

# INTERNATIONAL BUSINESS CYCLE COHERENCE AND PHASES

*- A spectral analysis of output fluctuations of G7 economies*

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Economics Working Paper No. 1206

November 2012

**Abstract** - This paper examines international linkages amongst G7 economies, in terms of co-movements in output growth and fluctuations, in the frequency domain. The paper has identified patterns in international business cycle co-movements among the G7, offering a general outlook of international business cycle co-movements. Moreover, the paper details the lower frequency, higher frequency and middle range characteristics of international co-movements in output growth and fluctuations. The main findings of the study are that co-movements among G7 economies are considerably stronger at lower frequencies, and G7 economies have become more synchronized considerably in episode 2 since the new millennium. The results and findings show support for real business cycle theory being extended to an international arena, with long effect real shocks impacting economies across borders. The three euro economies become more integrated since the millennium, against a background that all G7 economies have also become more synchronized.

**JEL No:** E32, F44, C32

**Key words:** business cycles, frequency domain, coherence, phase, coincidence

## 1. Introduction

This paper examines international business cycle amongst major industrialized economies in terms of coherence and phases in the frequency domain. It analyzes the cross spectra of output of these economies in different cycle components or at different frequencies. In particular, it focuses on the patterns of co-movement in terms of coherence, coincidence and phase leads/lags. This is in contrast with lead/lag relations and correlations in the time domain and, therefore, offers another means of looking at international business cycle issues. Intuitively, the approach is to inspect the degree to which one output variable differs from another in time series behavior in the frequency domain. It examines the similarities and synchronous relations in the spectra of time series. Characteristically, as spectrum and cross spectrum components are depicted against frequencies of time series, spectral analysis is particularly helpful in the study of cyclical co-movements, such as international business cycles. Therefore, the approach in the frequency domain may present a fuller picture of international business cycle fluctuations with the same (amount of) information available to us in the time domain, which is utilized in a way more appropriately and effectively for this type of investigation.

Research on international business cycles is not as extensive as that on business cycles in closed economies. Many empirical studies are more about international comparisons of business cycle features in individual countries than international business cycle linkages and co-movements between national economies; and the co-

movement examined is overwhelmingly the correlation of output fluctuations between countries. The present study goes beyond a documentation of a relationship between national economies in terms of correlation. It attempts to identify patterns in the interaction between individual economies, covering the whole spectrum of short, medium and long cycles, (and trends at extremity) and the phase relations. The empirical investigation is further empowered by the frequency domain method to achieve the set objectives effectively.

The rest of the paper is organized as follows. The next section provides a review of business cycle studies with international perspectives, paying specific attention to research in the frequency domain. Section 3 discusses the methodological aspects of the approach, presenting the frequency domain representations of cycles, spectra and cross spectra. Section 4 reports empirical results and discusses the findings and their implications. Finally, section 5 concludes.

## **2. International perspectives of business cycles - a review**

The term “business cycle” is itself controversial in its definitions and measurement, arising from the differences in research methodologies, investigating techniques, application purposes, and policy considerations. Conventional definition states that business cycles are periodic but irregular up and down movements in economic activity, measured by fluctuations in real GDP and other economic variables. A full

business cycle is identified as a sequence of four phases: contraction, trough, expansion, and peak, whereas the time span between, for example, two peaks, varies from time to time, so do the magnitude of peaks or troughs. Further analysis involves more details of business cycles such as large peaks/troughs and small peaks/troughs occurring at different time intervals, indicating business cycle components.

The notion of business cycles started to attract attention from economists and governments alike in the early 20<sup>th</sup> century, in search of an understanding of the patterns in economic activity and a possible therapy for mitigating the damage caused by severe economic downturns. The early research on business cycles was virtually in the frequency domain. For example, Schumpeter's (1939) long waves and the accompanied notions of long cycles, medium cycles and short cycles are alternations of states of economic activity or business conditions in different lengths of time period, which amounts to a decomposition of business cycle components in accordance with their frequencies of occurrences. Less prominently but comes little earlier the Juglar (1862) cycle, probably the first formal presentation and illustration of the business cycle featured by the 7-11 year fixed investment cycle, where business cycles are explicitly linked to economic crises and examined together. Other business cycle studies preceding Schumpeter's includes the Kitchin cycle featured by the 3-5 year inventory cycle (Kitchin 1923), the Kondratiev long wave developed in the 1920s and featured by the 45-60 years technological cycle (*cf.* Barnett 1998), and the Kuznets cycle featured by the 15-25 year infrastructural investment cycle (Kuznets 1930). Hayek (1931) has also studied, amongst others, business cycles and the causes. Although most empirical

