income in each of the two regions and in the two regions together. Our approach highlights, for instance, that integration can make both regions worse off.

Second, there is the literature on the new economic geography (for instance, Krugman, 1991, Forslid and Ottaviano, 2003, Pfüger and Südekum, 2008, and Gallo, 2010). These papers analyse the location of economic activities and the agglomeration of physical, knowledge and human capital. As far as these papers consider human capital, they assume an exogenously fixed total stock of human capital. By contrast, we endogenise the accumulation of human capital. This enables us to study the interplay between agglomeration and human capital formation. As already argued above, the effects of agglomeration on human capital and income can work in opposite directions. Also, we can identify additional distortions caused by agglomeration. In particular, agglomeration changes the individual incentives to invest in human capital; it induces some talented but immobile individuals to relinquish education, while it encourages less talented but mobile individuals to become skilled. By considering agglomeration in a framework
with endogenous human capital formation, we contribute to bridge the gap between the 
literature on brain drain and on the new economic geography.

The paper proceeds as follows: In the next section, the model is introduced. Sections 3 
and 4 analyse and characterise the equilibria in the cases of non-integrated and integrated 
labour markets, respectively. In section 5, the impact of labour market integration on 
total human capital and total real income is assessed. Also, we explore how the level 
of inter-regional labour mobility affects the implications of integration and present two 
numerical examples to illustrate the general conclusions. In section 6, we discuss two 
extensions. Section 7 concludes the paper.

2 The Model

We start by presenting the characteristics of the regions, industry, agriculture, and indi-
viduals.

Regions, Industry, and Agriculture  Consider two ex-ante identical regions Up-
stream and Downstream, each of them containing an urban area and a rural area. In 
the rural area of each region, unskilled workers live and produce an agricultural good, 
using a Ricardian technology. More precisely, a fixed number of unskilled workers is 
needed to produce one unit of the agricultural good. This agricultural good is, in turn, 
freely traded at the exogenous world market price. Thus, each unskilled worker in the 
agricultural sector generates an income of \( w \), which is equal to the constant output per 
worker evaluated at the world market price.

In the urban area of each region, there is a continuum of symmetric, profit maximising 
firms on the unit interval. These firms produce a high-quality good with skilled workers 
only. This industrial good is freely traded in the world market at the normalised price 
of unity. The production technology exhibits constant returns-to-scale at the firm level 
and increasing returns-to-scale at the local level. More precisely, the output of firm \( k \) of 
region \( i \)'s industrial sector is

\[
y^k_i = A(H_i) h^k_i, \tag{1}
\]

where \( h^k_i \) and \( H_i = \int_0^1 h^k_i dt \), denote the number of skilled workers employed by firm 
\( k \) of region \( i \) and the number of all skilled workers employed by the local industry. We 
also refer to \( H_i \) as the regional human capital. Regional productivity \( A(H_i) \) is a strictly 
increasing and concave function of the regional human capital \( H_i \) over the domain \([0, 2]\), 
with \( A(0) > w \). This function is assumed to be at least twice-continuously differentiable.

The production function (1) captures the notion that the close proximity of skill-
intensive firms and skilled workers in an urban area generates agglomeration benefits.
These benefits are modelled as human capital spillovers. On the downside, urban agglomeration also causes congestion costs \( C_i \) to those living in the urban area, i.e., to the skilled workers, who constitute the urban population. For instance, a larger urban population implies that individuals suffer from more pollution, spend more time commuting, are more likely to be a victim of a crime and face higher living costs. Let us denote the pecuniary equivalent to all such congestion costs by \( C_i \), with

\[
C_i = C(H_i) = \lambda f(H_i) .
\]

The function \( f(H_i) \) is strictly increasing and strictly convex in the size of the urban population \( H_i \), and fulfils the properties \( f(0) = 0 \) and \( f'(0) = 0 \). For analytical convenience, it is assumed to be at least twice-continuously differentiable. The positive parameter \( \lambda \) is simply a scale parameter which captures the strength of the congestion effects.\(^1\)

**Individual Characteristics and Options** The number, or mass, of people native to each of the two regions Upstream and Downstream is normalised to unity. In each population, individuals differ in their ability and inter-regional mobility. Ability is captured by individual education costs, i.e., the costs of becoming a skilled worker. Let \( e_{ij} \) denote the education costs of native \( j \) of region \( i \), and let us assume that these education costs \( e_{ij} \) are uniformly distributed over the interval \([0, \bar{e}]\).

All individuals are perfectly mobile between the rural and the urban area within their native region. By contrast, only \( \gamma \) natives of each region are perfectly mobile between the regions and can move to the other region at no cost, with \( \gamma \in (0, 1) \). The remaining \((1 - \gamma)\) natives are perfectly immobile between the regions and will never leave their home region. The distribution of education costs is the same across the inter-regionally mobile and immobile groups.

Individuals make two decisions. First, each individual chooses whether to become skilled or not (education choice). Skilled workers are employed in the industrial sector, while unskilled individuals work in the agricultural sector. Second, and only if labour markets are integrated, each inter-regionally mobile individual chooses whether to stay in his home region or to migrate to the other region (migration choice). Within each region, skilled workers work and live in the urban area, where the industrial sector is located; unskilled workers do so in the rural area, which is home to the agricultural sector.

\(^1\)Urban congestion effects could have also been introduced in a more microfounded fashion by, for instance, explicitly adding the costs of commuting to a central business district and a housing market to the model (see, e.g., Tabuchi, 1998). However, such a more detailed modelling strategy would lead to the same key feature: congestion costs increase, as the urban population grows.
No individual has any market power. When making the decisions, individuals take the choices of the others and of firms, and thus the wages and congestion costs, as given. They maximise their individual ‘net’ wage, which is defined as gross wage net of education costs and congestion costs.

**Remark** The assumptions that the industrial sector only employs skilled workers and the agricultural sector only uses unskilled workers are crude simplifications. In section 6, we therefore explore an extension in which unskilled individuals work in both the industrial and the agricultural sector. As we show, this extension does not change our key conclusions. However, it enables us to explain the stylised fact that the percentage of migrants among skilled workers is greater than among unskilled workers (see, for instance, Docquier and Marfouk, 2006). This is particularly interesting, since ability and mobility are not correlated in our framework. In a further extension, we also discuss the case of land as a second production factor in the agricultural sector.

# 3 Non-Integrated Labour Markets

Consider the benchmark case of non-integrated labour markets. With perfect competition, unskilled workers earn $w$ in the agricultural sector. Skilled workers receive a skilled wage $w_i$ according to their marginal product $A(H_i)$, which is exogenous from the perspective of each single firm in the industrial sector. The resulting inverse aggregate demand for skilled labour in region $i$ is

$$w_i = \frac{\partial y_i}{\partial h_i} = A(H_i).$$

The skilled wage increases with the regional human capital, reflecting the positive spillover effect.\(^2\) However, this does not necessarily imply that firms will pay a higher skill premium $p_i$, defined as $p_i = w_i - w - C(H_i)$, if the regional human capital rises. This skill premium $p_i$ captures the effective wage gain of a skilled worker compared to an unskilled worker in the agricultural sector once the congestion costs are taken into account. It enables us to alternatively express the inverse aggregate demand for skilled labour in

\(^2\)There is empirical evidence that the skilled wages increase with the number of skilled workers - at least for some range of the size of the skilled population. Dustmann et al. (2005) show, with evidence from the UK, that an inflow of immigrants with a high level of education (A-levels or college/university degree) has, if anything, a positive effect on wages of natives with the same educational background. Similarly, Friedberg (2001) establishes for the Israeli labour market that when immigrants enter high-skilled jobs, native wages rise. Grossmann and Stadelmann (2013) confirm these results, which have been derived for individual countries, with data on bilateral migration.
region $i$ as
\[ p_i = A(H_i) - [w + C(H_i)] =: B(H_i). \quad (4) \]

The regional skill premium $p_i = B(H_i)$ is a strictly concave function of the regional human capital $H_i$. It will increase (decrease) with the number of skilled workers if the positive productivity effect of more skilled workers dominates (is dominated by) the negative congestion effect of a larger urban population. In figure 1a, the threshold level $H^{max}$ constitutes the turning point. For low levels of human capital, the congestion effect is rather weak, and a strong productivity effect will drive the skill premium up if the number of skilled workers grows. By contrast, for high levels of human capital, and thus of urban population, a strong congestion effect will lead to a decline in the skill premium if even more skilled workers move to the urban area.$^3$

In the case of non-integrated labour markets, individuals can decide on their education only. Native $j$ of region $i$ will become skilled if and only if the skill premium $p_i$ exceeds the individual education costs $e^j_i$, i.e., $p_i \geq e^j_i$. Thus, the aggregate supply of skilled workers in region $i$ is
\[ S_i = \frac{p_i}{\bar{c}} \quad \Leftrightarrow \quad p_i = \bar{c} S_i. \quad (5) \]

In equilibrium, the labour markets in the two regions are cleared, and all education choices are optimal, given the resulting skill premium. Jointly, the demand function (4) and the supply function (5) determine the equilibrium in the regional skilled labour market. To ensure an ‘interior’ outcome with some individuals remaining unskilled and working in the agricultural sector, we assume that $\bar{c} > B(H_{max})$. Then, the equilibrium number, or mass, of skilled workers and the corresponding skill premium are implicitly given by
\[ H_i^* = S_i^*, \quad A(H_i^*) - [w + C(H_i^*)] = \bar{c} H_i^*, \quad \text{and} \quad p_i^* = \bar{c} H_i^*. \quad (6) \]

The equilibrium number of unskilled workers simply equals
\[ L_i^* = 1 - H_i^*. \]

We can state proposition 1.

**Proposition 1 Non-Integrated Labour Markets.**

*In the case of non-integrated labour markets, an equilibrium exists and is unique.*

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$^3$There are various channels which make skilled wages increase with the number of skilled workers. In a monopolistic competition framework, a larger number of skilled workers positively affects the number of differentiated firms. This in turn raises skilled wages. Grossmann and Stadelmann (2011), for instance, follow this line of reasoning. Our approach has the advantage that it is more tractable. In contrast to Grossmann and Stadelmann (2011), we can capture both agglomeration economies and diseconomies in a tractable model.
Figure 1: Labour Market Equilibrium - Part 1

(a) Non-Integrated Economy

(b) Integrated Economy

\[
\begin{align*}
\text{Non-Integrated Economy} & \\
\text{Integrated Economy} & \\
H_i^* & \\
B() & \\
H_{max} & \\
H_i & \\
\end{align*}
\]
Throughout the paper, we relegate the proofs to the appendix and focus on the economic intuition in the main text. Figure 1a illustrates the equilibrium in the skilled labour market in the benchmark case (ignore the dotted line for the time being). It depicts the inverse labour demand and supply functions, which are the same for the two symmetric regions. The intersection of the two curves gives the equilibrium number of skilled workers $H_i^*$ and the corresponding skill premium $p_i^*$ in each region.

The equilibrium number, or mass, of skilled workers $H_i^*$ can, in principle, be greater or smaller than the threshold $H^{max}$. This is again illustrated in figure 1a. As the congestion parameter $\lambda$ increases, the skill premium curve rotates inwards (see the dotted curve). Its peak is to the left (right) of the intersection of the demand and supply curves for sufficiently large (small) values of $\lambda$. More precisely, we can show that there exists a critical value $\lambda^{max} > 0$ such that $H_i^* \leq H^{max}$ if $\lambda \leq \lambda^{max}$.

If the congestion costs are not too large (i.e., if $\lambda < \lambda^{max}$), then labour market integration sets the stage for inter-regional migration and increasing agglomeration of skilled workers in one of the two urban areas, as will be shown in section 4. This is the insightful scenario on which we focus in this paper. Therefore, we make assumption 1.

