HOW DO EXCHANGE RATES MOVE FOLLOWING AN EXPANSIONARY MONETARY POLICY?

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This paper makes inquiries into the adjustment and evolution of the exchange rate towards its new long-run equilibrium level following a change in money supply. A pattern in exchange rate adjustments and movements has emerged that is featured by reverse movements of the exchange rate in the short-term, which is largely regulated by interest rate parities, prior to its eventual convergence to the new long-run equilibrium, which is confined to the working of PPP. Calibrations of the model with actual exchange rate data are performed in the study. Effects of recent quantitative easing exercises fit the featured pattern delicately well.

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Key words: exchange rate; monetary policy; interest rate parity; purchasing power parity; sticky price

1. INTRODUCTION

Foreign exchange is an important instrument in international finance and trade. Determination of exchange rates and understanding of exchange rate behavior are therefore crucial to an orderly international financial market to facilitate international economic co-operations. Its importance can partly be reflected by the scope and coverage of the statistics and surveys that the Bank for International Settlements compiles and carries. Global foreign exchange market activity is one of the two parts in its Triennial Central Bank Survey, alongside derivatives market activity, due to their massive turnovers that move the financial world. While facilitating international trade and related activities, disturbances generated in one part of this interlinked global economy can also be transmitted and magnified through foreign exchange as one of the direct and major channels to impact on the financial markets around the world. The significance of foreign exchange to national and global economies won't be confined to the imaginations of human beings.

Major models of exchange rate determination include Mundell-Fleming (*cf.* Mundell 1960, 1961, 1962, 1963, 1964; Fleming 1962, 1971), the monetary models of Frenkel (1976) and Dornbusch (1976), the real interest rate differential model of Frankel (1979), and the portfolio balance approach by Branson (1976), with follow-ups of Branson *et al.* (1977) and Branson and Henderson (1985). The Dornbusch (1976) model is one of the two monetary models of exchange rate determination that assumes sticky prices. That is, prices adjust to the effect of the new and changed economic fundamentals slowly, or it takes time for the price to settle down at the new level dictated by the new economic fundamentals. The Dornbusch (1976) model differs from the Frenkel (1976) model and its prior Mundell-Fleming model in that the price in the latter two models is either fully flexible or totally fixed. Due to its realistic assumptions about price adjustment processes, the Dornbusch (1976) model has attracted much attention in the field of exchange rate determination and adjustment.

One of the most famous features of the Dornbusch (1976) model, which is identified as a consequence of the sticky price assumption, is overshooting of the exchange rate in its adjustment process towards the new equilibrium pertinent to the new and changed economic fundamentals. Due to its prominence and influence, the overshooting proposition has been tested empirically over time. Overshooting in the Dornbusch model is observed by incorporating uncovered interest rate parity (UIRP) into the demand for money function, in conjunction with a kind of exchange rate expectations formation. As the new long-run equilibrium is not attainable immediately and in the short-term, targeting the new long-run equilibrium, as in Dornbusch (1976), in exchange rate adjustment in the short-term does not seem rational. If the agents foresee a transition path towards the new long-run equilibrium, adjustments based on the transition path may be reasonable and realistic in a competitive foreign exchange market. With simplicity, the present paper proposes that, there is a new long-run equilibrium exchange rate, which is determined by the eventual and total price change owing to a change in money supply and confined to purchasing power parity (PPP) in the long-run. In addition to that, there is a new short-term exchange rate target, which is mostly imposed by interest rate parities. While the exchange rate converges to the new long-run equilibrium exchange rate eventually in the long-run, it moves towards and around a short-term exchange rate target immediately after the change in money supply and in the short-term. The transition path thereby derived would then demonstrate a pattern that reflects the evolving effect of the two international parities as time passes by.

The rest of the paper is organized as follows. The next section provides a brief review of empirical studies on Dornbusch's overshooting model. Section 3 presents the construct and paradigm of this study, following notional remarks on the controversies cast by the overshooting model. Section 4 provides several illustrating cases and calibrates the model, while Section 5 summarizes this study.

2. A BRIEF REVIEW OF THE EMPIRICAL LITERATURE IN OVERSHOOTING

Following Driskill (1981), Mussa (1982) also offers support to the Dornbusch model, while working on exchange rate dynamics in the early days of the recent float. His study can produce the results of the Dornbusch model almost exactly if the exogenous monetary factor is assumed to be observable and follows a random walk and expectations about the behavior of this factor is assumed to be formed and revised rationally. In Mussa's model, the exchange rate also overshoots in response to monetary shocks, when the domestic price of goods is sticky. The unexpected change in the actual exchange rate upon shocks, in response to new information about the monetary factor, is always greater than the unexpected change in the long-run equilibrium exchange rate.

However in Obstfeld and Rogoff (1995), the exchange rate, upon an unexpected permanent increase in money stock in the domestic country, jumps immediately to its new post-shock long-run equilibrium level despite the inability of the price to adjust in the short-term, in contrast to the exchange rate behaviour predicted by the Dornbusch model. They, inheriting much of the Mundell-Fleming-Dornbusch approach to foreign exchange rate behavior, develop an inter-temporal model of international policy transmission incorporating short-term nominal price rigidities and microeconomic foundations of aggregate supply. The model offers direct welfare implications of international monetary and macroeconomic policies and institutions. Regarding exchange rate dynamics, it produces an equation almost identical to that of the flexible monetary model of exchange rates despite that the sticky nominal price is assumed, when the output differential is replaced by the consumption differential in the equation. Since all shocks have permanent effects on the difference of domestic and foreign per capita consumption, real interest rates have the same effect on domestic and foreign consumption growth, relative consumptions follow a random walk whatever patterns the consumptions in the two countries possess, which removes the sticky price effect so the model behaves as though the model worked under a flexible price environment. It follows that the exchange rate, upon an unexpected permanent increase in money stock in the domestic country, jumps immediately to its new post-shock long-run equilibrium level despite the inability of the price to adjust in the short-term. It is because both consumption differentials and money supply differentials follow random walks, implying a one-jump to the right level feature in any adjustment processes. In other words, since consumption differentials and money supply differentials are expected to be constant, the exchange rate is expected to be constant – it adjusts only once and stops making any further adjustments after the money supply shock. Chang and Lai (1997) have found that whether the domestic currency will depreciate or appreciate following a balanced-budget fiscal expansion is sensitive to plausible specification in the money demand function. In addition, it is also found that the mis-adjustment pattern of exchange rates can be observed in response to a balanced fiscal expansion, even if the system displays the saddle point stability rather than the global instability.

Proposing a real interest rate differential model as an alternative to the flexible price monetary model and Dornbusch's sticky price monetary model, findings in Frankel (1979) reject both models based on the results of coefficient restrictions. Following Frankel (1979), the results in Meese and Rogoff (1988) also fail to lend support to the functional relationship between real exchange rates and real interest rate differentials implied by the Dornbusch and Frankel models. They develop a modified version of the models by Dornbusch (1976), Frankel (1979) and others to investigate the relationship between real exchange rates and real interest rate differentials between the US, Germany, Japan and the UK. Adopting the assumptions of Dornbusch (1976) and Frankel (1979) including the sticky price assumption, they derive a function for real exchange rates and real interest rate differentials that real exchange rates are negatively associated with real interest rate differentials and changes in real exchange rates are less than proportionate to changes in real interest rate differentials. Then they use the derived functional relationship implied by the Dornbusch and Frankel models in empirical investigations by regressing real exchange rates on real interest rate differentials. Although the coefficient on the real interest rate differential variable is negative and smaller than one in its absolute value, the coefficient is statistically insignificant in all the three exchange rates of the German mark, the Japanese yen and the British pound vis-à-vis the US dollar. Further, they find little evidence of a long-run relationship between the real exchange rate and the real interest rate differential. Based on these results that fail to support the model and contradict the joint hypothesis that the domestic price is sticky and monetary disturbances are predominant, they claim that real disturbances may be a source of exchange rate volatility. Tu and Feng (2009) claim that exchange rate overshooting is not an intrinsic characteristic of the foreign exchange market and that it depends on a set of specific assumptions. As long as the interest elasticity of demand for money is infinity, the adjustment of the exchange rate in short-term is equal to that in the long-run. In addition, the exchange rate undershoots its long-run value with imperfect capital mobility. They conclude that the exchange rate overshooting hypothesis has proven of relatively limited value empirically.

A few of more recent studies are application based. Hwang (2003) employs the Dornbusch model and Frankel's real interest rate differential model to forecast the exchange rate between the US dollar and the Canadian dollar and claims that the random walk model outperforms the Dornbusch-Frankel model at every forecasting horizon. Zita and Cupta (2008) investigate the performance of the Dornbusch model for the monthly nominal exchange rate between the Mozambican metical and the South African rand over the period 1994-2005. Compared with a range of naïve and classical models for out-of-sample forecast errors, the fitted sticky price model performs unfavorably. Verschoor and Wolff (2001) investigate expectations formation using the Mexican peso vis-à-vis the US dollar exchange rate and a survey dataset containing market participants' forecasts of the exchange rate and of the interest differential between the peso and the dollar. Their findings suggest that the survey expectations were in the wrong by a large and significant constant. Regarding the expectations formation mechanism, market participants tended to react to the current unanticipated depreciation by expecting future depreciation at the 3-, 6-, and 12-month horizons, and the overshooting effect was present in the Mexican data. However, "news" about the interest differential could be replaced by a risk premium term. Bjørnland (2009) imposes a long-run neutrality restriction on the real exchange rate, thereby allowing for contemporaneous interaction between the interest rate and the exchange rate. He finds that a contractionary monetary policy shock has a strong effect on the exchange rate. The domestic currency appreciates on impact but it takes 1-2 quarters for the effect to maximize. Kim (2001) examines the international transmission of US monetary policy shocks in a VAR model. The paper claims that US expansionary monetary policy shocks lead to booms in the other G-7 countries. In this transmission, changes in trade balance seem to play a minor role while a decrease in the world real interest rate seems more important. On the other hand, US expansionary monetary policy shocks worsen the US trade balance in about a year, but the trade balance subsequently improves. The results suggest that the basic versions of the Mundell-Flemming-Dornbusch framework and the sticky price inter-temporal model are not found to be consistent with the details of the transmission mechanism, and appropriate modification and extensions are necessary to fit the data.