studies since then have been in the time domain, the business cycle is more an issue in the frequency domain arising from its two features: cycle components and phases. In an extreme case of (the decomposition of) cycle components, the longest “cycle” is the trend and the rest is the cycle, as in Beveridge and Nelson (1981). Furthermore, Nelson and Plosser (1982) classify models of decomposing economic fluctuations into two entirely different groups: models for deterministic trends and models for stochastic trends, and they favor the latter. Prior to the 1980s, the general view on economic time series was that economic variables could be decomposed into a secular or growth component and a cyclical component. A less restricted version of deterministic trends is obtained through applying the HP filter (Hodrick and Prescott 1980, 1997) where the trend, though deterministic, can be non-linear and track the time series to varied degrees depending on the chosen values of the filter’s parameter. Nelson and Plosser (1982) have questioned this approach to decomposing trends from cycles. Using an unobserved component model that decomposes fluctuations into a secular or growth component and a cyclical component, they find that the time series of the US economy used in the study are non-stationary stochastic processes with no tendency to return to a trend line. Watson (1986) and Clark (1987) are in line with the stochastic trend approach and also resort to the state space to decompose output into two unobserved components of the trend and the cycle, being executed with the help of Kalman filters. Another approach to examining trends and cycles is the measurement of persistence in output data. Since, by definition, the effect of shocks to trends is permanent or persistent and that to cycles is temporary or transitory, the degree of persistence amounts to an assessment of the relative composition of trend

components and cycle components in output data. Campbell and Mankiw (1987a,b) and Cochrane (1988) put forward the concept of persistence in economic time series analysis. Their persistence measure is the ratio of two variances of a time series: the variance in a longer period and a one period variance, which is achieved through different means. While Campbell and Mankiw (1987a,b) apply ARIMA procedures, Cochrane (1988) employs a non-parametric method for the same purpose. Cochrane's (1988) persistence measure appears to be in the time domain but is indeed in the frequency domain, since the measure is a special case of spectral analysis at the zero frequency point.

In the last three decades, the world economy has become increasingly integrated. As such, fluctuations in output in individual economies are increasingly influenced by fluctuations in the world economy in international business cycles. This has added an additional dimension to the study of business cycles, namely, interactions and co-movements between national economies. Such interactions and co-movements, though existed well before the emergence of the interest in business cycles and business cycle theory in the early 20<sup>th</sup> century, have become non-negligible in their roles and, consequently, in their research, only fairly recently. Backus *et al.* (1992), extending Kydland and Prescott (1982), are among the first to study real business cycles in a two-country setting. In their model, they allow residuals in the shocks to be correlated across countries, and there is diffusion of technology shocks between countries. They perform empirical work on diffusion and correlation for the US and an aggregate of European countries, based on estimates of Solow residuals. One of their findings that is particularly relevant is that openness substantially alters the nature of

some of the closed economy co-movements. In a similar framework, Ambler *et al.* (2002) propose a theoretical model for international transmission of business cycles that is simulated to study and predict the cross-country correlation of economic activity. Employing a Bayesian dynamic latent factor model, Kose *et al.* (2008) examine the changes in world business cycles during the period 1960-2003 by estimating common and country-specific components in the main macroeconomic aggregates of the G-7 countries. They suggest that the common G-7 factor explains a larger fraction of output, consumption and investment volatility in their globalization period beginning in 1986 than it does before the collapse of the Bretton Woods system. Lumsdaine and Prasad (2003) incorporate a time-varying weighting scheme for the construction of the common component. It involves estimating univariate models of time-varying conditional variances for output growth fluctuations in each country. The time-varying weights for each country are then derived as a function of the estimated conditional variances. They claim to have found evidence for a “world business cycle” as well as evidence for a distinct European common component. In line with the findings in Kose *et al.* (2008), their results indicate that macroeconomic fluctuations have become more closely linked across industrial economies in the period after 1973. Using a multicountry Bayesian panel VAR model, Canova *et al.* (2007) examine the properties of G-7 cycles with time variations, unit specific dynamics and cross-country interdependences. They detect an increase in synchronicity in the late 1990s, but find little evidence of major structural changes. Contrary to Lumsdaine and Prasad (2003), they find no evidence of the existence of a euro specific cycle or of its emergence in the late 1990s.