**Assumption 1** $\lambda < \lambda^{max}$.

At the very end of section 4.4, we will briefly discuss the implication of large congestion parameters, which violate assumption 1. Intuitively, high congestion costs act as a barrier to agglomeration and can prevent any inter-regional migration, as will be explored in more detail below.

## 4 Integrated Labour Market

Next, we analyse the equilibria in the case of integrated labour markets and compare them with the outcome under non-integrated labour markets.

### 4.1 Education and Migration Intertwined

Labour market integration does not affect unskilled workers, since they can still earn $w$ in the rural area of their home region and cannot achieve a higher income in the

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4Let us define $\lambda^{crit}$ such that $B'(1) = 0$ for $\lambda = \lambda^{crit}$, with $\lambda^{crit} > 0$. Then, congestion costs (2) and skill premium (4) imply that $B'(1) > 0$ for $\lambda < \lambda^{crit}$. Thus, $H^{max} > 1$ for $\lambda \leq \lambda^{crit}$, while $H_i^* < 1$ is satisfied (see proof of proposition 1). That is, $H^{max} > H_i^*$ holds at least for $\lambda \leq \lambda^{crit}$. Additionally, we can show that $H^{max} < H_i^*$ for sufficiently large values of $\lambda$ (details are provided upon request). Then, as both $H^{max}$ and $H_i^*$ are continuous functions of $\lambda$, there exists a $\lambda^{max} > \lambda^{crit}$ such that (i) $H_i^* = H^{max}$ for $\lambda = \lambda^{max}$ and (ii) $H_i^* < H^{max}$ if $\lambda < \lambda^{max}$. 

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other region. Hence, unskilled workers have no incentive to migrate. For simplicity, and without affecting our results, we assume that mobile unskilled individuals who are indifferent between the rural areas of the two regions stay at home. (In section 6, we consider an extension in which some unskilled workers move to the urban area of their neighbouring region to work there in the industrial sector.)

By contrast, labour market integration has substantial implications for skilled workers. As we will analyse in detail, it gives rise to an inter-regional differential in the skill premia and thus provides incentives for mobile skilled workers to migrate from the urban area of the low-premium (‘losing’) region to the urban area of the high-premium (‘winning’) region. For convenience, we refer to the ‘winning’ (‘losing’) region as region 1 (2), i.e., \( p_1 > p_2 \). As moving to the winning region enables individuals to reap higher returns on human capital, education and integration choices are now intertwined, with the decision on education depending on both ability and mobility.

Consider mobile natives of the losing region who can now easily move to the high-premium region. They base their education decision on the skill premium in the winning region. More precisely, such a mobile individual \( j \) of region 2 will become skilled if and only if the premium \( p_1 \) exceeds the education costs \( e_j^2 \), i.e., \( p_1 \geq e_j^2 \). Thus, skilled labour supply of immigrants in the winning region is \( M = m \gamma p_1 / \bar{\varepsilon} \), where \( m \in [0, 1] \) stands for the share of mobile skilled workers who migrate from region 2 to region 1.

In contrast to these mobile individuals, the immobile individuals of the losing region stay put. All those individuals remaining in region 2 will become skilled if and only if the skill premium \( p_2 \) exceeds their education costs \( e_j^2 \), i.e., \( p_2 \geq e_j^2 \). Overall, the number of skilled workers native to the losing region is \( S_2 = [m \gamma p_1 + (1 - m \gamma) p_2] / \bar{\varepsilon} \), of whom \( S_2 - M = (1 - m \gamma) p_2 / \bar{\varepsilon} \) stay in region 2.

In the winning region, individuals have no incentive to leave, and the domestic supply of skilled labour of the natives is given by the same function as in the case of non-integrated markets, i.e., \( S_1 = p_1 / \bar{\varepsilon} \). Thus, the aggregate skilled labour supply in the winning region is \( S_1 + M = (1 + m \gamma) p_1 / \bar{\varepsilon} \). To sum up,

\[
S_1 + M = (1 + m \gamma) \frac{p_1}{\bar{\varepsilon}} \quad \Leftrightarrow \quad p_1 = \frac{\bar{\varepsilon}}{(1 + m \gamma)} (S_1 + M) \quad \text{and} \quad (7)
\]

\[
S_2 - M = (1 - m \gamma) \frac{p_2}{\bar{\varepsilon}} \quad \Leftrightarrow \quad p_2 = \frac{\bar{\varepsilon}}{(1 - m \gamma)} (S_2 - M). \quad (8)
\]

The inverse demand for skilled labour in each region is still given by (4).

4.2 Agglomeration Equilibria

In an equilibrium, labour markets are cleared in the two regions and all education and migration choices are optimal for the resulting skill premia. Then, the demand function
(4) and the respective supply functions (7) and (8) yield the equilibrium conditions

\[ H_1^{**} = S_1^{**} + M^{**}, \quad A(H_1^{**}) - [w + C(H_1^{**})] = \frac{\bar{v}}{(1 + m^{**}\gamma)} H_1^{**}, \] (9)

\[ H_2^{**} = S_2^{**} - M^{**}, \quad A(H_2^{**}) - [w + C(H_2^{**})] = \frac{\bar{v}}{(1 - m^{**}\gamma)} H_2^{**}. \] (10)

and the equilibrium skill premia

\[ p_1^{**} = \frac{\bar{v}}{(1 + m^{**}\gamma)} H_1^{**}, \] (11)

\[ p_2^{**} = \frac{\bar{v}}{(1 - m^{**}\gamma)} H_2^{**}. \] (12)

As mobile skilled natives of region 2 move to the urban area in region 1 as long as there is an inter-regional differential in the skill premia in favour of region 1, the equilibrium migration share is either \( m^{**} = 1 \) if \( p_1^{**} > p_2^{**} \) or \( m^{**} \in [0, 1] \) if \( p_1^{**} = p_2^{**} \). Finally, the equilibrium number, or mass, of unskilled workers is

\[ L_1 = 1 - (H_1^{**} - M^{**}) \]

and

\[ L_2 = 1 - (H_2^{**} + M^{**}). \]

Labour market integration leads to a particular form of equilibria, to which we refer as agglomeration equilibria, defined as follows:

**Definition 1** An agglomeration equilibrium is an equilibrium in which (i) at least some skilled workers native to region 2 migrate to the urban area of region 1 and (ii) human capital in region 1 exceeds that in region 2, i.e., \( m^{**} > 0 \) and \( H_1^{**} > H_2^{**} \).

Importantly, the term ‘agglomeration’ refers here to inter-regional agglomeration, i.e., to the fact that the urban area in one region is more agglomerated than the urban area in the other region.

We cannot identify whether Upstream or Downstream is the winning (losing) region in an agglomeration equilibrium. Such an equilibrium is thus only determined up to the permutation of the two ex-ante identical regions Upstream and Downstream across the two indices. Nevertheless, we refer to any bundle \((m^{**}, H_1^{**}, H_2^{**})\) as ‘one’ equilibrium, although in fact this bundle stands for two equilibria. With this qualification in mind, we state proposition 2.

**Proposition 2** Integrated Labour Markets and Agglomeration Equilibria.

In the case of integrated labour markets, at least one agglomeration equilibrium exists. Three types of agglomeration equilibria are possible regarding the characteristics of the skilled labour markets.

\[ ^5 \text{For notational convenience, we omit the equilibrium quantities } L_1^{**} \text{ and } L_2^{**} \text{ when referring to equilibrium bundles.} \]
(i) Type 1: All mobile skilled natives of region 2 migrate to the urban area of region 1. Compared to the benchmark with non-integrated labour markets, human capital and the skill premium are larger in region 1 and smaller in region 2. That is, $m^{**} = 1$, $H_2^{**} < H_1^{**}$, and $p_2^{**} < p_1^{**}$.

(ii) Type 2: All mobile skilled natives of region 2 migrate to the urban area of region 1. Compared to the benchmark with non-integrated labour markets, human capital is larger in region 1 and smaller in region 2. The skill premium is larger in region 1 than it is in region 2, but weakly smaller than it is in the benchmark with non-integrated markets. That is, $m^{**} = 1$, $H_2^{**} < H_1^{**}$, and $p_2^{**} < p_1^{**}$.

(iii) Type 3: In general, only some mobile skilled natives of region 2 migrate to the urban area of region 1. Compared to the benchmark with non-integrated labour markets, human capital is larger in region 1 and smaller in region 2. The skill premia are identical in the two regions and lower than in the benchmark. That is, $m^{**} \in (0,1]$, $H_2^{**} < H_1^{**}$, and $p_2^{**} = p_1^{**} < p_1^i$.

Figures 1b, 2a and 2b illustrate agglomeration equilibria of type 1, 2 and 3, respectively. Whereas the labour demand function is still identical for the two regions, skilled labour supply is now higher in the winning region than in the losing region. Consequently, each of the figures 1b, 2a and 2b depicts two intersections of demand and supply curves, showing equilibrium human capital $H_1^{**}$ and $H_2^{**}$ and the corresponding skill premia $p_1^{**}$ and $p_2^{**}$ in the two regions.

Let us discuss the economic intuition for these agglomeration equilibria. Small levels of skilled migration from region 2 to 1 lead to an increase in the skill premium in the winning region (see, e.g., figure 1b). By contrast, the skill premium declines in the losing region. Hence, migration generates an inter-regional skill premium differential in favour of region 1, which triggers further migration to this region. If the initial inflow of skilled workers causes human capital in the winning region to grow above the critical level $H_{max}^{*}$, then further immigration will drive down the skill premium even in this region, as the productivity gains cannot compensate any more for surging congestion costs. Whether such a situation arises hinges on the magnitudes of the congestion effect and mobility. Depending on these magnitudes, the three different types of agglomeration equilibria can emerge.

In an agglomeration equilibrium of type 1 (see figure 1b), even though all mobile skilled individuals of region 2 move to the urban area of region 1, the resulting premium $p_1^{**}$ is still above the benchmark premium $p_1^i$, reflecting the fact that the overall increase in productivity and thus the skilled wage outweighs the rise in congestions costs.\textsuperscript{6} Compared to the case of non-integrated markets, human capital in region 1 increases for two

\textsuperscript{6}The number of skilled workers employed in region 1 might be below or above the threshold level
Figure 2: Labour Market Equilibrium - Part 2

(a) Type-2 Equilibrium

(b) Type-3 Equilibrium
reasons: first, mobile skilled natives of region 2 emigrate to region 1 to take advantage of a higher skill premium there. Second, a higher premium incentivises not only more natives of region 1, but also more mobile individuals of region 2, to invest in education and become skilled, thereby reinforcing the supply of both skilled natives and skilled immigrants in the winning region. Conversely, human capital in region 2 declines for two reasons: first, mobile skilled natives leave the losing region, as already mentioned above. Second, the decline in the local skill premium as a result of the outflow of human capital discourages the remaining natives of region 2 to become skilled.

In an agglomeration equilibrium of type 2 (see figure 2a), the skill premium falls below the benchmark $p^*_i$ even in the winning region. While integration still raises the productivity of skilled workers in region 1, the resulting increase in the skilled wage is now more than compensated by rising congestion costs. Still, all mobile skilled natives of the losing region move to the urban area of the winning region, as the skill premium drops even further in the losing region. Even more drastically, in an agglomeration equilibrium of type 3, immigration and the induced congestion costs cause the skill premium in region 1 to fall so sharply that it reaches the level of the skill premium in region 2. As the premia are then the same in the two regions, migration ceases and some mobile skilled natives of region 2 remain in their home region (see figure 2b).

In an agglomeration equilibrium of type 2 or 3, human capital again increases in region 1 compared to the case of non-integrated markets, but only because of skilled immigration, and despite the fact that a smaller premium discourages natives of region 1 and mobile individuals of region 2 to invest in education. Obviously, human capital again falls in region 2, as its mobile skilled labour force at least partly emigrates and fewer natives find it beneficial to become skilled. Importantly, the incentives to invest in education are now depressed across the board, and not only for immobile individuals in the losing region.