The extent to which exchange rates overshoot or possibly undershoot has been under scrutiny. Examining permanent and transitory components in real exchange rates, Cavaglia (1991) demonstrates that rational agents will price the real exchange rate as the sum of three unobservable components of future expected one period real interest differentials, future expected risk premia, and the infinite horizon forecast of the real exchange rate. The findings of the paper contrast with the exchange rate overshooting hypothesis, since its empirical analysis for three currencies over the 1979-1987 period shows that exchange rate risk premia represent a large transitory and short lived component of real exchange rate movements, but expected real interest differentials only represent a relatively small component. Akiba (1996) investigates the rebalancing effect on exchange rate overshooting, accommodating different degrees of capital mobility and emphasising gradual adjustment of commodity prices so that purchasing power parity holds as a long-run proposition. The paper demonstrates that the rebalancing effect unambiguously reduces exchange rate volatility regardless of the degree of capital mobility, by reducing the extent of exchange rate overshooting. Levin (1994) develops a model of exchange rate dynamics that incorporates sluggish output adjustment. In this more complex system, a monetary expansion initially lowers interest rates because of sluggish output adjustment but can still produce either overshooting or undershooting of the exchange rate as in the basic Dornbusch model. In an earlier study, Levin (1989) introduces output adjustment lags and trade flow lags into the Dornbusch model in order to analyze the dynamic effects of monetary and fiscal policy under floating exchange rates. Despite the fact that a monetary expansion at first leads to a reduced interest rate, the exchange rate may undershoot its new long-run equilibrium level. In addition, a fiscal expansion always causes the exchange rate to overshoot its new long-run equilibrium level. Finally, the system converges with oscillations if the output and trade-flow lags are relatively long. Kiguel and Dauhajre (1988) also incorporate sluggish output adjustment into the Dornbusch model and consider the cases of expansionary and contractionary real depreciation of the currency. They show that the exchange rate is likely to overshoot in both cases. It is claimed that when the real depreciation is expansionary, the model generates comovements in prices and output that are opposite to those presented by Dornbusch (1976). In the contractionary depreciation case, there is a unique stable equilibrium path and the economy is likely to experience cycles in output and the real exchange rate. Natividad-Carlos (1994) considers intervention, imperfect capital substitution, and sluggish aggregate demand in the framework of the Dornbusch model. The paper imposes perfect foresight directly in solving the model. The results show that intervention may eliminate overshooting arising from monetary expansions but may only dampen overshooting resulting from fiscal expansions.

It can be observed that a substantial share in the literature does not lend support to the overshooting model. Many studies show that the overshooting model compares unfavorably to other models of exchange rate determination and forecast, with all the initial intention to back the model and verify it empirically. In the next section, it can be shown that the disappointment in finding support for the overshooting model in empirical studies is due to the controversy cast by the model: while the sticky price rises over time and increasingly noticeably in the long-run, the exchange rate decreases, or the domestic currency appreciates over time. The exchange rate does not have to overshoot as stated in Dornbusch (1976); it has to if the exchange rate has to approach its new long-run equilibrium level in an appreciating fashion for the domestic currency following a monetary expansion, which is implausible. However, regardless the initial responses of the exchange rate, the exchange rate should approach the new long-run equilibrium level in a fashion of gradual depreciation, instead of appreciation, of the domestic currency, in the long-run. With many empirical studies having found no favorable results for the overshooting model already, this paper makes an effort to illuminate the exchange rate adjustment mechanism under the influence of pertinent parities of international finance. It is not the intention of this paper to add another piece of empirical evidence.

Regardless of whether it supports or rejects the overshooting model, it won't change much the balance in the empirical literature. What we lack are studies that scrutinize the reasons behind the failure of the overshooting model in the real world, alongside the workings of the foreign exchange market that conform to international parities under the assumed circumstances. This study is an attempt for achieving this ultimate purpose.

3. EXCHANGE RATE ADJUSTMENTS IN THE SHORT-TERM AND THE LONG-RUN

This section first brings in a simplified version of the overshooting model. Remarks are then made on the controversies cast by the overshooting model, which offers notional explanations to those disapproving results in the above reviewed empirical literature. Departing from the overshooting model, the final part in this section develops a construct and paradigm based on the proposition of the paper that the exchange rate, following a change in money supply, moves towards a new short-term exchange rate target in the short-term and converges to a new long-run equilibrium exchange rate in the long-run.

3.1. *The making of overshooting*

UIRP states that the expected change in the foreign exchange rate is equal to the interest rate differential between the domestic country and the foreign country. In the case of a small open economy, the expected change in the foreign exchange rate is equal to the difference between the prevailing domestic interest rate and its long-run equilibrium rate – the world's interest rate:

$$E\left(\frac{de_t}{dt}\right) = r_t - r^* \qquad (1)$$

where e_t is the exchange rate in logarithms, r_t is the domestic interest rate, and r^* is the long-run equilibrium interest rate where a time subscript is not relevant. Exchange rate expectations are formed in the following way:

$$E\left(\frac{de_t}{dt}\right) = \theta(\bar{e} - e_t) \tag{2}$$

where $\bar{e}$ is the long-run exchange rate and $\theta > 0$ is a coefficient. Equation (2) states that the expected change in the exchange rate follows a dynamic adjustment process that the exchange rate will revert to its long-run rate, with the speed of adjustment being decided by the value of θ. The domestic currency is expected to depreciate when the exchange rate is below its long-run level and is expected to appreciate when it is above its long-run level. The adjustment is swift with θ is large and slow when θ is small. Combining equation (1) with equation (2) leads to:

$$r_t = r^* + \theta(\bar{e} - e_t) \tag{3}$$

The demand for money equation is the standard version given below:

$$m_t - p_t = \eta y_t - \lambda r_t \tag{4}$$

in which m_t is demand for money, p_t is price of goods, y_t is income, and $\varphi > 0$ and $\lambda > 0$ are coefficients Inserting equation (3) into equation (4) yields:

$$m_t - p_t = \eta y_t - \lambda r^* - \lambda\theta(\bar{e} - e_t) \tag{5}$$

Assume that the system is in equilibrium at $t = 0$, with $p_0 = \bar{p}$, $e_0 = \bar{e}$, $y_0 = \bar{y}$ and $r_0 = r^*$. Assume also there is an increase in money supply dm at $t = 0$. Then, equation (6a) is for the system prior to the increase of money supply and equation (6b) represents the system upon the increase of money supply:

$$m_0 - p_0 = \eta y_0 - \lambda r^* \tag{6a}$$

$$m_0 + dm - p_{0^+} = \eta y_{0^+} - \lambda r^* - \lambda\theta(\bar{e}_n - e_{0^+}) \tag{6b}$$

which corresponds to $m_0 + dm - p_{0^+} = \eta y_{0^+} - \lambda r_{0^+}$ where $r_{0^+} = r^* + \theta(\bar{e}_n - e_{0^+})$. Since the price is fixed in the short-term and output is not supposed to be affected, subtracting equation (6a) from equation (6b) yields:

$$dm = -\lambda\theta(\bar{e}_n - e_{0^+}) = -\lambda\theta(\bar{e}_n - e_0) + \lambda\theta(e_{0^+} - e_0) \qquad (7)$$

where $\bar{e}_n$ is the new long-run equilibrium exchange rate to be established consistent with the new quantity of money supply. Since $r_{0^+} = r^* + \theta(\bar{e}_n - e_{0^+})$, this implies that $r_{0^+} = r^* - \dfrac{dm}{\lambda}$. Dornbusch (1976) argues rightfully that the increase in the long-run equilibrium exchange rate is equal to the increase of money supply. Equation (7) can be re-arranged as follows:

$$e_{0^+} = e_0 + \left(\bar{e}_n - e_0\right) + \frac{dm}{\lambda\theta} = e_0 + dm + \frac{dm}{\lambda\theta} = e_0 + \left(1 + \frac{1}{\lambda\theta}\right)dm > e_0 + dm \qquad (8)$$

Equation (8) demonstrates exchange rate overshooting, i.e., depreciation in the exchange rate is greater than what is justified by the amount of increase in money supply.

3.2. *Remarks on overshooting*

Having produced the overshooting effect, the Dornbusch model generates the evolution path for the price and the exchange rate, in a continuous-time fashion, as follows (equations 12 and 13, Dornbusch 1976, page 1165):

$$p_t = \bar{p} + (p_{0^+} - \bar{p})e^{-\nu t} \qquad (9)$$

$$e_t = \bar{e} + (e_{0^+} - \bar{e})e^{-\nu t} \qquad (10)$$

where $\nu > 0$ is a parameter for the speed of movements towards the new equilibrium. For the price equation, p_{0^+} is almost p_0 since the price is sticky and, is therefore lower than $\vec{p}$; but for e_{0^+}, it is the exchange rate upon overshooting in the Dornbusch model, so e_{0^+} is greater than $\bar{e}$. This means that while the sticky price rises over time and increasingly noticeably in the long-run, the exchange rate decreases, or the domestic currency appreciates over time, which is implausible. It is this implausibility that leads to the non-negligible disappointment in finding support for the overshooting model in empirical studies. The reasons why the overshooting model casts controversies in empirical studies can be partially explained by Dornbusch's own account:

> *"At the initial level of prices, the monetary expansion reduces interest rates and leads to the anticipation of a depreciation (of the domestic currency) in the long-run and, therefore, at the current exchange rate, to the expectation of a depreciating exchange rate (domestic currency). Both factors serve to reduce the attractiveness of domestic assets, lead to an incipient capital outflow, and thus cause the spot rate to depreciate. The extent of that depreciation has to be sufficient to give rise to the anticipation of appreciation at just sufficient a rate to offset the reduced domestic interest rate. The impact effect of a monetary expansion is, therefore, to induce an immediate depreciation in the spot rate and one that exceeds the long-run depreciation, since only under these circumstances will the public anticipate an appreciating exchange rate and thus be compensated for reduced interest on domestic assets."* (Dornbusch 1976, page 1168)

It is clear that the long-run in the above paragraph is not long enough. The period ends before the price, which is sticky, starts to rise noticeably to erode effectively the purchasing power of the domestic currency. The appreciation of the domestic currency is a relatively short phase in the entire process of exchange rate adjustment and movement. The exchange rate does not have to overshoot when the exchange rate approaches the new long-run equilibrium rate in a depreciating manner in the later and final phase of exchange rate adjustment and movement. If the exchange rate ever overshoots, the appreciating process would not end when the exchange rate has touched the new equilibrium from the above. The domestic currency would continue to appreciate and the exchange rate would continue to move down to a point below the new equilibrium, which allows the domestic currency to depreciate in response to a rising domestic price in the real long-run. A decreasing exchange rate (appreciating domestic currency) should not be the end of story of this monetary expansion. With the progressively rising domestic price, there should be a period in which the domestic currency is depreciating. This takes place when the domestic price, which is sticky and does not increase noticeably initially, starts to rise considerably, proportional to the monetary expansion, and erodes the domestic currency's purchasing power eventually.