From a historical perspective, Mumtaz *et al.* (2009) investigate international co-movements in business cycles and inflation, covering the period 1821-2007. They depict a picture with a declining contribution of world co-movements and an increasing contribution of regional co-movements to domestic output growth since the World War II. Backus and Kehoe (1992) document business cycle evidence in ten countries with more than 100 years' annual data from around the 1860s to the 1980s. Fluctuations in real output, expenditure, price levels and monetary aggregates in the individual countries are analyzed, and correlations in output between the countries are presented. Following Backus and Kehoe (1992), Basu and Taylor (1999) provide an international historical perspective on business cycles, employing annual time series data running from approximately 1870 to the 1990s. They present volatility and first order autocorrelation for output, consumption, investment, the current account, and prices for a pool of 15 countries, and correlation between the US and the pool of the rest 14 countries. Further, exchange rate volatility is examined for a pool of 20 countries and real wage cyclicalities is inspected for a pool of 13 countries.

Research on business cycles, in particular in the frequency domain, appears to experience a kind of cycle as well. There are indeed long cycles in frequency domain research on business cycles. The work by Sargent and Sims (1977) is probably the last business cycle study in the last century that contains influential frequency domain analysis, among others. Then in the new century, A'Hearn and Woitek (2001) pick up the frequency domain approach to studying business cycles, using annual historical industrial output (industrial production) data of 13 countries from around 1865 to 1913



for empirical univariate and bivariate analysis. When the analysis is multivariate, other dimensions of investigation are introduced to assess the closeness or the degree of co-movement of two or more output time series. While such closeness or co-movement can be evaluated in both time domain and frequency domain, the features of business cycles, as pointed above, suggest that analysis in the frequency domain will be more advantageous for multivariate cases compared with univariate cases. 10 years on, Poměnková and Maršálek (2011) adopt the frequency domain method and several time domain filters to conduct univariate analysis of business cycle features in the main sectors of the Czech economy, in a univariate manner. Most recently, Wang (2012) applies spectral analysis to pairwise co-movements between UK sectoral output, scrutinizing business cycle phases and coherence in the sectors of the UK economy. Multivariate though, it is between various sectors in an economy rather than business cycle co-movements between various economies.

It is evident that research on international business cycles is not as extensive as that on business cycles in closed economies. Many empirical studies are more about international comparisons of business cycle features in individual countries than international business cycle linkages and co-movements between national economies; and the co-movement examined is overwhelmingly the correlation of output fluctuations between countries. The present study goes beyond of documenting a relationship between national economies in terms of correlation. It attempts to identify patterns in the interaction between individual economies, covering the whole spectrum of short, medium and long cycles, (and trends at extremity) and the phase relations.

The empirical investigation is further empowered by the frequency domain method to achieve the set objectives effectively. This study contributes to the literature in three ways. Firstly, it opens up a new channel of research to gain knowledge in such important aspects of international business cycle coherence and phases that are either overlooked or unable to be quantified previously. Secondly, unlike most empirical studies in the area that use long, historical, and annual data, our data set covers the last quarter century in the quarterly frequency. Consequently, the present study is able to render empirical implications that are more relevant to contemporary welfare and has a more dynamic feature too. Finally, as the co-movement between individual economies is investigated from all the perspectives of short, medium, and long runs, instead of contemporary and led/lagged correlations, the present study is able to explain and encompass some of the rival views, such as whether the British economy is more close to the US economy or the Continental European economy in business cycles.

### **3. The frequency domain approach**

Spectral analysis, or studies in the frequency domain, is one of the unconventional subjects in time series econometrics. Analysis in the frequency domain does not bring in new or additional information, it is simply an alternative method with which information is observed, processed and abstracted. This is sometimes helpful. Depending on the characteristics of the issues, analysis in one domain may be more

powerful than in the other. For example, cycles are better and more explicitly observed and represented in the frequency domain. Correlations in the time domain and cross spectra in the frequency domain deal with the relationship between two time series from different perspectives and have defined links. In the following, we briefly introduce the ideas of the Fourier transform and spectra, cross spectra, coherence, and phases.