For convenience, table 1 shows the characteristics of the three types of agglomeration equilibria at a glance (see upper half of table 1; the lower part will be explained later). At the regional level, the key difference between the equilibria is whether the skill premium in the winning region is above or below the benchmark premium $p^*_i$.

\[ H^{max} \]

This issue is irrelevant, since the key feature of this equilibrium type is the property that the premium $p^*_i$ exceeds the benchmark $p^*_i$.

\[ ^7 \]

Strictly speaking, the term ‘winning region’ is not quite correct in the case of an agglomeration equilibrium of type 3, given that we have introduced this term to label the region with a (strictly) higher skill premium. We ignore this slight inaccuracy. Also, with identical premia in the two regions, migration in two directions is possible. That is, a share $m_1 (m_2)$ of the mobile skilled workers native to region 1 (2) might move to region 2 (1). Then, the equilibrium share $m^{**} = m_2 - m_1$ would capture net migration from region 2 to 1, which would still be uniquely determined.
Table 1: Characteristics of the Three Types of Agglomeration Equilibria

<table>
<thead>
<tr>
<th>Outcomes</th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Migration</td>
<td>$m^{**} = 1$</td>
<td>$m^{**} = 1$</td>
<td>$m^{**} \in (0, 1]$</td>
</tr>
<tr>
<td>Human capital</td>
<td>$H_{2}^{<strong>} &lt; H_{i}^{*} &lt; H_{1}^{</strong>}$</td>
<td>$H_{2}^{**} &lt; H_{i}^{<em>} &lt; H_{1}^{</em>}$</td>
<td>$H_{2}^{<strong>} &lt; H_{i}^{*} &lt; H_{1}^{</strong>}$</td>
</tr>
<tr>
<td>Skill premium</td>
<td>$p_{2}^{**} &lt; p_{i}^{<em>} &lt; p_{1}^{</em>}$</td>
<td>$p_{2}^{**} &lt; p_{i}^{<em>} \leq p_{1}^{</em>}$</td>
<td>$p_{2}^{**} = p_{1}^{<em>} &lt; p_{i}^{</em>}$</td>
</tr>
<tr>
<td>Total premium</td>
<td>$H_{1+2}^{**} &gt; H_{1+2}^{*}$</td>
<td>$NP_{1+2}^{**} &gt; NP_{1+2}^{*}$</td>
<td>$H_{1+2}^{**} &lt; H_{1+2}^{*}$</td>
</tr>
<tr>
<td>Human capital</td>
<td>$H_{1+2}^{**} &lt; H_{1+2}^{*}$</td>
<td>$NP_{1+2}^{**} &lt; NP_{1+2}^{*}$</td>
<td>$H_{1+2}^{**} \leq H_{1+2}^{*}$</td>
</tr>
</tbody>
</table>

Interestingly, labour market integration distorts the education decision if an agglomeration equilibrium of type 1 or 2 materialises. Then, the skill premia differ across regions. As a result, talented but immobile natives of the losing region shy away from education, whereas less talented natives of the winning region, and less talented but mobile natives of the losing region, become skilled.\(^8\) This kind of distortion does not occur in an agglomeration equilibrium of type 3, since the skill premia are equalised across regions in this case.

4.3 The Role of Mobility and Congestion

Having characterised the possible agglomeration equilibria in general, we now relate the agglomeration equilibria more precisely to the congestion cost and mobility parameters, $\lambda$ and $\gamma$.

**Proposition 3** Mobility, Congestion, and Agglomeration Equilibria.

For given regional productivity and congestion cost functions, $A(H_i)$ and $C(H_i)$, and education cost parameter $\bar{e}$, two patterns of agglomeration equilibria can emerge:

(i) **Weak congestion pattern:** a single agglomeration equilibrium of type 1, and none of type 2 or 3, exists for all mobility parameters $\gamma \in (0, 1)$. This pattern emerges if the congestion cost parameter is strictly below a threshold value $\lambda > 0$.

(ii) **Strong congestion pattern:** a single agglomeration equilibrium of type 1, and none of type 2 or 3, exists for all $\gamma \in (0, \widehat{\gamma})$, $\widehat{\gamma} < 1$, whereas at least one agglomeration equilibrium of type 2 or 3, and none of type 1, exists for all $\gamma \in [\widehat{\gamma}, 1)$. This pattern emerges if the congestion cost parameter exceeds a threshold value $\overline{\lambda}$, with $\lambda \leq \overline{\lambda} < \lambda^{\text{max}}$.

\(^8\)We do not question the empirical observations that mobile workers tend to be high-skilled (see, for instance, Ehrenberg and Smith, 1994, for the US, Uebelmesser, 2006, for Germany, and Coniglio and Prota, 2007, for Italy). However, we cast some doubt on the suggested causality that skilled workers become mobile. In contrast, we argue that mobile workers become skilled. This argument is in line with Eggert et al. (2010).
For ‘well-behaved’ specifications, such as the ones that will be used in our examples in section 5.3, there exist coinciding threshold values $\Lambda$ and $\bar{\lambda}$. This situation is illustrated in figure 3, which supports our explanation of proposition 3.

In general, higher mobility swells the ranks of skilled workers willing to migrate. It thus drives up skilled labour supply in the winning region, which in turn makes the negative congestion effects of a larger urban population more likely to prevail over the positive productivity effect of more skilled workers. After all, the marginal productivity gains decline, while the marginal congestion costs increase with the number of skilled workers employed in the urban area. Hence, as mobility increases, the resulting skill premium initially rises and then may possibly fall. This implies that we may move from an agglomeration equilibrium of type 1 (i.e., $p_{1i}^{**} > p_{i}^*$) to one of type 2 or 3 (i.e., $p_{1i}^{**} \leq p_{i}^*$), but never vice versa. (See table 1 for a summary of the equilibrium characteristics.)

However, in the case of weak congestion effects, only an agglomeration equilibrium of type 1 exists, as stated in part (i) of proposition 3 (see figure 3). For $\lambda < \bar{\lambda}$, the congestion effects are so small that even if the whole population of the losing region is perfectly mobile and all its skilled workers move to the urban area of the winning region, the induced rise in the congestion costs will still fall short of the increase in regional productivity. Thus, labour market integration raises the skill premium in region 1 for all levels of the mobility parameter $\gamma$.

The impact of integration is less clear-cut in the case of strong congestion effects (i.e., $\lambda > \bar{\lambda}$), as stated in part (ii) of proposition 3 (see, again, figure 3). Then, the productivity gains in the winning region still outweigh the rise in the congestion costs, but only if the inflow of skilled workers from the losing region is small. That is, an agglomeration equilibrium of type 1 still exists, but only if mobility is low (i.e., $\gamma < \hat{\gamma}$). Once mobility is high (i.e., $\gamma > \hat{\gamma}$) and the winning region faces a substantial level of immigration, the strong congestion effects dominate any productivity gains. The skill premium falls below its benchmark level $p_{i}^*$, and an agglomeration equilibrium of type 2 or 3 emerges.

4.4 Uniqueness and Stability

There might be more than one agglomeration equilibrium, but then these equilibria are of type 2 or 3. However, we can specify sufficient conditions that rule out multiple agglomeration equilibria altogether. To show this, let us consider symmetric $B(H_i)$-functions; that is, for all $\Delta H \in [0, H_{max}]$, $B(H_{max} - \Delta H) = B(H_{max} + \Delta H)$ and thus $B'(H_{max} - \Delta H) = -B'(H_{max} + \Delta H)$ hold. Then, we can state:
**Proposition 4** Agglomeration Equilibrium and Symmetric $B(H_i)$-Functions.

Assume that the $B(H_i)$-function is symmetric. Then, a single agglomeration equilibrium exists. This equilibrium is either of type 1 or 2, but never of type 3.

The symmetry assumption excludes too strong congestion effects and thus agglomeration equilibria of type 3. We will later use a specific symmetric $B(H_i)$-function as one of our examples to illustrate our arguments.

We conclude Section 4 with two brief remarks. First, the equilibrium in the benchmark case of non-integrated labour markets also constitutes an equilibrium in the case of integrated labour markets, irrespective of whether a general or symmetric $B(H)$-function is considered. However, this non-agglomeration equilibrium with $m^{**} = 0$ and $H_1^{**} = H_2^{**} = H_i^*$ is locally unstable in the case of integrated labour markets. In contrast, at least one of the existing agglomeration equilibria is locally stable. When analysing the outcome with integrated labour markets, we thus focus on the agglomeration equilibria and ignore the unstable non-agglomeration equilibrium. Details on the stability of the equilibria are provided in appendix B.

Second, if assumption 1 were violated and the human capital $H_i^*$ already exceeded the threshold level $H^{max}$, then the equilibrium in the case of non-integrated labour markets would also constitute a locally stable equilibrium in the case of integrated markets. Such a locally stable non-agglomeration equilibrium requires a rather large cost parameter $\lambda$. Then, starting from the equilibrium with non-integrated markets, high congestion costs dominate the productivity gains of any increase in the number skilled workers in one of the urban areas from the outset and thus prevent inter-regional agglomeration. As this
case is not particularly insightful, we focus on the agglomeration equilibria that arise under assumption 1.

5 Total Human Capital and Net Skill Premium

We are now able to assess how labour market integration affects the two regions as a whole, and how this overall impact is driven by the level of mobility and the strength of the congestion effects. In our framework, the ‘net’ wage, defined as gross wage net of congestion and education costs, captures individual welfare. (In section 1, we referred to this net wage as real income.) The net unskilled wage equals $w$ in both regions, irrespective of whether labour market integration has taken place. By contrast, the net skilled wage is $w_i + p_i^* - e_i^1$, $w_1 + p_1^{**} - e_1^2$, $w_2 + p_2^{**} - e_2^2$ or $w + p_i^* - e_i^2$, depending on whether labour markets are integrated, and whether the respective skilled native of region 2 migrates. As the constant component $w$ emerges in all wages, we can ignore it and focus instead on the ‘net’ skill premium, defined as the skill premium net of the individual education costs. This net skill premium is trivially zero for unskilled workers and, again depending on integration and migration, $p_i^* - e_i^1$, $p_1^{**} - e_1^1$, $p_2^{**} - e_2^2$ or $p_i^* - e_i^2$ for skilled workers. (Recall that the ‘gross’ skill premia $p_i^*$, $p_1^{**}$ and $p_2^{**}$ already take account of congestion costs, but not of individual education costs.)

5.1 The Impact of Labour Market Integration

In the case of an agglomeration equilibrium of type 1, labour market integration generates winners and losers. All skilled natives of the winning region benefit from integration, and so do all skilled natives of the losing region who are mobile and thus move to the winning region, since all their net skill premia increase—irrespective of whether those individuals would or would not have become skilled in the case of non-integrated markets. By contrast, all skilled workers who stay in the losing region suffer from a lower skill premium. Additionally, those immobile unskilled natives of the losing region who would have become skilled without integration are also worse off because they lose the net skill premium $p_i^* - e_i^1$. All other unskilled workers remain unaffected.

In the case of agglomeration equilibria of type 2 or 3, there are no winners at all. All skilled workers face declining net skill premia. Those unskilled workers who would have been skilled without integration lose out as well. Only the other unskilled workers are as well off as without integration.

As integration potentially leads to winners and losers, we are interested in whether the winners can at least hypothetically compensate the losers. Therefore, we explore the impact of integration on the total skill premium, $P_{1+2} = p_1 H_1 + p_2 H_2$, and, more
importantly, the total net skill premium, \( NP_{1+2} = P_{1+2} - \text{total education costs} \) (i.e., education costs of all skilled workers as a whole). Additionally, we analyse how integration affects total human capital, \( H_{1+2} = H_1 + H_2 \), and whether the impact on human capital is systematically related to the impact on the total net skill premium.