3.3. *Rethinking on interest rate parities of interest rate changes*

According to interest rate parities, a currency would depreciate vis-à-vis the other currency if the former is associated with a higher interest rate in the economy than the latter. Notwithstanding, the effect of an interest rate change can be perplexing, on the exchange rate "now" and "future". We use UIRP for example to illustrate. We inspect how the exchange rate may adjust to interest rate changes, with UIRP being upheld always:

$$E_0\{e_t\} - e_0 = r_{0,t} - r^* \tag{11}$$

where $E_0\{e_t\}$ is the current expectations of the logarithmic exchange rate at *t*, e_0 is the currently prevailing logarithmic spot exchange rate, $r_{0,t}$ is the domestic interest rate offered at time 0 for a loan in the domestic currency that matures at time *t*, and r^* is the interest rate in the foreign country. r^*, as a benchmark, does not have a time subscript and is assumed constant. The domestic currency is the numerator currency in all denominations. It is a common reaction on the currency market that an increase in the interest rate in the domestic country boosts the domestic currency that the domestic currency appreciates or the exchange rate decreases; and that a reduction in the interest rate in the domestic country prompts the domestic currency to depreciate or the exchange rate increases. We consider the latter case to coincide with the monetary expansion effect in Dornbusch (1976). The depreciation due to a lowered domestic interest rate, however, applies to the current spot exchange rate at the time of an interest rate reduction. The domestic currency would strengthen over time and, the exchange rate would return to its original level if it is a pure monetary manoeuvre without any changes in growth, productivity, and so on. The exchange rate would decrease, or the domestic currency is expected to appreciate in the future – the doctrine indicated by UIRP. There can be a number of responses to an interest rate change of the current exchange rate and the expected future exchange rate.

The following demonstrates the analysis and effect of an interest rate reduction. Equation (11) upholds in the following ways with the domestic interest rate being reduced:

$$E_0\{e_t\} - e_{0^+} \uparrow = r_{0,t} \downarrow - r^* \tag{11a}$$

$$E_0\{e_t\}\downarrow - e_{0^+} = r_{0,t} \downarrow - r^* \tag{11b}$$

$$E_0\{e_t\}\downarrow - e_{0^+} \uparrow = r_{0,t} \downarrow - r^* \tag{11c}$$

Equation (11a) is a case where a reduction in the domestic interest rate weakens the domestic currency immediately and the effect is entirely on the current exchange rate. Whereas the expected future exchange rate remains unchanged, it represents a kind of expected appreciation relative to the newly adjusted exchange rate. Equation (11b) is a case where a decrease in the domestic interest rate fails to make the domestic currency depreciate right now and the effect is entirely on the expectations about the future exchange rate, which is expected to decrease, or the domestic currency would appreciate in the following period. This is an indication that a currency cannot be weakened and sustained by monetary manoeuvres. With equation (11c), the effect is felt on, and shared by, both the current exchange rate and the expected future exchange rate. The domestic currency depreciates to a smaller extent than that in equation (11a) right now, and it is expected to appreciate to a smaller extent than that in equation (11b) in the following period. If the effect on the current exchange rate is beyond what indicated by equation (11a), overshooting seems to show up.

$$E_0\{e_t\}\downarrow - e_{0^+}\uparrow = r_{0,t}\downarrow - r^* \tag{11d}$$

The reaction represented by equation (11c) is most commonly observed in the real world, followed by those of equation (11a) and equation (11b). Nevertheless, all the above adjustment patterns in the current exchange rate upon an interest rate change and the expected exchange rate movement that follows are short-term adjustments largely confined to interest rate parities. How short is this adjustment period? It depends on the strength of influence asserted by interest rate parities but, regardless, it is much shorter than the period in which the sticky price has eventually increased by almost the same amount of the increase in money supply.

3.4. *Exchange rate adjustments in the short-term and in the long-run*

The analysis below departs from that of Dornbusch (1976). Instead of targeting the new long-run equilibrium exchange rate immediately upon the increase of money supply and all along subsequently, there is a transition period in exchange rate adjustments. The exchange rate, following a change in money supply, moves towards a new short-term exchange rate target, $\bar{e}_{n1}$[1], which is mostly confined to interest rate parities, in the short-term. The exchange rate then converges to a new long-run equilibrium exchange rate, $\bar{e}_{n2}$, in the long-run. A weighting function is applied so the weight allocated to the short-

[1] See equation (17) for its value.

term adjustment process is gradually decreasing over time and the weight allocated to the long-run adjustment process is gradually increasing over time:

$$E\left(\frac{de_t}{dt}\right) = e^{-\varkappa t}\theta(\bar{e}_{n1} - e_t) + \left(1 - e^{-\varkappa t}\right)\theta(\bar{e}_{n2} - e_t) \tag{12}$$

where $e^{-\kappa t}$ is the weight allocated to the short-term adjustment and $\left(1 - e^{-\varkappa t}\right)$ is that allocated to the long-run movement. A larger $\varkappa$ signifies a faster diminishing influence of interest rate parities and a less sticky price, and vice versa. So the weight allocation to the short-term adjustment and the long-run movement in the exchange rate depends on the strength of interest rate parities and stickiness of the price. It turns into the expression of equation (2) in Dornbusch (1976, page 1163) when t becomes sufficiently large; and the larger the parameter κ, the faster the exchange rate converges to its new long-run equilibrium level.

The demand for money equation is the standard version given by equation (4). Incorporating the analysis in Sub-section 2.3, we set three periods or phases for the immediate response, short-term reversion and long-run movement. For immediate responses, e_{0^+} would be in the ranges suggested in Sub-section 2.3 and typically some kind of immediate depreciation of the domestic currency. The domestic interest rate, corresponding to the monetary expansion, is reduced by:

$$r_{0^+} - r_0 = -\frac{dm}{\lambda} \tag{13}$$

according to equation (4). The short-term begins with e_{0^+} to endure the interest rate parity effect, and lasts until the effect confined to interest rate parities diminishes to be negligible. The domestic interest rate is reverting to its equilibrium level, the level of the world interest rate[2], and the appreciation of the domestic currency becomes smaller and

[2] This is a typical small country assumption, which can be relaxed. For a large country, the interest rate in the rest of the world, or part of the rest of the world that is closely linked to this large country, would move towards to its interest rate.

smaller as the interest rate differential becomes narrower and narrower. The long-run is the period when the sticky price rises noticeably eventually until the price increase reaches the full of the monetary expansion.

Given the upon impact exchange rate e_{0^+}, the domestic currency is expected to appreciate due to interest rate parity effects. If, for example, the domestic interest rate is to remain at this level for one year, and then reverts to the level of the world interest rate, the amount of appreciation would be $\frac{dm}{\lambda}$; if the domestic interest rate is to remain at this level for half a year, and then reverts to the level of the world interest rate, the amount of appreciation would be $\frac{dm}{2\lambda}$. Let the domestic interest rate now assume a function form with which it rises gradually in reverting to the world level of interest rate:

$$r_t = r^* - \frac{dm}{\lambda} e^{-\varphi t} \tag{14}$$

where φ determines the speed of interest rate reversal. The domestic interest rate would have almost reverted to that of the world level in one year with a φ of 2.5 ($e^{-2.5} = 0.0821$) and in half a year with a φ of 5. The interest rate differential at time t is:

$$r_t - r^* = -\frac{dm}{\lambda} e^{-\varphi t} \tag{14'}$$

Accumulated appreciation of the domestic currency at time t is then:

$$E_{0^+}(e_t) - e_{0^+} = \int_{\tau=0^+}^{t} -\frac{dm}{\lambda} e^{-\varphi\tau} d\tau = \frac{dm}{\lambda\varphi}\left(e^{-\varphi t} - 1\right) \tag{15}$$

and total expected appreciation owing to interest rate differentials is given by:

$$E_{0^+}\left(de^{ird}\right) = \int_{\tau=0^+}^{\infty} -\frac{dm}{\lambda} e^{-\varphi\tau} d\tau = \frac{dm}{\lambda\varphi} e^{-\varphi\tau}\Big|_{0^+}^{\infty} = -\frac{dm}{\lambda\varphi} \tag{16}$$

The short-term target is therefore the exchange rate upon the initial response plus the total appreciation due to the working of interest rate parities:

$$\bar{e}_{n1} = e_{0^+} - \frac{dm}{\lambda\varphi} \tag{17}$$

The new long-run equilibrium level of the exchange rate, by definition, is:

$$\bar{e}_{n2} = (\bar{e} + dm) = (e_0 + dm) \tag{18}$$

Replacing $\bar{e}_{n1}$ and $\bar{e}_{n2}$ in equation (12) by their values in equation (17) and equation (18) leads to:

$$E\left(\frac{de_t}{dt}\right) = e^{-\varkappa t}\theta\left(e_{0^+} - \frac{dm}{\lambda\varphi} - e_t\right) + \left(1 - e^{-\varkappa t}\right)\theta\left(e_0 + dm - e_t\right) \tag{19}$$

The general solution to the above equation is derived as[a]:

$$E(e_t) = \frac{\theta}{\varkappa - \theta} e^{-\varkappa t}\left(e_0 - e_{0^+} + dm + \frac{dm}{\lambda\varphi}\right) + e_0 + dm + Ce^{-\theta t} \tag{20}$$

Given the boundary or initial conditions, C is solved by the relationship:

$$e_{0^+} = \frac{\theta}{\varkappa - \theta}\left(e_0 - e_{0^+} + dm + \frac{dm}{\lambda\varphi}\right) + e_0 + dm + C \tag{21}$$

That is:

$$C = \frac{\varkappa}{\varkappa - \theta} e_{0^+} - \frac{\varkappa}{\varkappa - \theta}(e_0 + dm) - \frac{\theta}{\varkappa - \theta}\frac{dm}{\lambda\varphi} \tag{22}$$

Re-arrangements by combining equation (20) with equation (22) yield[b]:

$$\begin{aligned} E(e_t) = {} & \frac{\theta e^{-\varkappa t} - \varkappa e^{-\theta t}}{\theta - \varkappa} e_{0^+} + \frac{\theta\left(e^{-\varkappa t} - e^{-\theta t}\right)}{\theta - \varkappa}\left(-\frac{dm}{\lambda\varphi}\right) \\ & + \left(1 - \frac{\theta e^{-\varkappa t} - \varkappa e^{-\theta t}}{\theta - \varkappa}\right)(e_0 + dm) \end{aligned} \tag{23}$$

Notice that setting $\varkappa = 0$ in the above is a special situation for pure interest parity effects, which should be identical to equation (15). Given:

$$E(e_t)\big|_{\varkappa=0} = e_{0^+} + -\frac{dm}{\lambda\varphi}\left(e^{-\theta t} - 1\right) \tag{24}$$

it is inferred that φ= θ.