### 3.1. The spectrum, phase and coherence

The spectrum of a time series is the frequency domain representation of the time series, which reveals the characteristics of the time series from its frequency domain, rather than its time domain, perspectives. The spectral density function of a discrete random process  $\Delta X_t = X_t - X_{t-1}$  ( $t=1, \dots, N$ ) is:

$$h(k) = \sum_{\tau=-(N-1)}^{N-1} R(\tau) e^{-j\tau \frac{2\pi k}{N}} \quad (1)$$

where  $R(\tau)$  is the autocovariance function of  $\Delta X_t$ , i.e.,  $R(\tau) = E\{(\Delta X_t - \mu)(\Delta X_{t-\tau} - \mu)\}$  and  $\mu = E\{\Delta X_t\}$ . The inverse Fourier transform of equation (1) is:

$$R(\tau) = \frac{1}{N} \sum_{k=-(N-1)}^{N-1} h(k) e^{jk \frac{2\pi \tau}{N}} \quad (2)$$

Setting  $\tau=0$  in equation (2), we have:

$$R(0) = E\{(\Delta X_t)^2\} = \frac{1}{N} \sum_{k=-(N-1)}^{N-1} h(k) e^{jk \frac{2\pi \tau}{N}} \quad (3)$$

It is the mean squared value of the process and has the meaning of power of the process, so equation (1) is called the power spectrum.  $R(\tau)$  usually takes real values and is an even function, i.e.,  $R(-\tau) = R(\tau)$ . Accordingly, the spectral density function can be written as:

$$h(k) = \sigma_x^2 + 2 \sum_{\tau=1}^{N-1} R(\tau) \cos\left(\frac{2\pi\tau k}{N}\right) \quad (4)$$

When  $R(\tau)$ , the autocovariance function of  $\Delta X_t$ , is replaced by the covariance between two time series, i.e.,  $Cov_{X,Y}(\tau) = E\{(\Delta X_t - \mu_X)(\Delta Y_{t-\tau} - \mu_Y)\}$ ,  $\mu_X = E\{\Delta X_t\}$  and  $\mu_Y = E\{\Delta Y_t\}$ , the cross spectrum of the two time series is derived in the form of:

$$h_{X,Y}(k) = \sum_{\tau=-(N-1)}^{N-1} Cov_{X,Y}(\tau) e^{-j\tau \frac{2\pi k}{N}} \quad (5)$$

$Cov(\tau)$  is in general not an even function, so equation (5) cannot take the form of equation (4), and  $h_{X,Y}(k)$  is in general a complex number:

$$h_{X,Y}(k) = c(k) \cos\left(\frac{2\pi k}{N} \tau\right) + jq(k) \sin\left(\frac{2\pi k}{N} \tau\right) \quad (6)$$

Unlike the univariate Fourier transform where the imaginary part is zero, the cross spectrum has both magnitude and phase as follows:

$$m(k) = \sqrt{c^2(k) + q^2(k)} \quad (7)$$

and

$$p(k) = \tan^{-1} \frac{q(k)}{c(k)} \quad (8)$$

Equations (7) and (8) are called magnitude spectrum and phase spectrum respectively.

It can be seen, from the above analysis, that if  $Cov_{X,Y}(\tau)$  is an even function, then the phase spectrum is zero, i.e., there is no overall lead of series  $X_t$  over series  $Y_t$ , and vice versa. With equations (7) and (8), the cross spectrum can also be expressed as:

$$h_{X,Y}(k) = m(k)e^{ip(k)} \quad (9)$$

so that both magnitude and phase are shown explicitly.

The cross-spectrum, like covariance in the time domain, does not provide an indication of the closeness of two time series. Coherence is then derived, in a similar way to the correlation coefficient, as:

$$Coh_{X,Y}(k) = \frac{h_{X,Y}(k)}{h_{X,X}^{1/2}(k)h_{Y,Y}^{1/2}(k)} \quad (10)$$

Let us contrast the measures in the frequency domain with those in the time domain. The cross spectrum of equation (5) corresponds to covariance in the time domain, which is not standardized; then the coherence as with equation (10) is corresponds to correlation in the time domain, which are standardized by the square roots of the two time series' spectra and the two time series' standard deviations respectively. The phase of equation (8) addresses leads and lags. The closeness of two time series is straightforwardly observed with the standardized measures of coherence, together with the phase measure, which we adopt in this study.