The total net skill premium is given by

\[
NP_{1+2}^* = 2 \left[ p^*_i H_i^* - \frac{\bar{\tau}}{2} (H_i^*)^2 \right]
\]

(13)

\[
NP_{1+2}^{**} = \left[ p_i^{**} H_i^{**} - \frac{\bar{\tau}}{2(1 + m\gamma)} (H_i^{**})^2 \right] + \left[ p^{**}_2 H_2^{**} - \frac{\bar{\tau}}{2(1 - m\gamma)} (H_2^{**})^2 \right]
\]

(14)

in the case of non-integrated and integrated labour markets, respectively. To facilitate the further analysis, we describe the relationship between the total skill premium and the total net skill premium in lemma 1.

**Lemma 1** With both non-integrated and integrated labour markets, the total net skill premium is exactly half as high as the total skill premium, i.e., \( NP_{1+2} = (1/2) P_{1+2} \) and \( NP_{1+2}^{**} = (1/2) P_{1+2}^{**} \).

As a result of the relationship established in lemma 1, there is no need to consider the total net skill premium and the total skill premium separately. This makes it simpler to compare the equilibrium in the case of non-integrated markets with the agglomeration equilibrium in the case of integrated markets. We summarise our conclusions in proposition 5.

**Proposition 5** The Impact of Labour Market Integration.

Part (a): If an agglomeration equilibrium of type 1 materialises, then labour market integration leads to one of the following three outcomes: (i) Total human capital increases, i.e., \( H_{1+2}^{**} \geq H_{1+2}^* \). Then, the total net skill premium also rises, i.e., \( NP_{1+2}^{**} > NP_{1+2}^* \). (ii) Total human capital declines, but the total net skill premium increases, i.e., \( H_{1+2}^{**} < H_{1+2}^* \) and \( NP_{1+2}^{**} > NP_{1+2}^* \). (iii) The total net skill premium decreases, i.e., \( NP_{1+2}^{**} < NP_{1+2}^* \). Then, human capital also declines, i.e., \( H_{1+2}^{**} < H_{1+2}^* \).

Part (b): If an agglomeration equilibrium of type 2 or 3 materialises, then labour market integration causes a decline in total human capital and the total net skill premium. That is, \( H_{1+2}^{**} < H_{1+2}^* \) and \( NP_{1+2}^{**} < NP_{1+2}^* \).

The lower half of table 1 shows these results at a glance. Let us first discuss part (a) of proposition 5. If agglomeration causes a substantial increase in the skill premium in region 1 relative to the losses in region 2, then integration will strengthen the overall incentives to invest in education, and total human capital will grow. If this happens,
the total skill premium will inevitably increase, as will the total net skill premium. This scenario is illustrated in figure 4a, with an almost linear (inverse) labour demand function \( B(H_i) \), which can stem, for instance, from a linear productivity function combined with a very weak congestion effect (i.e., very small congestion parameter \( \lambda \)). Here, the labour supply curves in the case of non-integrated and integrated labour markets are plotted in a single diagram. As the intersections between the demand curve and the supply curves show, labour market integration causes a substantial increase in the skill premium in region 1, from \( p_1^* \) to \( p_1^{**} \), but only a minor decline in the skill premium in region 2, from \( p_2^* \) to \( p_2^{**} \).

Importantly, the total net skill premium may increase even if human capital declines. As a consequence, we cannot infer from the fact that there are fewer skilled people that there are no overall benefits from agglomeration. In this scenario, the rise of the skill premium for some individuals not only compensates for the loss of others, but also makes up for the fall in the total number of skilled workers.

However, if the premium decline in region 2 is sufficiently drastic relative to the increase in region 1, the total net skill premium will decrease, and such a drop will always be accompanied by a decrease in total human capital. This scenario is illustrated in figure 4b, which shows a strongly curved (inverse) labour demand function \( B(H_i) \). Now, labour integration induces only a minor rise of the skill premium in region 1, from \( p_1^* \) to \( p_1^{**} \), but a very pronounced reduction in the skill premium in region 2, from \( p_2^* \) to \( p_2^{**} \).

Part (b) of proposition 5 is evident. If the skill premia drop in both regions below their level in the case of non-integrated markets, the incentives to invest in education will fall as well. Then, total human capital and the total net skill premium will decline compared to the case of non-integrated markets, although more skilled workers will be employed in region 1.

5.2 The Role of Mobility and Congestion

Next, we explore how the overall impact of integration depends on the degree of mobility and the extent of congestion. The following proposition assesses the implications of a highly mobile labour force. It follows immediately from proposition 3.

**Proposition 6** High Mobility and the Impact of Labour Market Integration.

(i) Consider the weak congestion pattern, as described in proposition 3. Then a critical threshold \( H_{i}^{max} \) exists such that if the skill premium in region 1 exceeds \( H_{i}^{max} \), both total human capital and the total net skill premium can decline.

---

9The potentially negative impact of labour market integration on total human capital and the total net skill premium does not at all rely on the fact that \( B(H_i) \) eventually declines with regional human capital \( H_i \). Even if equilibrium human capital \( H_{1i}^{**} \) falls short of the threshold \( H_{i}^{max} \), both total human capital and the total net skill premium can decline.
Figure 4: Impact of Integration on Human Capital and Skill Premium

(a) (Almost) Linear B(H)-Function

(b) Strongly Curved B(H)-Function
value $\gamma < 1$ exists such that integration boosts total human capital and the total net skill premium for all $\gamma > \gamma$. That is, $H_{1+2}^{**} > H_{1+2}^*$ and $NP_{1+2}^{**} > NP_{1+2}^*$ if $\gamma > \gamma$.

(ii) Consider the strong congestion pattern, as described in proposition 3. Then a critical value $\hat{\gamma} < \gamma$ exists such that integration reduces total human capital and the total net skill premium for all $\gamma > \hat{\gamma}$. That is, $H_{1+2}^{**} < H_{1+2}^*$ and $NP_{1+2}^{**} < NP_{1+2}^*$ if $\gamma > \hat{\gamma}$.

Recall that if weak congestion effects are at work (see proposition 3 and figure 3 for $\lambda < \lambda$), only an agglomeration equilibrium of type 1 exists. That is, the skill premium in the winning region always exceeds its level in the benchmark case of non-integrated labour markets (see proposition 2 and table 1). Consequently, integration will always generate overall gains if individuals are sufficiently mobile, i.e., if enough individuals are able to move to region 1 and join the winners. Then both total human capital and the total net skill premium will rise. However, the overall benefits conceal severe distributional conflicts. While a larger share $\gamma$ of mobile individuals increases the gains in the net skill premium of those employed in the winning region, it also means larger losses in the net skill premium of those working in the losing region.

Obviously, the situation is very different in the presence of strong congestion effects. Then, for sufficiently high levels of mobility, an agglomeration equilibrium of type 2 or 3 emerges (see proposition 3 and figure 3 for $\lambda > \lambda$). That is, the skill premia of both regions fall below the level in the case of non-integrated markets (see proposition 2 and table 1). Thus, labour market integration makes skilled workers in both regions worse off and depresses both total human capital and the total net skill premium.\(^{10}\)

Complementing proposition 6, we now analyse the overall implications of integration in the case of low mobility levels.

**Proposition 7** Low Mobility and the Impact of Labour Market Integration.

A critical value $\gamma^{crit} > 0$ exists such that integration reduces total human capital and the total net skill premium (i.e., $H_{1+2}^{**} < H_{1+2}^*$ and $NP_{1+2}^{**} < NP_{1+2}^*$) for all $\gamma \in (0, \gamma^{crit})$ if

$$\left[2 + \varepsilon_{B,H}(H_i^*)\right] \left[1 - \varepsilon_{B,H}(H_i^*)\right] < -\eta(H_i^*),$$

(15)

where $\varepsilon_{B,H}(H_i^*) = B'(H_i^*)H_i^*/B(H_i^*)$ and $\eta(H_i^*) = B''(H_i^*)H_i^*/B'(H_i^*)$.

As an agglomeration equilibrium of type 1 materialises for sufficiently low levels of mobility (see proposition 3 and figure 3), the overall impact of integration is ambiguous.

\(^{10}\)This conclusion is somewhat reminiscent of a result in Baldwin et al. (2003, ch. 17). They allow for congestion effects in a two-region model with agglomeration, capital mobility and endogenous growth. Baldwin et al. (2003, ch. 17) argue that low mobility costs of capital might be detrimental to growth rates. However, as their model still predicts a positive ‘level’ effect in one of the two regions, it is not clear whether lower growth rates indeed lead to lower welfare at any point in time.
without further specifications (see proposition 5). To understand proposition 7, note that $\eta(H)$ captures the elasticity of the slope of the (inverse) labour demand function with respect to changes in the number of skilled workers; it thus reflects the curvature of $B(H_i)$. If this curvature is sufficiently pronounced, the scenario plotted in figure 4b prevails. Then, labour market integration moderately raises the skill premium in region 1, but sharply decreases the skill premium in region 2. As mobility is low, the relatively small increase in the number of better-off skilled workers in the winning region cannot compensate for the decline in both the skill premium and the number of skilled workers in the losing region. Thus, integration reduces both total human capital and the total net skill premium. For the elasticity condition (15) to be fulfilled, $|\eta(H)| > 2$ is sufficient, since the left-hand side of inequality (15) is smaller than 2.$^{11}$

Proposition 7 implies that even under the weak congestion pattern, labour market integration can reduce total human capital and the total net skill premium for low levels of mobility. This outcome occurs despite the fact that under this pattern both total human capital and the total net skill premium increase for sufficiently high levels of mobility.

Such a situation is depicted in figure 5, which is consistent with one of the examples analysed in detail in the next section. Consider the lower part of the diagram ($\lambda < \bar{\lambda}$). As the mobility parameter $\gamma$ increases, we move from an agglomeration equilibrium with a lower level of both total human capital and the total net skill premium than in the case of non-integrated markets (for $\gamma < \gamma_{crit}$) to one with less total human capital and a higher total net skill premium (for $\gamma_{crit} < \gamma < \bar{\gamma}$) to one with a higher level of both total human capital and the total net skill premium (for $\gamma > \bar{\gamma}$).

5.3 Examples

To illustrate the propositions, we consider the following function for the regional skill premium (see (4))

$$p_i = A(H_i) - [w + C(H_i)] = 2H_i + 0.1 - \lambda H_i^2 =: B(H_i),$$

(16)

which results from a linear regional productivity function and a quadratic congestion function, specified as $A(H_i) = 2H_i + 0.1 + w$ and $C(H_i) = \lambda H_i^2$, respectively. In the following two examples, $\bar{\gamma}$ is set to 1.6.

$^{11}$The assumptions about $A(H_i)$ and $C(H_i)$ and assumption 1 imply that $B(H_i^\ast) > 0$, $B'(H_i^\ast) > 0$, $B''(H_i^\ast) < 0$ and $B(H_i^\ast)/H_i^\ast > B'(H_i^\ast)$. Thus, while $\eta(H_i^\ast) < 0$, $\varepsilon_{B,H}(H_i^\ast) = B'(H_i^\ast)H_i^\ast/B(H_i^\ast) \in (0,1)$ holds. Then, simple calculations reveal that $[2 + \varepsilon_{B,H}(H_i^\ast)] [1 - \varepsilon_{B,H}(H_i^\ast)] < 2$ for all $\varepsilon_{B,H}(H_i) \in (0,1)$. Hence, condition (15) is fulfilled for $|\eta(H)| > 2$.
Example 1  Let us set $\lambda = 1$. Then, the regional skill premium is given by

$$p_i = B(H_i) = -(H_i - 1)^2 + 1.1.$$  \hfill (17)

The function $B(H_i)$ is symmetric (as defined in section 4.4), positive for all $H_i \in [0, 2]$, and strictly concave. It peaks for $H_i = 1$.\footnote{For the specification in example 1 and $\bar{\pi} = 1.6$, $\bar{\pi} > B(H^{max}) = 1.1$ is satisfied. This guarantees an interior solution, i.e., non-negative numbers of both skilled and unskilled individuals.} Under this specification, a weak congestion pattern emerges: an agglomeration equilibrium of type 1 exists for all mobility levels, i.e., $m^{**} = 1$, $H_2^{**} < H_i^* < H_1^{**}$ and $p_2^{**} < p_i^* < p_1^{**}$ for all $\gamma \in (0, 1)$ (see propositions 2 and 3). This is illustrated in figure 6.