The features of the three terms are as follows. The first term on the right hand side of equation (23) is always positive, starting from e_{0^+} and gradually becoming zero. The second term is always negative except at $t = 0$. It starts with a value of zero, followed by gradual decreases and then gradual increases back to zero. That is, the domestic currency appreciates initially influenced by interest rate parities, and then the interest rate effect diminishes gradually when the domestic interest rate is converting to, and approaching, the world level of interest rate, while the PPP effect takes over. When the price is very sticky, i.e., $\varkappa$ is very small relative to θ, the second term is almost the interest rate parity effect of the exchange rate, $\frac{dm}{\lambda\theta}\left(1-e^{-\theta t}\right)$. The stickiness of the price plays a role also. The third term is always positive, starting from zero, and gradually increasing to $\left(e_0 + dm\right)$, the new long-run equilibrium level of the exchange rate at which the exchange rate has settled down. Figure 1 illustrates the evolution of the three terms on the right hand side of equation (23), which fits nicely to exchange rate movement and adjustment, after the monetary expansion, in the short-term and the long-run. Figure 1(a) shows that the initial reaction to the monetary expansion, some kind or degree of depreciation of the domestic currency, is gradually reduced to nil with time. The pre-expansion exchange rate is set to be one, so in logarithm it is zero. Figure 1(b) exhibits interest rate parity effects. Without growth or productivity changes, which are assumed for the case of a pure monetary exercise, the appreciation process of the domestic currency ought to stop and then revert, for the obvious reason that the appreciation of no currencies can be sustained by a pure monetary manoeuvre, without faster growth in the economy and greater increase in its productivity. Figure 1(c) demonstrates the evolution path to the long-run equilibrium exchange rate with a rising sticky price that gradually erodes the purchasing power of the domestic currency, with a 10 percent increase in money supply. A large $\varkappa$, or a less sticky price, means the price would rise faster and its erosion into the purchasing power of the domestic currency would take place sooner. The interest rate parity effect dominates in the earlier period, the second phase, and the PPP effect dominates in the later period, the third phase, of dynamic adjustment in the exchange rate. The relative sizes of $\varkappa$ and θ indicate the relative length of the period during which one effect is dominant over the other. In general $\varkappa$ is smaller than θ for the sake of sticky prices. The overall exchange rate adjustment and movement are displayed in Figure 2. The exchange rate in Figure 2(a) is logarithmic. Hence, Figure 2(b) is provided as an exponential version of Figure 2(a). That is, Figure 2(b) plots the exchange rate in its original form. With a pre-expansion exchange rate of one, the exchange rate depreciates to 1.02 upon the monetary expansion, and then it appreciates due to the interest rate parity effect. In its final phase of evolution, the exchange rate moves towards the new long-run equilibrium level of 1.10, confined to PPP with a 10 percent monetary expansion.

{Figure 1}

{Figure 2}

The following establishes an association between price adjustments and interest rate adjustments. A simple function reflecting the sticky feature of the price is also produced:

$$p_t = p_0 + \left(1 - e^{-\gamma t}\right)dm \tag{25}$$

Bringing the above price process and the interest rate process featured in equation (14) into equation (4) yields:

$$\begin{aligned} & m_t - \left[p_0 + \left(1 - e^{-\gamma t}\right)dm\right] \\ & = \varphi y_0 - \lambda r^* + dme^{-\varphi t} \end{aligned} \tag{26}$$

The system is in equilibrium at $t = 0$, with $p_0 = \bar{p}$, $e_0 = \bar{e}$ and $y_0 = \bar{y}$. So $m_0 - p_0 - \varphi y_0 + \lambda r^* = 0$ holds in equilibrium and equation (26) becomes:

$$dme^{-\gamma t} = dme^{-\varphi t} \tag{27}$$

Equation (27) indicates $\gamma = \varphi$, i.e., the speed of price increases is equal to that of interest rate increases, for equation (6) to hold while there is no change in output and there is no further change in money supply. Previously it has been inferred that $\varphi = \theta$. Therefore, the three speeds of adjustment parameters are equalized $\gamma = \varphi = \theta$.

{Figure 3}

Figure 3 demonstrates the shifting IS curves and LM curves in response to the monetary expansion. The LM curve is portrayed by equation (4), while the IS curve is represented by:

$$\beta y_t + \gamma r_t = g_t + hq_t \tag{28}$$

where q_t is the real exchange rate and g_t is government spending, and β, γ and h are positive coefficients. The real exchange rate, by definition, is:

$$q_t = e_t + p^* - p_t \tag{29}$$

where p^* is foreign price of goods. The LM curve shifts from its original position as at LM^{O} rightwards to a temporary position and becomes LM^{T} and the IS curve shifts from IS^{O} leftwards to IS^{T}, a move from the cross point of LM^{O} and IS^{O} to the cross point of LM^{T} and IS^{T} as illustrated in Figure 3(a), immediately following the monetary expansion, resulting in a lower domestic interest rate but no change in output[3]. The LM curve shifts gradually towards the left with a rising price, reverting to its original position LM^{O}, which is also the LM curve with the new equilibrium LM^{N}; whilst the interest rate keeps increasing alongside. Meanwhile, the IS curve shifts gradually towards the right, as well

[3] The indication in the graph is interpreted as one move, not two moves. The IS curve shifts simultaneously with the LM curve.

reverting to its original position IS^O, the same position for the new equilibrium IS^N. This process, moving (reverting) from the cross point of LM^T and IS^T to the cross point of LM^N and IS^N (cross point of LM^O and IS^O), is illustrated in Figure 3(b)[4]. Figure 3(c) combines the both processes.

Figure 4 extends the LM-IS analysis on the *y-r* plane to include the external sector of the economy on the *e-r* plane – the relationship between the exchange rate and the interest rate, with the given values for the rest variables. The *e-r* plane is arranged to the left of the *y-r* plane. The arrow of the horizontal axis indicates the direction of increase of the exchange rate. When mapping the LM curve onto the *e-r* plane, the curve is horizontal since the exchange rate is not a variable for the LM curve. The curve is named LX. For the IS curve on the *e-r* plane, equation (21) is re-arranged and equivalent to the following:

$$he_t - \gamma r_t = hp_t - hp^* + \beta y_t - g_t \tag{30}$$

The curve is named IX. Different from the closed IS-LM analysis, the IX curve is divided into two sections, one with the domestic interest rate being higher than the world interest rate r^*, and the other with the domestic interest rate being equal to and lower than the world interest rate. The IX curve is down-sloping; the curve is flatter when r_t is far away from r^* and steeper when r_t is closer to r^*. For $r_t - r^* < 0$, it means the larger the interest rate differential, the greater the decrease in the exchange rate, or the appreciation of the domestic currency; for $r_t - r^* > 0$, it means that the larger the interest rate differential, the greater the increase in the exchange rate, or the depreciation of the domestic currency.

[4] Similar to the previous note, the indication in the graph is interpreted as one move, not two moves. The IS curve shifts simultaneously with the LM curve.

{Figure 4}

The exchange rate depreciates and changes to e_{0^+} immediately following the monetary expansion, along the IX^O curve. It reverts to point T1 while the price has yet to rise noticeably, reflecting cumulative interest rate parity effects. With the sticky price rising slowly initially, the IX curve shift leftwards to IX^{T2}, and the exchange rate at T2 may increase or decrease further depending on the slope of the IX curve. The exchange rate finally settles down at point N. The horizontal distance between IX^O and IX^N is $\Delta p = \Delta m$, so is the overall depreciation of the domestic currency. The extended IS-LM analysis with an external sector of the economy as demonstrated by Figure 4 fits well with the derived exchange rate evolution process of equation (23), depicting the exchange rate adjustment and movement immediately upon a monetary expansion, and the aftermath in the short-term and the long-run.

4. ILLUSTRATING CASES AND CALIBRATION

The last financial crisis has provided us with the rare opportunities to inspect the patterns in exchange rate movements following an expansionary monetary policy. The intention to expand the monetary base in most economies during the crisis period was almost solely to prevent the economy from sliding into recession or, at best, to keep the economy as it was, with the outcome of virtually every intervention being just that. The scales of monetary policy intervention have been enormous and unprecedented. The policy tool adopted by most monetary authorities around the developed world is the most direct amongst the three major policy tools – large scale open market purchases of bonds and gilts or quantitative easing (QE). Unlike "conventional" monetary expansions where changes in a few other economic variables may influence exchange rates as much as money supply does, the effect of QE on exchange rates and exchange rate movements greatly dwarfs that of any other economic variables. For this reason, QE effectively isolates the impact of other economic variables on exchange rate movements from that of monetary expansions, offering an immaculate environment in which the effect of monetary expansions on exchange rate adjustment and movement is studied.

4.1. *Patterns in exchange rates movements of QE1 and QE2*

The recent two rounds of QE of the US are used for case analysis and illustrations. The first round of QE, QE1 started in December 2008 when the Federal Reserve announced it would purchase up to $100 billion in agency debt and up to $500 billion in agency mortgage-backed securities on November 25, 2008. Although the purchases spread over a period, that period was fairly short. The announcement effect would be also considerable, which Gagnon *et al.* (2010) scrutinize for QE1 in detail. Figure 5 and Figure 6 exhibit US dollar exchange rate movements for QE1 and QE2 respectively. The exchange rate used in the study is the US dollar effective exchange rate provided by the US Federal Reserve. The effective exchange rate is re-arranged so that an increase in it corresponds to the depreciation of the US dollar vis-à-vis the currencies of its trading partners, the same way as directly quoted bilateral exchange rates. Figure 5 exhibits US dollar effective exchange rate movements since the start of QE1 in a one-year frame, by which time the rest of the developed economies had also begun implementing their own QE programs and their asset purchases became sizeable. For example, the MPC of the UK announced a £75b asset purchase plan over a three-month period in March 2009; by the November MPC meeting asset purchases were extended to £200b (*cf.* Joyce *et al.* 2011 for the design and operation of QE in the UK). During this period, the ECB also adopted some kind of QE, albeit on a much smaller scale, including a €60b corporate bond purchase program made known in May 2009. Observing Figure 5 in conjunction with Figure 2, US dollar effective exchange rate movements in QE1 fit the theoretical curve delicately well. The US dollar effective exchange rate increased from an index number of around 118 at the beginning of December in 2008 to 129 by the middle of the month, causing 9 percent depreciation. Then the US dollar embarked on a reverse movement course and on March 9, 2009, the index decreased to less than 116, amounting to more than 11 percent of accumulated appreciation in nearly a quarter time period. Afterwards, the US dollar kept depreciating and by December 2009, the US dollar effective exchange rate reached 140. The US dollar depreciated by nearly 19 percent relative to its position a year ago, measured by its effective exchange rate. There are two periods when the exchange rate deviates from the evolution path on the theoretical curve. One is around the end of March 2009 when the Bank of England, following the announcement on March 5, purchased its first large chunk of corporate bonds, lasting for three weeks. This seems to be a counter effect of QE by other economies; the US dollar exchange rate stopped monotonic rising but fluctuated, for about three weeks. The second period is the end of May and June 2009, which coincided the move of the ECB and reflected another counter effect of QE in other economies. These deviations, contributed by other economies' QE, are fairly modest nonetheless.

{Figure 5}

{Figure 6}

QE2 was initiated in the fourth quarter of 2010. Starting in August 2010, the Federal Reserve started reinvesting principal payments from agency debt and agency mortgage-backed securities that it had acquired in QE1 in longer-term Treasury securities. On November 3, 2010, the Federal Reserve announced plans to purchase $600b of Treasury securities. So, QE2 officially started on November 3, 2010 but with a prelude two months. Figure 6 exhibits US dollar effective exchange rate movements for the period, with Figure 6(a) starting from the commencing date of QE2, November 3, 2010, and Figure 6(b) including the prelude two months. Once again, the effect of QE2 on US dollar effective exchange rate movements is observed to fit the theoretical curve practically perfectly. There was virtually no immediate depreciation of the US dollar at the beginning of QE2. However, the reverse movement of the exchange rate was evident. The US dollar effective exchange rate reversely moved from around 141 on November 4, 2010 to less than 134 by the end of the month; that is, the US dollar appreciated by nearly 5 percent in a matter of one month. Following the reverse movement period of the exchange rate, the US dollar steadily depreciated and by the time of the Greek sovereign debt crisis hitting the headline once again in early May 2011, the US dollar depreciated by almost 10 percent. The US dollar did not depreciate much in the months to come due primarily to the trouble in the euro area. The announcement effect and its impact on exchange rate movement also bear the features of monetary expansions interestingly. There is one disturbance that makes the exchange rate deviate from its original evolution path. On May 2, 2011, a three-year package worth €110b to rescue Greece was agreed and on May 19, 2011, the first funds were released, which caused sharp appreciation of the US dollar. Figure 6(c) shows US dollar effective exchange rate movements excluding the Greek debt crisis period.

It has been observed that the US dollar depreciated upon the expansion of money supply on all these occasions, but there were no signs of overshooting of exchange rates. While the initial depreciation was pulsating in QE1, that in QE2 was feeble for actual asset purchases as well as for the announcement. That being said, the initial reaction of exchange rates to an expansionary monetary policy is mostly incomprehensible to quantify, though initial depreciation has been broadly witnessed qualitatively. This kind of initial depreciation following an expansionary monetary policy is prevalent in the empirical literature, featured largely by undershooting of exchange rates, but alluded implicitly by the palpable failure to endorse exchange rate overshooting as well. Reverse movements of the exchange rate in the short-term, however, were manifest in both QE1 and QE2. The US dollar depreciated inevitably afterwards in the long-run and the exchange rate moved inevitably in a depreciating manner. The displayed patterns in US dollar effective exchange rate adjustments and movements mirror the theoretical analysis of this paper remarkably agreeably.

4.2. *Calibrations of the model with QE1 and QE2*

Given the remarkable matching between the theoretical curves of the model exhibited by Figure 2 and the patterns in real world exchange rate adjustments movements displayed by Figure 5 and Figure 6, this Sub-section presents the results of calibrations of the model parameters with the actual exchange rate data. Calibrations are performed by minimizing the standard deviation of the errors – the difference between the actual exchange rate data and the value produced by the model. Model parameters are accordingly obtained. Table 1 presents the calibrated parameters of the model, while Table 2 reports pertinent monetary data vis-à-vis changes in exchange rates and prices. Figure 7 and Figure 8 show the calibrated and actual exchange rate adjustments and movements for QE1 and QE2. We can observe from the graphs that the calibrated curves fit into the actual exchange rate data well.

{Table 1}

{Figure 7}

{Figure 8}

Turning to the calibrated parameters and inspecting QE1 first, it takes 131 business days, or about half a year, for the weight allocated to the long-run adjustment process to rise to 90 percent from zero, with a $\varkappa$ of 0.017572 ($1 - e^{-\varkappa t} = 1 - e^{-0.017572 \times 131} = 0.9$). By the time, the interest rate parity effect has diminished sufficiently and the PPP effect sets off to play a dominant role, and the currency depreciates substantially. Now Let us consider the interest rate parity effect and the PPP effect respectively. Reverse movement of the exchange rate due to UIRP, the second term on the right hand side of equation (17), reaches the lowest level in 57 days. Whereas the depreciating movement due to PPP, the third term on the right hand side of equation (17) rises to its 90 percent level in 221 days[5]. Observing Figure 7, the calibrated reverse movement of the exchange rate stops in 55 days while the actual reverse movement of the exchange rate ends in 62 days. The US money stock figures at the time are \$1,505b for monetary base, \$1,517b for M1 and \$8,058b for M2 in November 2008. Such created money as in QE1 is beyond the scope of M1 but still much smaller than M2. While monetary base has increased by 38 percent, M1 has increased by 12 percent and M2 by just under 6 percent between November 2008 and December 2009. $\lambda\varphi$ is jointly derived as 0.644444. Inagaki (2009) has estimated the interest rate semi elasticity in low interest rate environments and the figure is around -30 when the US Treasury bill rate is close to 1%. Given such interest rate semi elasticity and a 21% increase in money supply with the calibration, the initial drop in the interest rate would be 0.7% as per equation (7). φ is worked out as 0.021481 accordingly, with which the price evolution path can be projected using equation (18). Estimation in Table 2 indicates the price would have almost adjusted to the full by December 2009. The actual depreciation of the US dollar in terms of effective exchange rates is 17% in the period between November 2008 and December 2009, which is smaller than the increase in monetary base but greater than both increases in M1 and M2 in the same time period.

{Table 2}

[5] Detailed results are available upon request.

Exchange rate adjustments are swifter and more forceful in QE2. Reverse movement of the exchange rate, the second term on the right hand side of equation (17), reaches the lowest level in 29 days, compared with 57 days in QE1. The depreciating movement due to PPP, the third term on the right hand side of equation (17) rises to its 90 percent level in 114 days, compared with 221 days in QE1. Figure 8 shows that the calibrated reverse movement of the exchange rate stops in 24 day, and the actual reverse movement of the exchange rate reaches the bottom in 19 days. As reported in Table 2, between November 2010 and September 2011, monetary base has increased by 35 percent, M1 has increased by 20 percent and M2 by 9 percent. Given a figure of 0.304234 for $\lambda\varphi$, an estimate of φ is 0.010141. The price would have risen by 88% of the amount of increase in money supply. The actual depreciation of the US dollar in terms of effective exchange rates is just over 4% in this period, whereas increases in monetary base, M1 and M2 are as large as, or greater than, those in the QE1 period. Nonetheless, this period has witnessed various QE exercises not only in Europe and the developed world, but also in large emerging economies, so the net effective increases in money supply in the US, being offset by other economies' QEs, should be much smaller. Moreover, the world outside the US, especially the euro area, has been hit hard during this period by one crisis followed by another, propelling the US dollar in many ways.

5. SUMMARY

This paper analyzes the adjustment and evolution path of the exchange rate following a change in money supply. The paper proposes that there is a transition period in exchange rate adjustments, which is largely regulated by interest rate parities, and the transition period is featured by reverse movements of exchange rates in the short-term. The exchange rate then converges to the new long-run equilibrium exchange rate, the working that is contributed normally to PPP in the long-run. This pattern in exchange rate adjustments and movements is resulted from the joint and sequential effects of interest rate parities and sticky prices on the rise, from the short-term through to the long-run horizon. Following an expansionary monetary policy, the currency in concern would typically depreciate upon the announcement and implementation of the policy, albeit the extent of depreciation depends largely upon the environment in which the policy change is introduced. With a kind of regime change or break, the extent of immediate depreciation remains unpredictable, as evidenced by all kinds of findings in the empirical literature, ranging from undershooting to overshooting. What is discernible is the currency's subsequent appreciation in the short-term before the exchange rate finally moving towards its new long-run equilibrium rate definitely in a depreciating manner in the long-run.

Recent two rounds of QE of the US, the rare opportunities conferred to us to inspect the ways in which exchange rates adjust and move following an expansionary monetary policy, are then scrutinized to ratify the proposition and model of the paper. A clear and well defined pattern has evidently emerged in US dollar effective exchange rate adjustments and movements in both rounds of QE. Calibrations of the model work well and reinforce graphical illustrations. The observed pattern in exchange rate adjustments and movements in both rounds of QE mirror the theoretical analysis of this paper remarkably agreeably, and the actual exchange rate movements fit the theoretical curve delicately well.

ENDNOTES

[a] The solution to the following first order differential equation:

$$\frac{dy}{dx} = P(x)y + Q(x) \tag{A1}$$

is:

$$y = e^{\int P(x)dx}\left[\int Q(x)e^{-\int P(x)dx}dx + C\right] \tag{A2}$$

Let $y = E(e_t)$ and $x = t$, with:

$$\begin{aligned} \frac{dE(e_t)}{dt} &= E\left(\frac{de_t}{dt}\right) = e^{-\varkappa t}\theta\left(e_{0^+} - \frac{dm}{\lambda} - e_t\right) + \left(1 - e^{-\varkappa t}\right)\theta(e_0 + dm - e_t) \\ &= -\theta e_t + e^{-\varkappa t}\theta\left(e_{0^+} - \frac{dm}{\lambda}\right) + \left(1 - e^{-\varkappa t}\right)\theta(e_0 + dm) \end{aligned} \tag{A3}$$

$P(x)$ and $Q(x)$ are given as:

$$P(x) = -\theta \tag{A4}$$

$$Q(x) = e^{-\varkappa t}\theta\left(e_{0^+} - \frac{dm}{\lambda}\right) + \left(1 - e^{-\varkappa t}\right)\theta(e_0 + dm) \tag{A5}$$

Therefore:

$$\begin{aligned} E(e_t) &= e^{\int -\theta dt}\left\{\int\left[e^{-\varkappa t}\theta\left(e_{0^+} - \frac{dm}{\lambda}\right) + \left(1 - e^{-\varkappa t}\right)\theta(e_0 + dm)\right]e^{\int \theta dt}dt + C\right\} \\ &= \frac{\theta}{\varkappa - \theta}e^{-\varkappa t}\left(e_0 - e_{0^+} + dm + \frac{dm}{\lambda}\right) + e_0 + dm + Ce^{-\theta t} \end{aligned} \tag{A6}$$

[b] When $\varkappa = \theta$, equation (23) becomes:

$$\begin{aligned} E(e_t) &= \frac{(1 + \theta t)e^{-\theta t}}{1 - \theta}e_{0^+} + \frac{(1 + t)\theta e^{-\theta t}}{1 - \theta}\left(-\frac{dm}{\lambda}\right) \\ &+ \left(1 - \frac{(1 + \theta t)e^{-\theta t}}{1 - \theta}\right)(e_0 + dm) \end{aligned} \tag{A7}$$

Proofs are as follows:

$\frac{\theta e^{-\varkappa t} - \varkappa e^{-\theta t}}{\theta - \varkappa}$ and $\frac{e^{-\varkappa t} - e^{-\theta t}}{\theta - \varkappa}$ become $\frac{0}{0}$ when $\varkappa \rightarrow \theta$, the ratios are derived as follows:

$$\begin{aligned} &\underset{\varkappa \to \theta}{Lim}\left(\frac{\theta e^{-\varkappa t} - \varkappa e^{-\theta t}}{\theta - \varkappa} \right) = \underset{\varkappa \to \theta}{Lim}\left(\frac{-\theta t e^{-\varkappa t} - e^{-\theta t}}{\theta - 1} \right) = \frac{-\theta t e^{-\theta t} - e^{-\theta t}}{\theta - 1} \\ &= \frac{(1 + \theta t) e^{-\theta t}}{1 - \theta} \end{aligned} \tag{A8}$$

$$\begin{aligned} &\underset{\varkappa \to \theta}{Lim}\left(\frac{e^{-\varkappa t} - e^{-\theta t}}{\theta - \varkappa} \right) = \underset{\varkappa \to \theta}{Lim}\left(\frac{-t e^{-\varkappa t} - e^{-\theta t}}{\theta - 1} \right) = \frac{-t e^{-\theta t} - e^{-\theta t}}{\theta - 1} \\ &= \frac{(1 + t) e^{-\theta t}}{1 - \theta} \end{aligned} \tag{A9}$$

All the features of equation (17) are preserved and the analysis with equation (17) applies.