Figures 6a and 6b show the human capital employed in the two regions and the corresponding skill premia in the agglomeration equilibrium as a function of the mobility parameter $\gamma$. Since the agglomeration equilibrium converges towards the equilibrium in the case of non-integrated markets as the mobility parameter approaches zero, the starting points of the curves capture the values of human capital and the skill premium in each region in the economic benchmark (see the grey horizontal line). For all mobility levels, the levels of human capital and the skill premium in the winning (losing) region are above (below) the corresponding levels in the case of non-integrated markets, as figures 6a and 6b clearly illustrate.

Figures 6c and 6d depict total human capital and the total net skill premium in the agglomeration equilibrium as a function of mobility. Again, the starting points
Figure 6: Weak Congestion Pattern

(a) Regional Human Capital

(b) Skilled Wages

(c) Total Human Capital

(d) Total Net Wage Sum
of the curves give the levels of these variables in the case of non-integrated markets. Comparing these benchmark values with the corresponding values in the agglomeration equilibrium shows that, depending on the mobility level, all three outcomes described in part (a) of proposition 5 occur. First, for high levels of mobility (i.e., $\gamma > \bar{\gamma} = 0.62$), both total human capital and the total net skill premium are larger with integrated markets than with non-integrated ones. This outcome is consistent with proposition 6 (see also figure 5). Second, for intermediate mobility levels (i.e., $\gamma \in (\gamma^{\text{crit}}, \bar{\gamma})$, $\gamma^{\text{crit}} = 0.30$), agglomeration reduces total human capital compared to the case of non-integrated markets, whereas it increases the total net skill premium. Third, both total human capital and the total net skill premium are smaller with integrated labour markets than with non-integrated ones for low mobility levels (i.e., $\gamma < \gamma^{\text{crit}} = 0.30$). This is consistent with proposition 7 because $\varepsilon_{A,H}(H^*_1) = 0.53$ and $-\eta(H^*_1) = 1.35$ imply that condition (15) is fulfilled.

**Example 2** Let us now set $\lambda = 3$. $B(H_i)$ is no longer a symmetric function. It is positive for all $H_i \in [0, 0.71]$ and strictly concave. It peaks for $H_i = 1/3 + 0.1$.\(^{13}\)

This specification generates a strong congestion pattern. An agglomeration equilibrium of type 1 emerges for relatively low mobility levels (i.e., $\gamma < \hat{\gamma} = 0.55$), whereas an agglomeration equilibrium of type 2 prevails for high mobility levels (i.e., $\gamma \geq \hat{\gamma} = 0.55$). This is in line with proposition 3, and is captured in figure 7, the counterpart of figure 6 (see also figure 5 for an overview of the different types of equilibria as functions of $\lambda$ and $\gamma$).

As figure 7b illustrates, the skill premium in region 1 now exceeds the skill premium in the non-integrated economy only for low mobility levels. For high mobility, the skill premia in both regions fall below the benchmark level (indicated by the grey horizontal line). Compared with the levels in the case of non-integrated markets, the number of skilled workers employed in region 1 is still higher in the agglomeration equilibrium (see figure 7a), but total human capital is lower for all levels of mobility (see figure 7c). Similarly, the total net skill premium is lower with integrated markets than with non-integrated ones, irrespective of the mobility level (See figure 7d). This is consistent with propositions 5 and 6, with the critical level $\gamma = 0$. It is also in line with proposition 7, since the values of the elasticities $\varepsilon_{A,H}(H^*_1) = 0.27$ and $-\eta(H^*_1) = 3.61$ again imply that condition (15) is fulfilled.

\(^{13}\)For this specification with $\lambda = 3$ and $\bar{\sigma} = 1.6$, the condition for an interior solution $\sigma > B(H^{\text{max}})$ is again satisfied.
Figure 7: Strong Congestion Pattern

(a) Regional Human Capital

(b) Skilled Wages

(c) Total Human Capital

(d) Total Net Wage Sum
5.4 Labour Market Integration - Curse or Blessing?

In the presence of agglomeration and congestion effects, labour market integration can have rather complex implications. We are inclined to assume that at least one region benefits from agglomeration, even if its gain is at the expense of another region. As our model highlights, this need not be the case if congestion effects are sufficiently strong. Then, labour market integration and the ensuing agglomeration can reduce human capital and skill premia in both regions, and this outcome is the more likely the higher the level of mobility. Hence, high mobility can be detrimental to the welfare of both regions, instead of promoting prosperity. Indicative of such an outcome is that the skill premia are fairly equal in the two regions despite substantial net migration from one region to the other.

However, the impact of mobility is very ambiguous. If congestion effects are weak, high mobility ensures that labour market integration does not only benefit the winning region, but also increases both overall human capital and the total net skill premium. However, even under these circumstances, total human capital and the total net skill premium can decline for low levels of mobility, as illustrated in example 1 above. In this scenario, low mobility is an obstacle to overall gains from labour market integration. In any case, every overall gain comes at the expense of distributional conflicts, with one region clearly losing out. Moreover, as example 2 nicely illustrates, labour market integration can lead to a fall in total human capital and the total net skill premium for all levels of mobility.

Finally, we show that there is no easy correlation between human capital and the net skill premium. Even if total human capital drops as a result of labour market integration, the total net skill premium can still be higher than in the case of non-integrated markets. Interestingly, labour market integration distorts the education choice. Open borders make it more attractive for mobile individuals to acquire education relative to immobile ones. While some talented but immobile individuals will invest less in education, some less talented but mobile individuals will increase their human capital investment. This distortion accelerates the gains in the winning region, if any, as well as the losses in the losing region.  

It does so, however, without necessarily implying a positive correlation between total human capital and the total net skill premium, as previously mentioned.

\footnote{Fershtman et al. (1996) discuss a somewhat similar distortion. In their model, individuals differ in ability and initial wealth. People with high wealth but lower ability crowd people with less wealth but higher ability out of activities that enhance growth if these activities are associated with social status. This is detrimental to growth, although it increases the steady-state number of skilled workers. In our model, with mobility distorting educational choices, the crowding out might not even go hand in hand with an increase in the number of skilled workers.}
6 Extensions

To check the robustness of our results, we explore two extensions of our basic model. First, we add unskilled labour as complementary input in industrial production. Second, we introduce land as a second factor of production in the agricultural sector. As we will show, our previous conclusions remain unaffected. However, these extensions provide some further insights that are consistent with stylised facts.

6.1 Unskilled Labour as Industrial Input Factor

Assume now that unskilled labour is not only used in the agricultural sector as in the basic model, but it also enters industrial production. More precisely, the output of firm $k$ of region $i$’s industrial sector is now given by

$$y_i^k = A(H_i) \min (h_i^k, \frac{1}{\alpha_i} l_i^{k}), \quad (18)$$

where $l_i^k$ denotes the number of unskilled workers employed by firm $k$ of region $i$.

The modified production function (18) reflects the fact that skilled and unskilled workers are complementary inputs in manufacturing. For instance, team assistants cannot replace engineers, but the more engineers there are, the more team assistants are needed. Consequently, profit maximising firms hire $\alpha$ unskilled workers per skilled worker, leading to a regional demand for unskilled industrial labour of $L_i = \int_0^1 l_i^k dk = \alpha H_i$. Then, the urban population of region $i$, which is now formed of skilled and unskilled industrial workers, amounts to $H_i + L_i = (1 + \alpha) H_i$, causing congestion costs $C ((1 + \alpha) H_i) = \lambda f ((1 + \alpha) H_i)$.

In the benchmark case of non-integrated labour markets, unskilled workers can work in the agricultural sector and earn an income of $w$, or they can work in the industrial sector. With perfect intersectoral mobility, unskilled workers are indifferent between the two choices in equilibrium, implying that unskilled industrial workers are exactly compensated for the congestion costs in the urban area. Thus, the unskilled wage in the industrial sector is

$$v_i = w + C ((1 + \alpha) H_i). \quad (19)$$

In the case of integrated labour markets, mobile unskilled individuals can also migrate and work in the other region. In equilibrium, they are indifferent between working in either sector in either region. Again, the unskilled wage is given by $w$ in the agricultural sector.

\[^{15}\text{In this and many other examples, the term ‘unskilled’ has to be interpreted as ‘relatively unskilled compared to other, more skilled workers’. In this vein, most of the studies surveyed by Hamermesh (1993) find that unskilled (blue collar) and skilled (white collar) workers are complements in production.}\]
sector and by (19) in the industrial sector. In line with empirical evidence, wages are higher in urban areas than in rural areas; i.e., $v_i > w$. They are also higher in bigger agglomeration centres than in smaller urban areas; i.e., $v_1 > v_2$ if $H_1 > H_2$.\footnote{For empirical evidence of this ‘urban wage premium’, see, e.g., Glaeser and Maré (2001).}

Perfect competition in the labour market implies that the regional skilled wage $w_i$ is equal to the marginal contribution of a skilled worker to profits, given that the share of skilled to unskilled workers is optimally chosen by each firm in the industrial sector. The resulting inverse aggregate demand for skilled labour in region $i$ is

$$w_i = A(H_i) - \alpha v_i = A(H_i) - \alpha [w + C((1 + \alpha)H_i)],$$  

yielding the regional skill premium

$$p_i = A(H_i) - (1 + \alpha) [w + C((1 + \alpha)H_i)].$$  

In contrast to the basic model (see (3)), the skilled wage will now decline with the number of skilled workers if their number exceeds a threshold level $H^\text{crit}$ (cf. (20)). As the population in the urban area grows, attracting the complementary unskilled workers becomes more and more expensive, which initially curbs and ultimately depresses the skilled wage.

Despite this difference between the basic and the extended model, the relationship between the skill premium and the number of skilled workers remains qualitatively unaltered, as comparing the skill premia (4) and (21) shows. Likewise, the supply of skilled labour is also qualitatively unaffected by the extension and still given by (5), (7) and (8). Thus, the previous propositions also correctly characterise the equilibria and the implications of integration on human capital and the (net) skill premia in this extended version of the model.

However, we can now provide an additional proposition about the migration pattern. To this aim, let $N$ denote the number of unskilled workers who migrate from the losing to the winning region. Moreover, let us confine our attention to ‘interior’ equilibria, that is, to equilibria in which at least some unskilled individuals still work in the agricultural sector.

**Proposition 8 Migration Patterns**

*In an agglomeration equilibrium of type 1 or 2, the share of migrants among skilled workers native to the losing region strictly exceeds the share of migrants among unskilled workers native to the losing region, i.e., $M/S_2 > N/(1 - S_2)$. This inequality will also be satisfied in an agglomeration equilibrium of type 3 if (i) vacant unskilled positions in the industry are first filled with natives and then with potential immigrants and (ii)*
unskilled workers in the agricultural sector prefer to work in their native region as long as wages are identical.