REFERENCES

Akiba, H., "Exchange-Rate Sensitive Demand for Money and Overshooting," *International Economic Journal* 10 (1996), 119-129.

Bjørnland, H.C., "Monetary Policy and Exchange Rate Overshooting: Dornbusch Was Right after All," *Journal of International Economics* 79 (2009), 64-77.

Branson, W.H., "Asset Markets and Relative Prices in Exchange Rate Determination," *IIES Seminar Paper* No. 66, 1976.

———, H. Halttunen and P. Masson, "Exchange Rates in the Short Run: The Dollar-Deutschemark Rate," *European Economic Review* 10 (1977), 303-324.

———, W.H. and D.W. Henderson, "The Specification and Influence of Asset Markets," in R.W. Jones and P.B. Kenen (eds.), *Handbook of International Finance 2, International Monetary Economics and Finance* (Amsterdam, North Holland: Elsevier, 749-806, 1985).

Cavaglia, S., "Permanent and Transitory Components in the Time Series of Real Exchange Rates," *Journal of International Financial Markets, Institutions and Money* 1 (1991), 1-44.

Chang, W.Y. and C.C. Lai, "The Specification of Money Demand, Fiscal Policy, and Exchange Rate Dynamics," *Journal of Macroeconomics* 19 (1997), 79-102.

Dornbusch, R., "Expectations and Exchange Rate Dynamics," *Journal of Political Economy* 84 (1976), 1161-1176.

Driskill, R., "Exchange Rate Overshooting, the Trade Balance, and Rational Expectations," *Journal of International Economics* 11 (1981), 361-377.

Fleming, J.M., "Domestic Financial Policies under Fixed and Floating Exchange Rates," *IMF Staff Papers* 9 (1962), 369–79.

Fleming, J.M., *Essays in International Economics*, (London: Allen and Unwin, 1971).

Frankel, J.A., “On the Mark: A Theory of Floating Exchange Rates Based on Real Interest Differentials,” *American Economic Review* 69 (1979), 610-622.

Frenkel, J.A., “A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence,” *Scandinavian Journal of Economics* 78 (1976), 200-224.

Gagnon, J., M. Raskin, J. Remache and B. Sack, “Large-Scale Asset Purchases by the Federal Reserve: Did They Work?” *Quantitative Easing Conference*, the Federal Reserve Bank of St. Louis, 2010.

Hwang, J.K., “Dynamic Forecasting of Sticky-Price Monetary Exchange Rate Model,” *Atlantic Economic Journal* 31 (2003), 103-114.

Inagaki, K., “Estimating the Interest Rate Semi-Elasticity of The Demand for Money in Low Interest Rate Environments,” *Economic Modelling* 26 (2009), 147-154.

Joyce, M., M. Tong and R. Woods, “The United Kingdom’s Quantitative Easing Policy: Design, Operation and Impact,” *Quarterly Bulletin* Q3, Bank of England, 2011.

Kiguel, M.A. and A. Dauhajre, “A Dynamic Model of the Open Economy with Sluggish Output,” *International Economic Review* 29 (1988), 587-606.

Kim, S., “International Transmission of US Monetary Policy Shocks: Evidence from VARs,” *Journal of Monetary Economics* 48 (2001), 339-372.

Levin, J.H., “On the Dynamic Effects of Monetary and Fiscal Policy under Floating Exchange Rates: Simulations with an Asset Market Model,” *Weltwirtschaftliches Archiv* 125 (1989), 665-680.

Levin, J.H., “On Sluggish Output Adjustment and Exchange Rate Dynamics,” *Journal of International Money and Finance* 13 (1994), 447-458.

Meese, R. and K. Rogoff, “Was It Real? The Exchange Rate-Interest Differential Relation over the Modern Floating-Rate Period,” *Journal of Finance* 43 (1988), 933-948.

Mundell, R.A., “The Monetary Dynamics of International Adjustment under Fixed and Flexible Exchange Rates,” *Quarterly Journal of Economics* 84 (1960), 227–257.

———, "The International Disequilibrium System," *Kyklos* 14 (1961), 154–172.

———, "The Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability," *IMF Staff Papers* 9 (1962), 70–79.

———, "Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates," *Canadian Journal of Economics and Political Science* 29 (1963), 475–485.

———, "A Reply: Capital Mobility and Size," *Canadian Journal of Economics and Political Science* 30 (1964), 421–431.

Mussa, M., "A Model of Exchange Rate Dynamics," *Journal of Political Economy* 90 (1982), 74-104.

Natividad-Carlos, F.B., "Monetary Policy, Fiscal Policy and Intervention in A Model of Exchange-Rate and Aggregate-Demand Dynamics," *Journal of Macroeconomics* 16 (1994), 523-537.

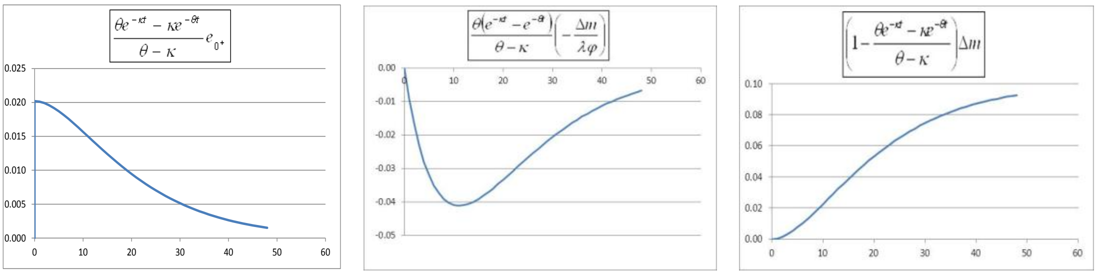
Obstfeld, M. and K. Rogoff, "Exchange Rate Dynamics Redux," *Journal of Political Economy* 103 (1995), 624-660.

Tu, W.W. and J.W. Feng, "An Overview on the Dornbusch Overshooting Hypothesis," *International Journal of Economics and Finance* 1 (2009), 110-116.

Verschoor, W.F.C. and C.C.P. Wolff, "Exchange Risk Premia, Expectations Formation and "News" in the Mexican Peso/US Dollar Forward Exchange Rate Market," *International Review of Financial Analysis* 10 (2001), 157-174.

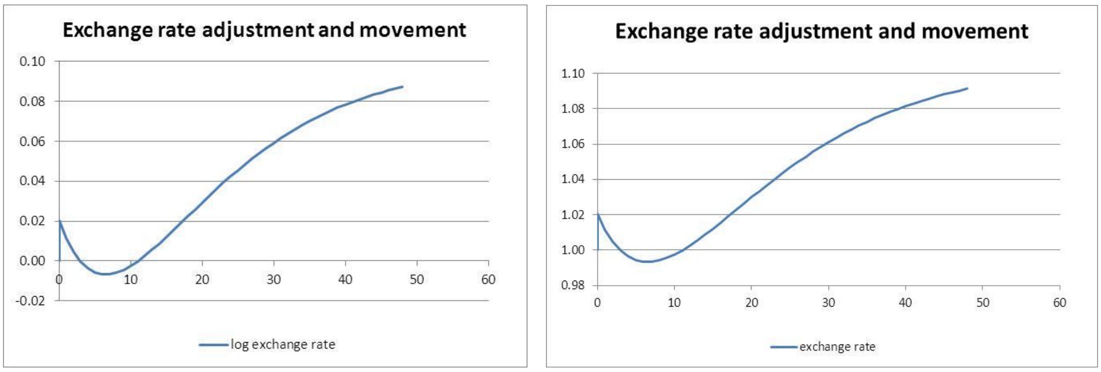
Zita, S. and R. Gupta, "Modeling and Forecasting the Medical-Rand Exchange Rate," *ICFAI Journal of Monetary Economics* 6 (2008), 63-90.

Figures



(a) (b) (c)

Figure 1. Evolution of three terms



(a) (b)

Figure 2. Exchange rate adjustment and movement

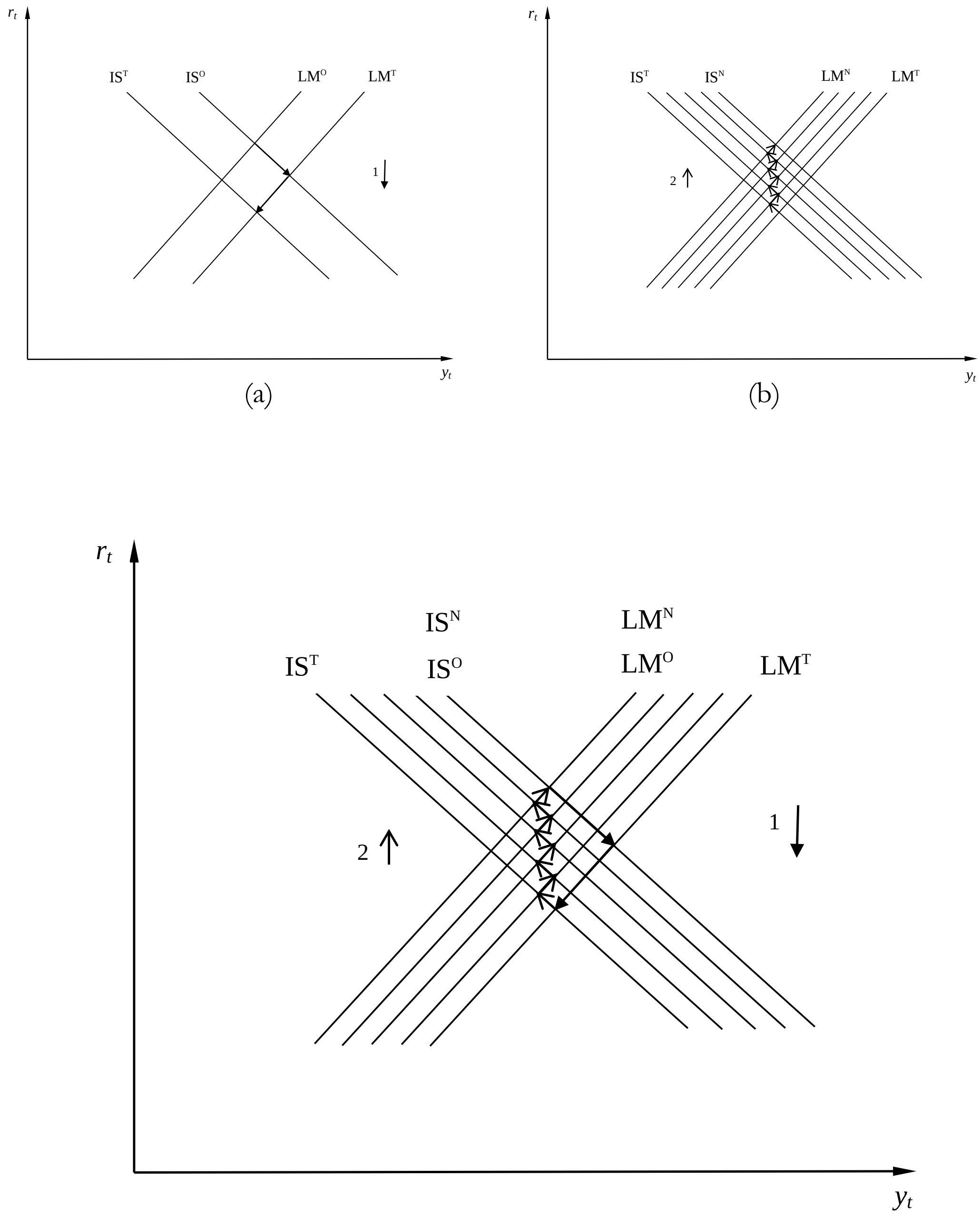


Figure 3. IS-LM analysis

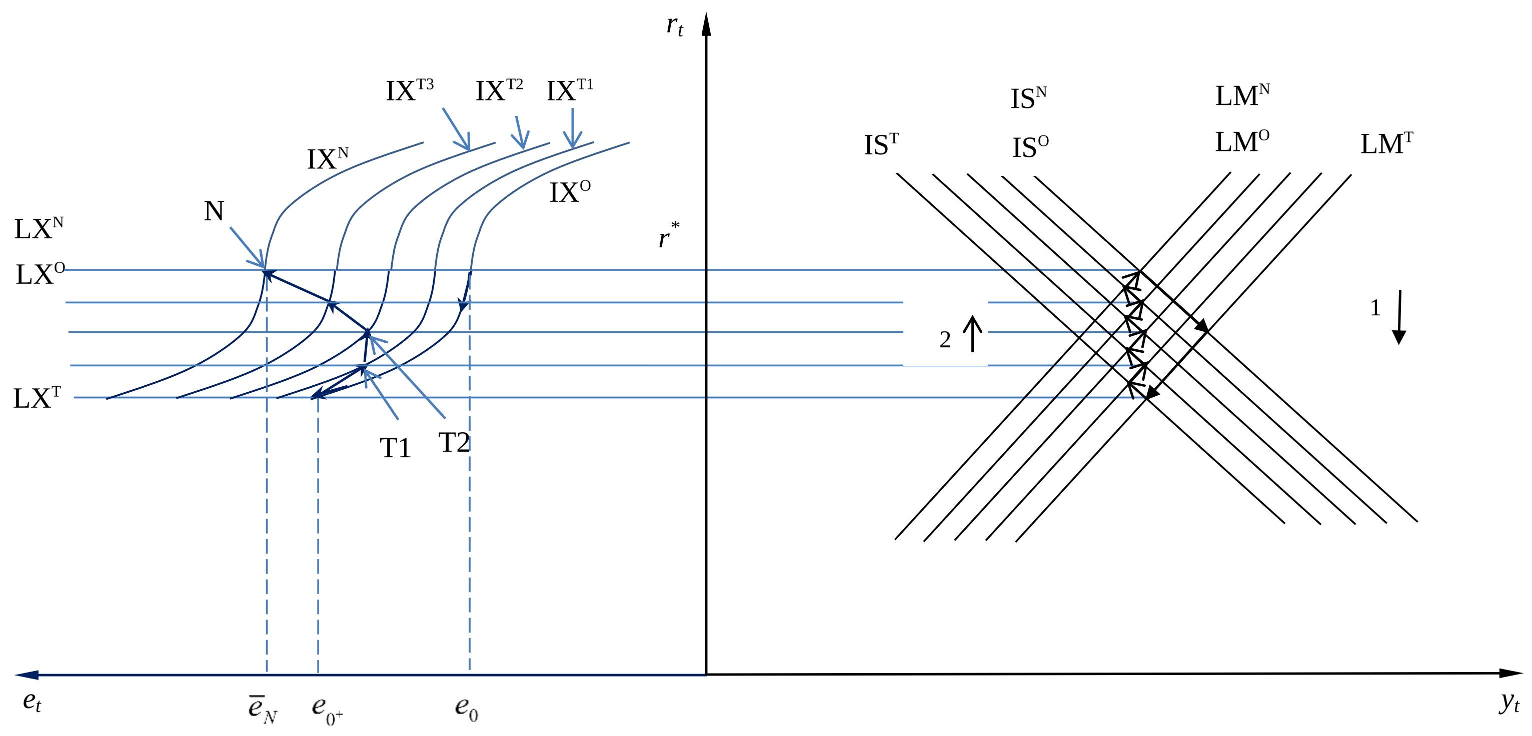


Figure 4. Extended IS-LM analysis

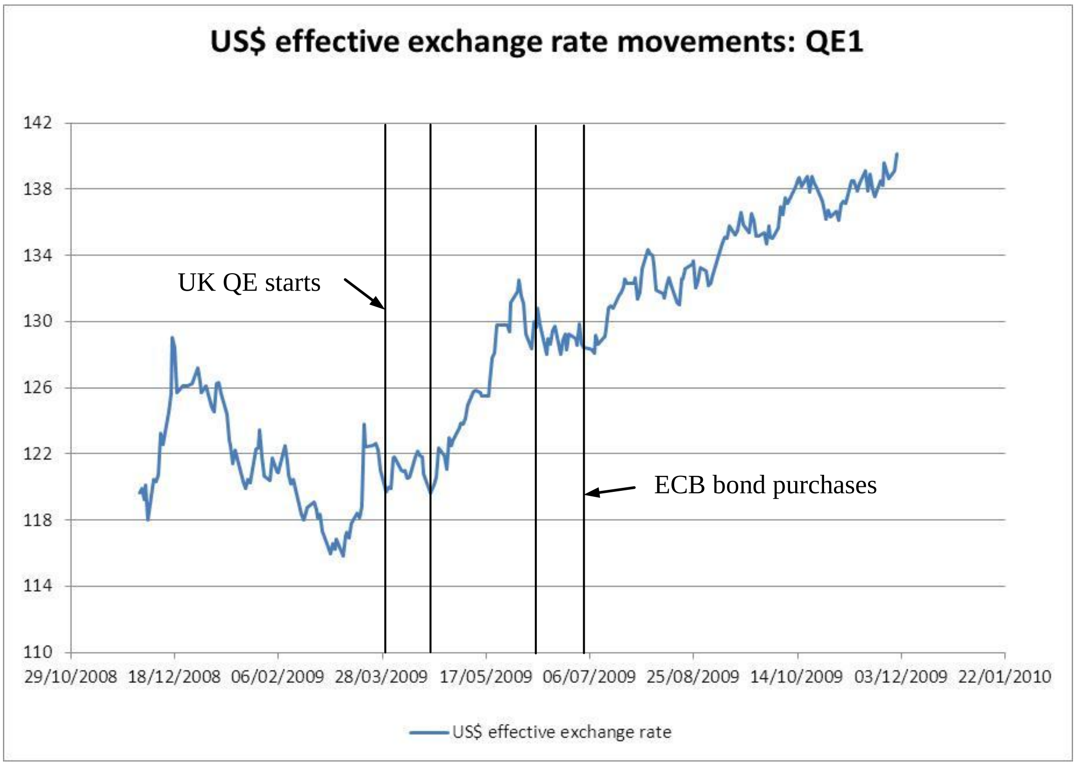
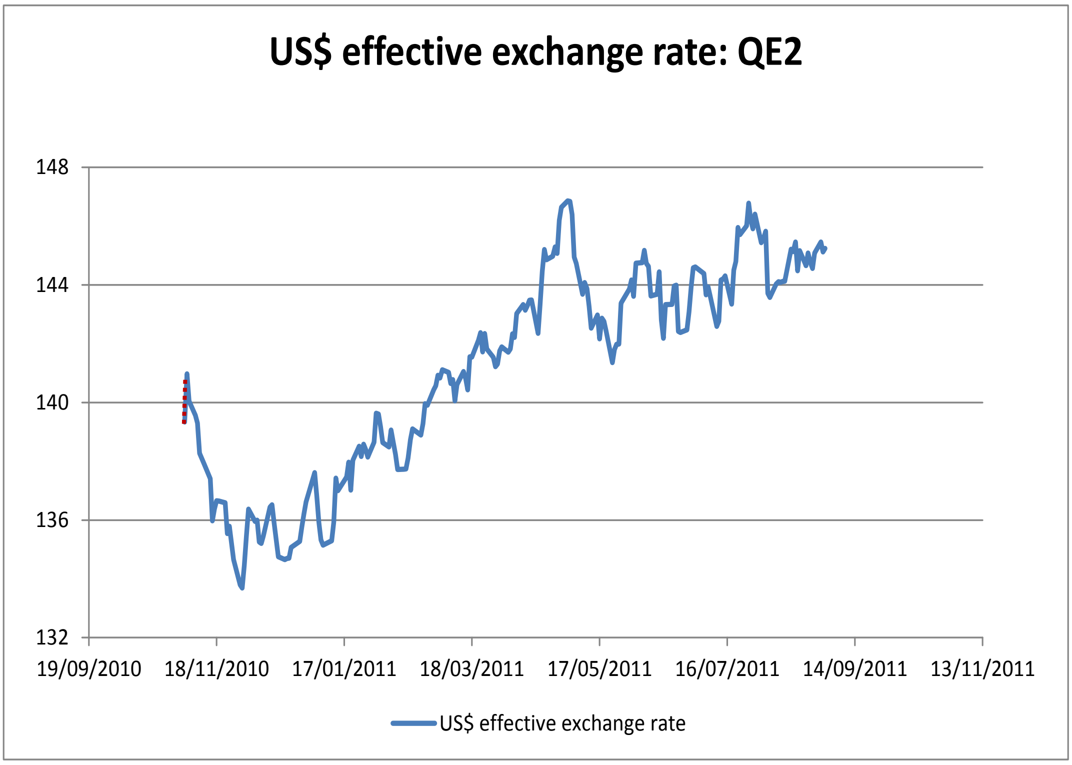
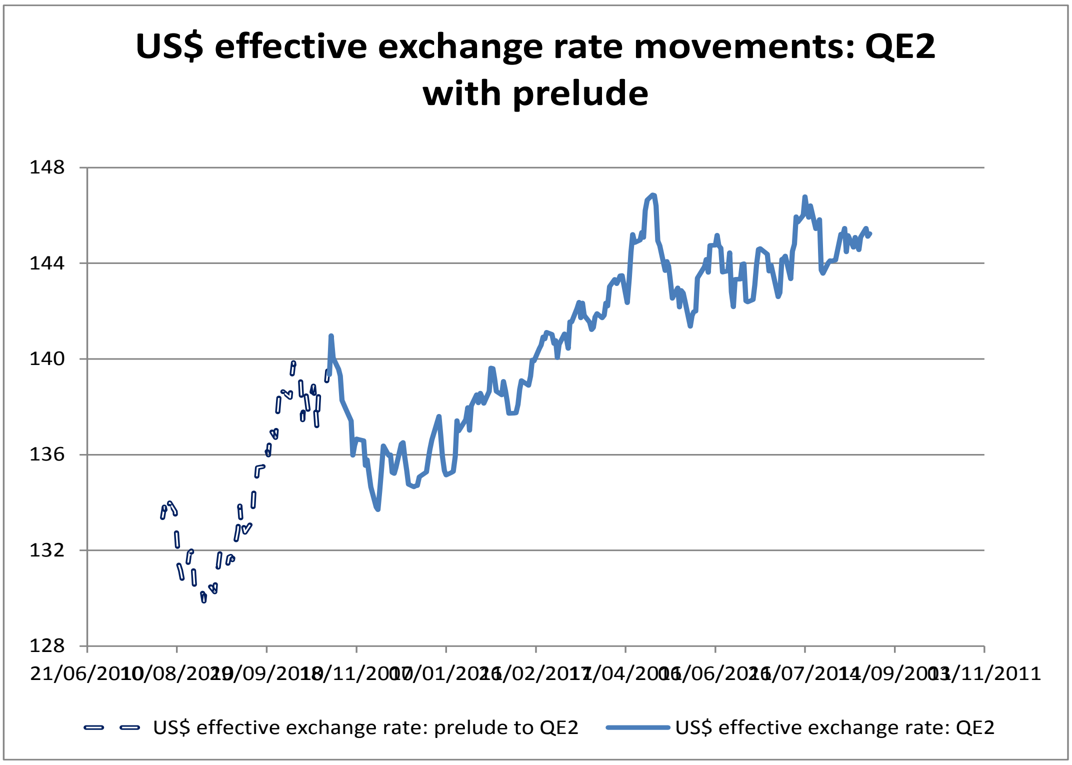