Recall that, in the losing region, the share of mobile natives who become skilled is greater than the corresponding share of immobile natives, simply because the skill premium in the winning region exceeds the one at home. This ‘overrepresentation’ of skilled workers among the mobile population then translates into a relatively higher migration rate of skilled individuals compared to unskilled ones. This outcome is in line with international migration data, which shows that in most countries the migration rates of high-skilled workers are higher than the corresponding rates of less skilled individuals.\(^\text{17}\)

### 6.2 Land as Production Factor in the Agricultural Sector

Finally, we briefly discuss how introducing land as a second input in the agricultural sector affects our results. Assume that land and unskilled labour are substitutes in agricultural production. To get a grasp of the implications of this extension, consider first an agglomeration equilibrium of type 1 in which labour market integration increases total human capital. Then, if mobility is sufficiently high, unskilled workers become scarcer in both regions and their wages rise. Consequently, most unskilled workers benefit from integration along with skilled workers in the winning region. Only some of those unskilled workers in the losing region who would have become skilled under non-integrated labour markets lose out, as do the skilled workers in the losing region. Also, the rise in unskilled wages curbs the increase in total human capital. If mobility is rather low, matters are more complicated. Then, it is possible that labour market integration raises the number of unskilled workers and lowers unskilled wages in the losing region, as fewer natives become skilled.

If total human capital declines in response to integration, which is certainly the case in an agglomeration equilibrium of type 2 or 3, then our previous arguments can be reversed. As unskilled labour tends to be more abundant, unskilled wages are likely to fall, and the unskilled workers join the ranks of those who lose out from integration.

\(^{17}\)For empirical evidence, see, e.g., Docquier and Marfouk (2006). They establish the emigration rates of low-, medium- and high-skilled individuals for a wide range of countries. In 2000, for instance, the emigration rates of high skilled natives exceed those of the other skill groups in 38 out of 44 analysed European countries.
7 Concluding Remarks

In the era of globalisation, it has become increasingly important to assess the consequences of economic integration. In this paper, we have focused on the impact of labour market integration on human capital formation and wage income. We have shown that integration and the ensuing agglomeration give rise to a complex picture. Whether labour market integration generates overall benefits and fosters human capital accumulation at the aggregate level depends very much on circumstances such as the costs of congestion and the mobility of the population. The political prospects of fine-tuning labour market integration might be limited. For instance, even if political decisions could affect mobility, it would be far from clear what degree of mobility would be optimal. Low, intermediate or high mobility levels could maximise the total net skill premium, depending on the strength of the agglomeration and congestion effects. This ambiguity cautions against simple conclusions and policy recommendations.

Appendix A

Proof of Proposition 1:

Let us define the excess inverse demand for skilled labour in region $i$ as $\Gamma(H_i) := B(H_i) - \bar{\pi}H_i$, which is a continuous function. In equilibrium, $\Gamma(H_i^*) = 0$ must hold. As $\Gamma(0) = B(0) = A(0) - \bar{\pi}C(0) > 0$ (recall that $A(0) > \bar{\pi}$ and $C(0) = 0$) and $\Gamma(1) = B(1) - \bar{\pi} < 0$ (recall that $B(H_{max}) < \bar{\pi}$), there exists at least one $H_i^* \in (0,1)$ such that $\Gamma(H_i^*) = 0$ and $\Gamma'(H_i^*) = B'(H_i) - \bar{\pi} < 0$. This follows from the intermediate value theorem. Moreover, this equilibrium value is unique, since $\Gamma''(H_i^*) = B''(H_i) = A''(H_i) - C''(H_i) < 0$ (recall that $A''(H_i) \leq 0$ and $C''(H_i) > 0$) and thus $\Gamma(H_i^*) < 0$ for all $H_i > H_i^*$. See figure 1a. (This equilibrium is also stable.)

Proof of Proposition 2:

Step 1 (labour market equilibrium): Let us now define the excess inverse demand functions for skilled labour in regions 1 and 2 as $\Gamma_1(H_1) := B(H_1) - [\bar{\pi}/(1 + m\gamma)]H_1$ and $\Gamma_2(H_2) := B(H_2) - [\bar{\pi}/(1 - m\gamma)]H_2$, which are continuous functions. For any given bundle $(m; \gamma)$, there exists a unique $\widetilde{H}_1(m; \gamma)$ such that $\Gamma_1(\widetilde{H}_1(m; \gamma)) = 0$, and a unique $\widetilde{H}_2(m; \gamma)$ such that $\Gamma_2(\widetilde{H}_2(m; \gamma)) = 0$. This follows from applying the line of reasoning in the proof of proposition 1 with minor amendments. For $\widetilde{H}_1(m; \gamma)$ and $\widetilde{H}_2(m; \gamma)$, the labour markets in the two regions are cleared. Denote the corresponding skill premia by $\tilde{p}_1(m; \gamma) = B(\widetilde{H}_1(m; \gamma))$ and $\tilde{p}_2(m; \gamma) = B(\widetilde{H}_2(m; \gamma))$, and the corresponding inter-regional premium differential by $\Delta\tilde{p}(m; \gamma) := \tilde{p}_1(m; \gamma) - \tilde{p}_2(m; \gamma)$.

Step 2 (implication of agglomeration for human capital): Note that $\Gamma_1(\widetilde{H}_1(m; \gamma)) =$
In this scenario, for all \( p_e \) and \( \Gamma_2(\tilde{H}_1(m;\gamma)) = 0 \) imply that \( d\tilde{H}_1/dm > 0 \) and \( d\tilde{H}_2/dm < 0 \). Furthermore, \( \tilde{H}_1(0;\gamma) = \tilde{H}_2(0;\gamma) = H_1^* \). This relationship and the derivatives \( d\tilde{H}_1/dm > 0 \) and \( d\tilde{H}_2/dm < 0 \) imply that \( \tilde{H}_2(m;\gamma) < H_1^* < \tilde{H}_1(m;\gamma) \) for all \( m \in (0, 1] \).

Step 3 (implication of agglomeration for skill premia): Note that \( \tilde{p}_1(0;\gamma) = \tilde{p}_2(0;\gamma) = w^*_1 \), \( \tilde{p}_1(0;\gamma) = B'(\tilde{H}_1) \frac{d\tilde{H}_1}{dm} \geq 0 \Leftrightarrow \tilde{H}_1(m;\gamma) \leq H^{max} \), and \( \tilde{p}_2(0;\gamma) = B'(\tilde{H}_2) \frac{d\tilde{H}_2}{dm} < 0 \). These relationships have two implications: First, \( \tilde{p}_2(m;\gamma) < p_i^* \) for all \( m \in (0, 1] \). Second, either (a) \( \tilde{p}_1(m;\gamma) > p_i^* \) for all \( m \in (0, 1] \) or (b) \( \tilde{p}_1(m;\gamma) \geq p_i^* \iff m \leq \tilde{m} \). In the latter case, there might exist a migration level \( m^{crit} \in (\tilde{m}, 1] \) such that \( \tilde{p}_2(m^{crit};\gamma) = \tilde{p}_1(m^{crit};\gamma) \).

Step 4 (agglomeration equilibria): In an agglomeration equilibrium, either (a) \( m^{**} = 1 \) and \( \Delta \tilde{p}_1(1;\gamma) \geq 0 \) or (b) \( m^{**} < 1 \) and \( \Delta \tilde{p}_1(m^{**};\gamma) = 0 \). Let us combine these equilibrium conditions with the results of the steps 1, 2, and 3. Then, only three different types of agglomeration equilibria can emerge:

First, if \( \tilde{p}_1(1;\gamma) > w^*_1 \), then \( m^{**} = 1 \), \( H_2^{**} = \tilde{H}_2(1;\gamma) < H_i^* < H_1^{**} = \tilde{H}_1(1;\gamma) \) and \( p_2^{**} = \tilde{p}_2(1;\gamma) < p_i^* < p_1^{**} = \tilde{p}_1(1;\gamma) \) constitute an agglomeration equilibrium of type 1.

Second, if \( \tilde{p}_2(1;\gamma) < \tilde{p}_1(1;\gamma) \leq p_i^* \), then \( m^{**} = 1 \), \( H_2^{**} = \tilde{H}_2(1;\gamma) < H_i^* < H_1^{**} = \tilde{H}_1(1;\gamma) \) and \( p_2^{**} = \tilde{p}_2(1;\gamma) < p_1^{**} = \tilde{p}_1(1;\gamma) \leq p_i^* \) constitute an agglomeration equilibrium of type 2.

Third, if \( \tilde{p}_2(m^{crit};\gamma) = \tilde{p}_1(m^{crit};\gamma) \) for some \( m^{crit} \in (\tilde{m}, 1] \), then \( m^{**} = m^{crit} \), \( H_2^{**} = \tilde{H}_2(m^{crit};\gamma) < H_i^* < H_1^{**} = \tilde{H}_1(m^{crit};\gamma) \) and \( p_2^{**} = \tilde{p}_2(m^{crit};\gamma) < p_1^{**} = \tilde{p}_1(m^{crit};\gamma) \leq p_i^* \) constitute an agglomeration equilibrium of type 3.

In section 5, we show examples of agglomeration equilibria of type 1 and 2. An agglomeration equilibrium of type 3 will emerge if the increasing part of the \( B(H) \)-function is very flat, the decreasing part is very steep, and each region’s number of skilled workers in the case of non-integrated markets is already close to \( H^{max} \).

**Proof of Proposition 3:**

In the proof of proposition 2, step 3, we show that either (a) \( \tilde{p}_1(m;\gamma) > p_i^* \) for all \( m \in (0, 1] \) or (b) \( \tilde{p}_1(m;\gamma) \geq p_i^* \iff m \leq \tilde{m} \), with \( \tilde{m} < 1 \). (See proof of proposition 2 for all definitions.) Applying the same line of reasoning leads to either (aa) \( \tilde{p}_1(1;\gamma) > p_i^* \) for all \( \gamma \in (0, 1) \) or (bb) \( \tilde{p}_1(1;\gamma) \geq p_i^* \iff \gamma \leq \tilde{\gamma} \). (i) Weak congestion pattern: Consider the case (aa) \( \tilde{p}_1(1;\gamma) > p_i^* \) for all \( \gamma \in (0, 1) \). In this scenario, \( m^{**} = 1 \), \( H_2^{**} = \tilde{H}_2(1;\gamma) < H_i^* < H_1^{**} = \tilde{H}_1(1;\gamma) \), and \( p_2^{**} = \tilde{p}_2(1;\gamma) < p_i^* < p_1^{**} = \tilde{p}_1(1;\gamma) \) constitute an agglomeration equilibrium of type 1 for all \( \gamma \in (0, 1) \) (cf. proposition 2). There is only one agglomeration equilibrium of type 1 because \( m^{**} = 1 \) yields unique levels of \( H_1^{**} \), \( H_2^{**} \), \( p_i^{**} \), and \( p_2^{**} \) for any given \( \gamma \). Moreover, if \( \tilde{p}_1(1;\gamma) > p_i^* \), then, too, \( \tilde{p}_1(m;\gamma) > p_i^* \) for all \( m \in (0, 1) \). That is, \( \tilde{p}_1(m;\gamma) \leq p_i^* \) is not possible, and thus any agglomeration equilibrium of type 2 or 3 is excluded.
The case (aa) certainly prevails for \( \lambda = 0 \). Then, \( B(H_i) > B(H_i^*) \) for all \( H_i > H_i^* \) and thus, as \( \tilde{H}_1(1; \gamma) > H_i^* \), \( B(\tilde{H}_1(1; \gamma)) = \tilde{p}_1(1; \gamma) > p_i^* = B(H_i^*) \) for all \( \gamma \in (0, 1) \). Using continuity arguments, we can conclude that the case (aa) also emerges for values of \( \lambda \) sufficiently close to zero. This guarantees the existence of a threshold level \( \lambda \).

(ii) Strong congestion pattern: Consider the case (bb) \( \tilde{p}_1(1; \gamma) \geq p_i^* \Leftrightarrow \gamma \leq \tilde{\gamma} \). As \( \tilde{p}_1(1; \gamma) > p_i^* \) for all \( \gamma \in (0, \tilde{\gamma}) \), we can again apply the arguments of part (i), yielding that a single agglomeration equilibrium of type 1, and none of type 2 or 3, exists for \( \gamma \in (0, \tilde{\gamma}) \). However, as \( \tilde{p}_1(1; \gamma) \leq p_i^* \) for all \( \gamma \in [\tilde{\gamma}, 1) \), no agglomeration equilibrium of type 1 can exist for any \( \gamma \in [\tilde{\gamma}, 1) \). There remain two possibilities.

First, \( \tilde{p}_1(1; \gamma) > \tilde{p}_2(1; \gamma) \) holds for all \( \gamma \in [\tilde{\gamma}, 1) \). In this case, \( m^{**} = 1 \), \( H_2^{**} = \tilde{H}_2(1; \gamma) < H_i^* < H_1^{**} = \tilde{H}_1(1; \gamma) \), and \( p_2^{**} = \tilde{p}_2(1; \gamma) < p_i^* = \tilde{p}_1(1; \gamma) \leq p_i^* \) constitute an agglomeration equilibrium of type 2 (cf. proposition 2). There is only one agglomeration equilibrium of type 2 because \( m^{**} = 1 \) yields again unique levels of \( H_1^{**}, H_2^{**}, p_1^{**}, \) and \( p_2^{**} \) for any given \( \gamma \).

Second, \( \tilde{p}_1(m^{\text{crit}}; \gamma) = \tilde{p}_2(m^{\text{crit}}; \gamma) \) holds for some \( m^{\text{crit}} \in (\tilde{\gamma}, 1] \) and \( \gamma \in [\tilde{\gamma}, 1) \) (see proof of proposition 2 for definitions). In this case, the corresponding \( m^{**} = m^{\text{crit}} \), \( H_2^{**} = \tilde{H}_2(m^{\text{crit}}; \gamma) < H_i^* < H_1^{**} = \tilde{H}_1(m^{\text{crit}}; \gamma) \), and \( p_2^{**} = \tilde{p}_2(m^{\text{crit}}; \gamma) = p_1^{**} = \tilde{p}_1(m^{\text{crit}}; \gamma) < p_i^* \) constitute an agglomeration equilibrium of type 3 (cf. proposition 2). As \( m^{\text{crit}} \) is not necessarily uniquely determined in this case, multiple equilibria of type 3 can exist, and they can co-exist with an agglomeration equilibrium of type 2.

Finally, we prove the existence of a threshold \( \overline{\lambda} \). If \( \lambda = \lambda^{\text{max}} \), then \( B(H_i) < B(H_i^*) \) for all \( H_i > H_i^* \), as \( H_i^* = H_i^{\text{max}} \) for \( \lambda = \lambda^{\text{max}} \) (see section 3 and footnote 4). Thus, as \( \tilde{H}_1(1; \gamma) > H_i^* = H_i^{\text{max}} \), \( B(\tilde{H}_1(1; \gamma)) = \tilde{p}_1(1; \gamma) < p_i^* = B(H_i^*) \) for all \( \gamma \in (0, 1) \). By definition of \( \lambda^{\text{max}} \) (see section 3 and footnote 4), and due to continuity arguments, the following will be true if \( \lambda \) is sufficiently close to \( \lambda^{\text{max}} \): there exists a threshold level \( \overline{\gamma} \) such that (i) \( \tilde{p}_1(1; \gamma) \geq p_i^* \) is now satisfied for \( \gamma \in (0, \overline{\gamma}) \) and (ii) \( \tilde{p}_1(1; \gamma) < p_i^* \) still holds for \( \gamma \in (\overline{\gamma}, 1) \). That is, the case (bb) exists for values of \( \lambda \) that are sufficiently close to \( \lambda^{\text{max}} \), guaranteeing the existence of a threshold level \( \overline{\lambda} \).

**Proof of Proposition 4:**

To prove proposition 4, we only have to show that no agglomeration equilibrium of type 3 exists. The other conclusions of proposition 4 follow directly from proposition 3.

Recall that the inter-regional premium differential \( \Delta \tilde{p}(m; \gamma) \) is positive for sufficiently small \( m \). Thus, if there exists an agglomeration equilibrium of type 3, where \( \Delta \tilde{p}(m^{\text{crit}}; \gamma) = 0 \), then \( d[\Delta \tilde{p}(m^{\text{crit}}; \gamma)]/dm = d\tilde{p}_1(m^{\text{crit}}; \gamma)/dm - d\tilde{p}_2(m^{\text{crit}}; \gamma)/dm \leq 0 \) must hold for at least one \( m^{\text{crit}} \). Consequently, if we can show that \( -d\tilde{p}_1(m; \gamma)/dm < -d\tilde{p}_2(m; \gamma)/dm \) for all \( m \) such that \( \tilde{p}_1(m; \gamma) = \tilde{p}_2(m; \gamma) \), then an agglomeration equilibrium of type 3 is
impossible:

\[-\frac{dp_1}{dm} < -\frac{dp_2}{dm} \Leftrightarrow -B'(H_1) \frac{dH_1}{dm} < -B'(H_2) \frac{dH_2}{dm}\]

\[\Leftrightarrow \frac{\gamma}{(1 + m\gamma)} \frac{-\varepsilon(H_1)}{[1 - \varepsilon(H_1)]} B(H_1) < \frac{\gamma}{(1 - m\gamma)} \frac{\varepsilon(H_2)}{[1 - \varepsilon(H_2)]} B(H_2)\]  

(22)

\[\Leftrightarrow -B'(H_1) B(H_1) - B'(H_2) B(H_2) < -B'(H_1) B(H_1) \varepsilon(H_2) - B'(H_2) B(H_2) \varepsilon(H_1),\]

where \(B'(H_i) = A'(H_i) - C'(H_i)\) and \(\varepsilon(H_i) = B'(H_i) H_i / B(H_i)\). To rearrange the inequalities, we have used the comparative statics results

\[\frac{dH_1}{dm} = \frac{1}{(1 + m\gamma)} \frac{\varepsilon(H_1)}{[1 - \varepsilon(H_1)]} B(H_1) / B'(H_1)\]

and

\[\frac{dH_2}{dm} = \frac{1}{(1 - m\gamma)} \frac{-\varepsilon(H_2)}{[1 - \varepsilon(H_2)]} B(H_2) / B'(H_2),\]

(23)

which follow from \(\Gamma_1(H_1 (m; \gamma)) = 0\) and \(\Gamma_2(H_2 (m; \gamma)) = 0\) (see proof of proposition 2).

Also, \(\Gamma_1(H_1 (m; \gamma)) = 0\) and \(\Gamma_2(H_2 (m; \gamma)) = 0\) have been used to derive the third line of (22).

In the case of a symmetric \(B(H)\)-function, \(p_1 = B(H_1) = B(H_2) = p_2\) implies that \(-B'(H_1) = B'(H_2)\). Thus, the left-hand side of the third line of (22) is zero. In contrast, the right-hand side is positive, since \(B(H_1) > 0\), \(B'(H_1) < 0\), \(\varepsilon(H_2) > 0\), \(B'(H_2) > 0\), and \(\varepsilon(H_1) < 0\). Thus, inequality (22) is fulfilled, and an agglomeration equilibrium of type 3 is impossible.

**Proof of Lemma 1:**

The total net skill premium is given by

\[NP_{1+2} = \left[ p_1 H_1 - \frac{\bar{\tau}}{2(1 + m\gamma)} H_1^2 \right] + \left[ p_2 H_2 - \frac{\bar{\tau}}{2(1 - m\gamma)} H_2^2 \right]\]

\[= \frac{1}{2} B(H_1) H_1 + \frac{1}{2} B(H_2) H_2 = \frac{1}{2} P_{1+2},\]

(24)

where we used the equilibrium conditions \(B(H_1) = \bar{\tau} H_1 / (1 + m\gamma)\) and \(B(H_2) = \bar{\tau} H_2 / (1 - m\gamma)\) (see (9) and (10)).

**Proof of Proposition 5:**

As \(NP_i = (1/2) P_i\) (see lemma 1), we can phrase the proof in terms of the total skill premium \(P_i\) instead of the total net skill premium \(NP_i\).

Part (a): Assume that an agglomeration equilibrium of type 1 materialises, i.e., \(m^{**} = 1\).

(i) First, we show that \(H_{1+2}^{**} > H_{1+2}^*\) is possible. To this end, we consider the boundary case where \(B(H_i) = A(H_i) - [w + C(H_i)] = d + \delta H_i\) for \(H \in [\underline{H}, \overline{H}]\), \(\underline{H} < H_{2}^{**}, \overline{H} > H_{1}^{**}\), and \(\bar{\tau} > d + 2\delta\) (see figure 4a). Then, \(H_{1+2}^* = 2d / (\bar{\tau} - \delta), H_{1}^{**} = [(1 + \gamma) d] / [\bar{\tau} - (1 + \gamma) \delta],\) and \(H_{2}^{**} = [(1 - \gamma) d] / [\bar{\tau} - (1 - \gamma) \delta]\), implying that \(H_{1+2}^{**} >\)
$H_{i+2}^* \iff \gamma^2 \delta \epsilon > 0$. Finally, we can slightly manipulate $A(H_i)$ or $C(H_i)$ such that $B(H_i)$ is strictly concave and still yields $H_{i+2}^{**} > H_{i+2}^*$.

Second, we show that $H_{i+2}^{**} \geq H_{i+2}^* \Rightarrow P_{i+2}^{**} > P_{i+2}^*$. Note that

$$P_{i+2}^{**} = (1 + \gamma) \left( \frac{(p_{i+2}^*)^2}{\epsilon} + (1 - \gamma) \left( \frac{p_{i+2}^*}{\epsilon} \right)^2 \right)$$

$$> (1 + \gamma) \left( \frac{(p_{i+2}^*)^2}{2\epsilon} + (1 - \gamma) \left( \frac{p_{i+2}^*}{\epsilon} \right)^2 \right) + (1 + \gamma) (1 - \gamma) \frac{p_{i+2}^* p_{i+2}^*}{\epsilon} = \frac{\epsilon}{2} (H_{i+2}^{**})^2$$

$$\iff (p_{i+2}^{**} - p_{i+2}^*)^2 > 0$$

Thus, $P_{i+2}^{**} > \left[ \frac{\epsilon}{2} (H_{i+2}^{**})^2 \right] / 2$. We also know that $H_{i+2}^{**} \geq H_{i+2}^* \iff \left[ \frac{\epsilon}{2} (H_{i+2}^{**})^2 \right] / 2 \geq \left[ \epsilon (H_{i+2}^{**})^2 \right] / 2 = 2 (p_i^*)^2 / \epsilon = P_{i+2}^*$. To sum up, $P_{i+2}^{**} > \left[ \frac{\epsilon}{2} (H_{i+2}^{**})^2 \right] / 2$ and, if $H_{i+2}^{**} \geq H_{i+2}^*$, we have $H_{i+2}^{**} \geq H_{i+2}^* \Rightarrow P_{i+2}^{**} > P_{i+2}^*$.

(iii) First, the logical relationship $H_{i+2}^{**} \geq H_{i+2}^* \Rightarrow P_{i+2}^{**} > P_{i+2}^*$ implies $P_{i+2}^{**} < H_{i+2}^*$, Second, we show that $P_{i+2}^{**} < P_{i+2}^*$ is possible. Comparing equilibrium values yields $p_i^{**} > p_i^* > p_{i+2}^{**}$. Thus we can write $p_i^{**} = p_i^* + \epsilon$ and $p_{i+2}^{**} = p_i^* - \eta$, with $\epsilon > 0$ and $0 < \eta < p_i^*$. By choosing the components $A(H_i)$ and $C(H_i)$ of $B(H_i)$ appropriately, $\epsilon$ can be infinitesimally small. Moreover, $H_i^{**} > (1 + \gamma)H_i$, $\lim_{\epsilon \to 0} H_i^{**} = (1 + \gamma)H_i$ and $H_{i+2}^{**} < (1 - \gamma)H_i$ follow from (6), (9), and (10). Thus, $\lim_{\epsilon \to 0} H_i^{**} = (1 + \gamma) p_i^* H_i$ and $p_{i+2}^{**} H_{i+2}^{**} < (1 - \gamma) (p_i^* - \eta) H_i^*$, implying that $\lim_{\epsilon \to 0} P_{i+2}^{**} < 2p_i^* H_i^* - (1 - \gamma) \eta H_i^* < 2p_i^* H_i^* = P_{i+2}^*$.