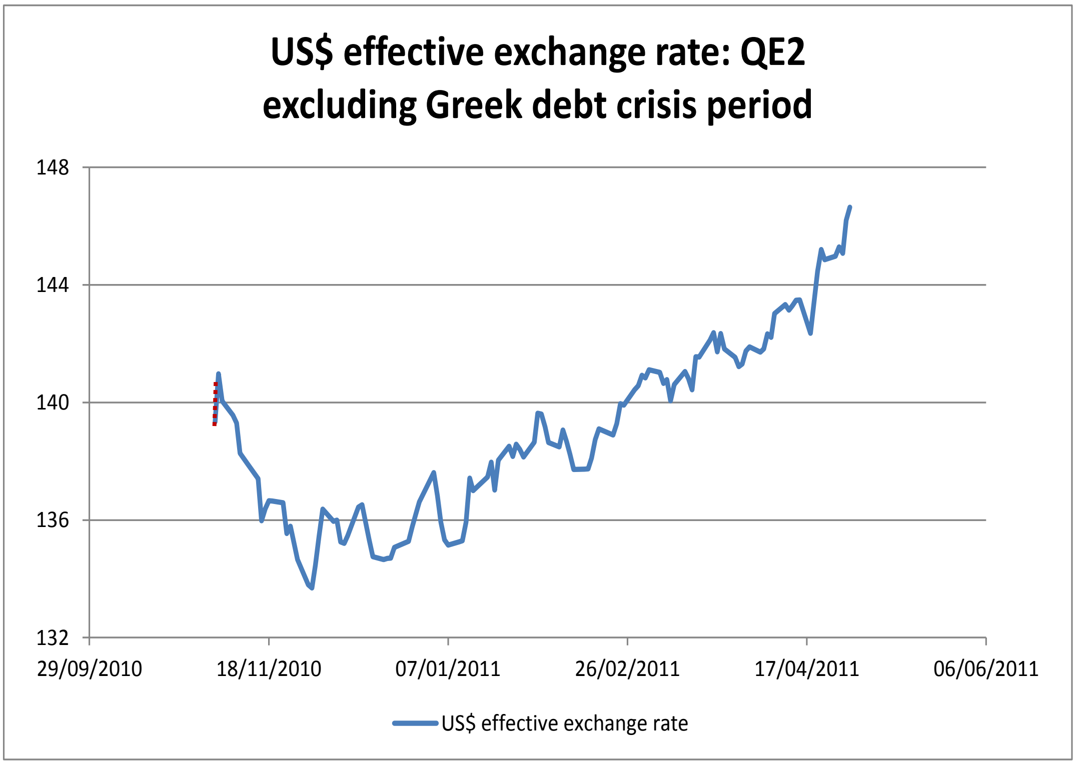
Figure 5. Exchange rate adjustments and movements: QE1



(a)



(b)



(c)

Figure 6. Exchange rate adjustments and movements: QE2

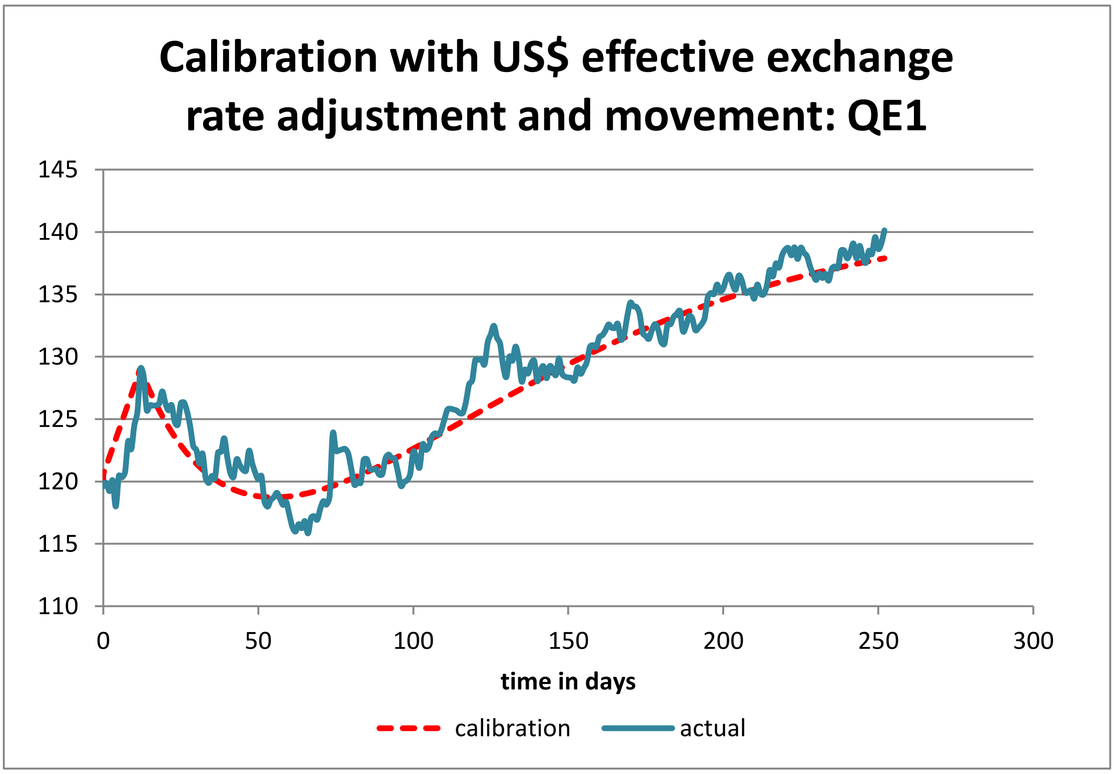


Figure 7. Calibrated and actual exchange rate adjustments and movements for QE1

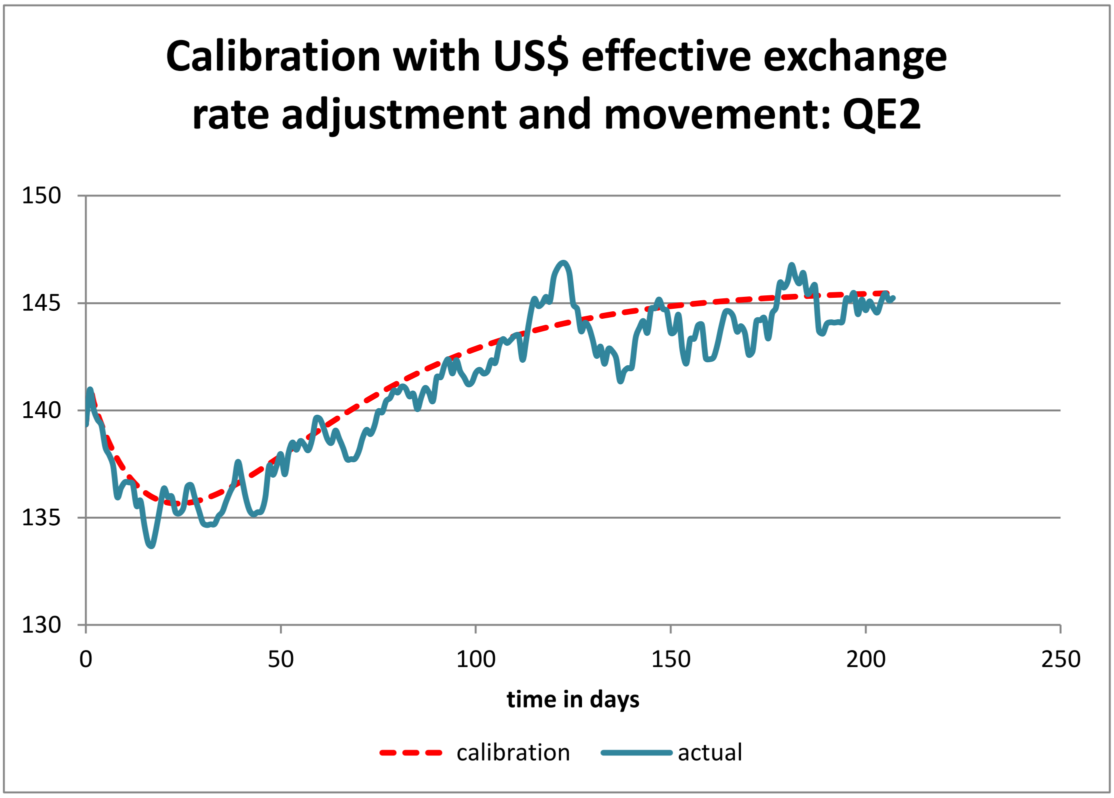


Figure 8. Calibrated and actual exchange rate adjustments and movements for QE2

Table 1. Calibration results

	θ	$\varkappa$	$\lambda\varphi$	Δm
QE1	0.017602	0.017569	0.644444	21.0211%
QE2	0.034033	0.034032	0.304234	5.3415%

Table 2. Monetary and exchange rate statistics, and price estimation

	ΔMB	$\Delta M1$	$\Delta M2$	Δe	Δp^*
Nov08 - Dec09	38.2908%	11.5022%	5.8230%	16.8980%	99.5638%
Nov10 - Sept11	35.4558%	20.1246%	8.7933%	4.1110%	87.8684%

* adopting λ = -30, φ = 0.021481 for the QE1 period, and φ = 0.010141 for the QE2 period