(ii) By choosing the components $A(H_i)$ and $C(H_i)$ of $B(H_i)$ appropriately, we have $P_{i+2}^{**} = P_{i+2}^*$, implying that $H_{i+2}^{**} < H_{i+2}^*$. But then, we can slightly manipulate $A(H_i)$ or $C(H_i)$ such that $p_i^{** \text{new}} = p_i^{**} + \mu$ (with an infinitesimally small $\mu$), $P_{i+2}^{** \text{new}} > P_{i+2}^{**} = P_{i+2}^*$, and still $H_{i+2}^{** \text{new}} < H_{i+2}^*$.

Part (b): Assume that an agglomeration equilibrium of type 2 or 3 materialises. Then, $p_2^{**} < p_1^*$ and $p_1^{**} < p_1^*$ (see proposition 2(ii) and (iii)). Hence, $S_2^{**} < S_2^*$ and $S_1^{**} \leq S_1^*$, implying $H_{i+2}^{**} < H_{i+2}^*$. Since both skill premia and total human capital decline after integration, $P_{i+2}^{**} \leq P_{i+2}^*$ must hold.

**Proof of Proposition 6:**

(i) As $NP_{i+2} = (1/2)P_{i+2}$ (see lemma 1), we can phrase the proof in terms of the total skill premium $NP_{i+2}$ instead of the total net skill premium $NP_{i+2}$. As $p_i^{**} > p_i^*$, we can write $p_i^{**} = p_i^* + \epsilon$, $\epsilon > 0$. Jointly with $H_i^{**} > (1 + \gamma)H_i$, this leads to $P_{i+2}^{**} > (1 + \gamma) (p_i^* + \epsilon) H_i$. Also, $(1 + \gamma) (p_i^* + \epsilon) H_i > 2p_i^* H_i \iff \gamma > (p_i^* - \epsilon) / (p_i^* + \epsilon)$. As $P_{i+2}^{**} = 2p_i^* H_i$ and $(p_i^* - \epsilon) / (p_i^* + \epsilon) < 1$, we can conclude that for all $\epsilon > 0$, $\exists \gamma < 1 : P_{i+2}^{**} > P_{i+2}^*$ for all $\gamma \geq \gamma$.

(ii) The second part can be proved by ‘reversing’ the arguments of part (i). Moreover, note that $H_{i+2}^{**} < H_{i+2}^*$ and $NP_{i+2}^{**} < NP_{i+2}^*$ must certainly be true if $\gamma \in [\gamma, 1)$, i.e., if
an agglomeration equilibrium of type 2 or 3 materialises (see proposition 3 (ii)). Due to continuity, \( H_{1+2}^* < H_{1+2}^* \) and \( NP_{1+2}^* < NP_{1+2}^* \) must then also hold for some \( \gamma \) which is slightly smaller than \( \hat{\gamma} \). Thus, \( \hat{\gamma} < \hat{\gamma} \).

**Proof of Proposition 7:**

Recall that for sufficiently small \( \gamma \), an agglomeration equilibrium of type 1 materialises (see proposition 3). Using this result, \( NP_{1+2}^*(\gamma) \) can be expressed as a function of \( \gamma \). Also, \( NP_{1+2}^*(0) = NP_{1+2}^* \) holds in the (hypothetical) case of \( \gamma = 0 \). As a consequence, if we can prove that either (i) \( dNP_{1+2}^*(\gamma)/d\gamma \big|_{\gamma=0} < 0 \) or (ii) \( dNP_{1+2}^*(\gamma)/d\gamma \big|_{\gamma=0} = 0 \) and \( d^2NP_{1+2}^*(\gamma)/d\gamma^2 \big|_{\gamma=0} < 0 \), then \( NP_{1+2}^*(\gamma) < NP_{1+2}^* \) for sufficiently small \( \gamma \). After some rearrangements, we find that \( dNP_{1+2}^*(\gamma)/d\gamma \big|_{\gamma=0} < 0 \) if and only if

\[
[B(H_{1}^*)]^2 \frac{B(H_{1}^*) + B'(H_{1}^*)H_{1}^*}{B(H_{1}^*)} \leq [B(H_{2}^*)]^2 \frac{B(H_{2}^*) + B'(H_{2}^*)H_{2}^*}{B(H_{2}^*)} - \frac{B_H(H_{1}^*)H_{1}^*}{B(H_{1}^*)}.
\]  

(26)

Let us denote the term on the left-hand side by \( \Phi(H_{1}^*(\gamma)) \) and the term on the right-hand side by \( \Phi(H_{2}^*(\gamma)) \). For \( \gamma = 0 \), \( \Phi(H_{1}^*(\gamma)) = \Phi(H_{2}^*(\gamma)) \) because \( H_{1}^*(\gamma) = H_{2}^*(\gamma) \). Thus, \( dNP_{1+2}^*(\gamma)/d\gamma \big|_{\gamma=0} = 0 \). Consequently, we have to prove that \( d^2NP_{1+2}^*(\gamma)/d\gamma^2 \big|_{\gamma=0} < 0 \).

It turns out that \( d^2NP_{1+2}^*(\gamma)/d\gamma^2 \big|_{\gamma=0} < 0 \) if and only if

\[
\left. \frac{d\Phi(H_{1}^*)}{dH_{1}^*} \right|_{\gamma=0} < 0 \Leftrightarrow \frac{d\Phi(H_{1}^*)}{dH_{1}^*} < 0
\]

\[\Leftrightarrow [2 + \varepsilon(H_{1}^*)] [1 - \varepsilon(H_{1}^*)] \leq -\frac{B_H(H_{1}^*)H_{1}^*}{B'(H_{1}^*)} = -\eta(H_{1}^*),\]

(27)

where we made use of \( H_{1}^*(0) = H_{1}^* \) and \( \varepsilon(H_{1}) = B'(H_{1})H_{1}/B(H_{1}) \). The final inequality (27) is the condition in proposition 7. This condition ensures that a critical value \( \gamma_{\text{crit}} \) exists such that \( NP_{1+2}^*(\gamma) < NP_{1+2}^* \) for \( \gamma \in (0, \gamma_{\text{crit}}) \). From proposition 5, part (a.iii), it follows then that \( H_{1+2}^*(\gamma) < H_{1+2}^* \) for \( \gamma \in (0, \gamma_{\text{crit}}) \).

**Proof of Proposition 8:**

First, consider an agglomeration equilibrium of type 1 or 2 (i.e., \( m^* = 1 \) and \( p_{1}^* > p_{2}^* \)). The maximum number of unskilled migrants \( \overline{N} \) equals the number of mobile natives of region 2, \( \gamma \), minus the number of skilled migrants, \( M \), i.e., \( \overline{N} = \gamma - M = \gamma(1 - p_{1})/\bar{\tau}, \) where we used \( M = \gamma p_{1}/\bar{\tau} \). Using \( N = \gamma(1 - p_{1})/\bar{\tau}, \) \( M = \gamma p_{1}/\bar{\tau} \) and \( S_{2} = (\gamma p_{1} + (1 - \gamma) p_{2})/\bar{\tau} \) yields \( M/S_{2} > \overline{N}/(1 - S_{2}) \). This proves the first part of proposition 8, since \( p_{1} > p_{2} \) is obviously satisfied.

Next, consider an agglomeration equilibrium of type 3 (i.e., \( m^* < 1 \) and \( p_{1}^* = p_{2}^* \)). Under the assumption that vacant unskilled positions are first filled with natives and then with potential immigrants, the number of unskilled immigrants who work in the industrial sector of the winning region is given by \( N = \alpha H_{1} - (1 - S_{1}) = \)

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\[ \alpha (1 + m \gamma) p + p - \bar{\tau} \] (i.e., demand for unskilled workers in region 1 minus domestic supply), where we used \( H_1 = (1 + m \gamma) p_1 / \bar{\tau} \) and \( S_1 = p_1 / \bar{\tau} \). Using the expressions for \( N, H_1 \) and \( S_1 \) as well as \( S_2 = p_2 / \bar{\tau} \) leads to \( M / S_2 \geq \bar{N} / (1 - S_2) \Leftrightarrow \bar{\tau} \geq (1 + \alpha) p_2 \). The latter inequality is fulfilled in the case of an ‘interior’ equilibrium (i.e., an equilibrium in which some unskilled individuals still work in the agricultural sector). In this case, the overall supply of unskilled workers has to exceed the total demand for unskilled workers in the industrial sector; that is, \( 2 - S_1 - S_2 > \alpha H_{1+2} = \alpha (S_1 + S_2) \) or, equivalently, \( \bar{\tau} > (1 + \alpha) p_2 \). This proves the second part of proposition 8.

**Appendix B**

As argued in section 4.4, the equilibrium in the case of non-integrated markets is also an equilibrium in the case of integrated markets in addition to the agglomeration equilibria (and at least one agglomeration equilibrium exists, as proposition 2 states). The following lemma considers the stability of these equilibria. More precisely, we consider a distortion of the equilibrium migration \( m^{**} \) and analyse the stability of the equilibrium for given numbers of mobile and immobile skilled workers.

**Lemma 2** (i) With integrated labour markets, a non-agglomeration equilibrium with \( m^{**} = 0 \) and \( H_1^{**} = H_2^{**} = H_1^{*} \) exists but is unstable. (ii) An agglomeration equilibrium of type 1 or 2, if it exists, is locally stable. Similarly, at least one equilibrium of type 3, if it exists and none of type 2, is locally stable.

**Proof of Lemma 2:**

(i) The equilibrium conditions are fulfilled for \( m^{**} = 0, p_2^{**} = \bar{p}_2 (0; \gamma) = \bar{p}_1^{**} = \bar{p}_1 (0; \gamma) = p_1^{*} \) and \( H_2^{**} = \bar{H}_2 (0; \gamma) = H_1^{**} = \bar{H}_1 (0; \gamma) = H_1^{*} \) (in particular, the equilibrium conditions \( m^{**} < 1 \) and \( \Delta \bar{p} (m^{**}; \gamma) = 0 \); see proof of proposition 2 for all definitions). However, an equilibrium is locally stable if and only if either (a) \( m^{**} = 1 \) and \( \Delta \bar{p} (m^{**}; \gamma) > 0 \) or (b) \( m^{**} < 1 \) and \( d [\Delta \bar{p} (m^{**}; \gamma)] / dm < 0 \). As \( d [\Delta \bar{p} (0; \gamma)] / dm = d \bar{p}_1 (0; \gamma) / dm - d \bar{p}_2 (0; \gamma) / dm > 0 \), the non-agglomeration equilibrium is unstable.

(ii) If an agglomeration equilibrium of type 1 or 2 materialises, then the stability conditions \( m^{**} = 1 \) and \( \Delta \bar{p} (m^{**}; \gamma) > 0 \) are fulfilled. These types of equilibrium are locally stable. Next, assume that at least one agglomeration equilibrium of type 3, and none of type 2, exists. (Recall that an agglomeration equilibrium of type 1 can never co-exist with an equilibrium of type 3, as proposition 3 implies.) Then, \( \Delta \bar{p} (1; \gamma) < 0 \) and, as \( \Delta \bar{p} (m; \gamma) > 0 \) for sufficiently small \( m \), at least one critical value \( m^{crit} \) exists such that \( \bar{p}_2 (m^{crit}; \gamma) = \bar{p}_1 (m^{crit}; \gamma) \) and \( d [\Delta \bar{p} (m^{crit}; \gamma)] / dm < 0 \). In this case, there is at least one locally stable agglomeration equilibrium of type 3 with \( m^{**} = m^{crit} \).

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References