While the row periodogram illustrated above produces an unbiased estimate of the spectrum, it is inconsistent, as the variance of the estimators does not go to zero as the number of data points grows. Therefore, it is usually to let the time series pass through a spectral window, which is a procedure called smoothing, to get a consistent estimate of the spectral density or cross-spectral density. There are two requirements for this to produce consistency. First, the window width must go to infinity as the

number of data points increases to ensure that the variance goes to zero. That is, a window should not be too narrow to produce imprecise estimates. Second, the window width must increase at a rate slower than that in the number of data points to ensure that the bias goes to zero. A too wide window will flatten the peaks and troughs too much. To express mathematically, it is:  $N \rightarrow \infty$ ,  $M \rightarrow \infty$ ,  $N \gg M$ ; where  $N$  is the number of data points and  $M$  is the window size. The design and choice of window types is also important. Although a window is essential for consistency, it can produce some spurious frequencies or ripples, especially when the window edges are sharp, e.g., a rectangular window. A window with curved edges mitigates this problem by scaling the ends of the data so they merge smoothly with the zeros on either side. Bartlett's window and the tent window are examples<sup>1</sup>.

$h_{x,y}(k)$  and  $Coh_{x,y}(k)$  follow normal distributions. With Bartlett's window (Bartlett 1950), the standard error of  $h_{x,y}(k)$  is:

$$Std[h_{x,y}(k)] = h_{x,y}(k) \sqrt{\frac{2M}{3N}}, \quad \text{for } k \neq 0 \quad (11)$$

and

$$Std[h_{x,y}(k)] = h_{x,y}(k) \sqrt{\frac{4M}{3N}}, \quad \text{for } k = 0 \quad (12)$$

The standard error of  $Coh_{x,y}(k)$  is:

$$Std[Coh_{x,y}(k)] = Coh_{x,y}(k) \sqrt{\frac{2M}{3N}}, \quad \text{for } k \neq 0 \quad (13)$$

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<sup>1</sup> Refer to one of the books on signals and systems, e.g., Ziemer, R.E., Tranter, W.H. and Fannin, D.R. (1993), *Signals and Systems: Continuous and Discrete* 3<sup>rd</sup> ed, MacMillan Publishing Company, NY; or Priestley, M.B., 1996, *Spectral Analysis and Time Series* 9<sup>th</sup> printing, Academic Press, London..

and

$$Std[Coh_{x,y}(k)] = Coh_{x,y}(k) \sqrt{\frac{4M}{3N}}, \quad \text{for } k = 0 \quad (14)$$

The window width should be appropriately chosen in relation to the sample size, so that the standard error of the coherence estimate is reasonably small and that the coherence estimate is statistically significant, in addition to the unbiasedness and consistency requirements.

**{Figure 1 about here}**

We demonstrate phase relations as adopted by RATS graphically with explanations. Figure 1 demonstrates several special cases of the relationship between time series. Coherence is plotted against the left hand side axis (in blue) with minimum being zero and maximum being one; and phases are plotted against the right hand side axis (in red) with minimum being  $-\pi$  (a half cycle lag) and maximum being  $\pi$  (a half cycle lead). Figure 1(a) shows perfect coherence but one time series has one phase lag vis-à-vis the other at all frequencies, and Figures 1(b), 1(c) and 1(d) show perfect coherence while there exist two phase lags, three phase lags and four phase lags, respectively, between them at all frequencies. Any lead between a half cycle and a full cycle ( $\pi < p(k) < 2\pi$ ) is regarded as a lag between a half cycle and zero lag (i.e.,  $-\pi < p(k) - 2\pi < 0$ ). Figure 2 exhibit phase corresponding lags and leads in the time domain. Figure 2(a) shows that a one-quarter lead/lag in quarterly changing time series data is a half cycle lead/lag, and it is point  $(1, \pi)$  in Figure 1(a) with 1 being the highest frequency and  $\pi$  being half cycle ( $2\pi$  is a full cycle). Similarly, Figure 2(b) shows that a one-quarter

lead/lag in semi-annually changing data is  $1/4$  of the cycle, and its corresponding point is  $(0.5, \pi/2)$  in Figure 1(a) with 0.5 being half of the highest frequency and  $\pi/2$  being  $1/4$  cycle; and Figure 2(c) shows that a one-quarter lead/lag in annually changing data is  $1/8$  of the cycle, and it is point  $(0.25, \pi/4)$  in Figure 1(a) with 0.25 being  $1/4$  of the highest frequency and  $\pi/4$  being  $1/8$  cycle. A two-quarter lead/lag in quarterly changing data of Figure 2(a) is equivalent to zero lead/lag, which is point  $(1, 0)$  in Figure 1(b). A two-quarter lead/lag in semi-annually changing data of Figure 2(b) is equivalent to a half cycle lead/lag, which is point  $(0.5, \pi)$  or  $(0.5, -\pi)$  in Figure 1(b) (notice that a half cycle lead and a half cycle lag have the same meaning with regard to phases). A two-quarter lead/lag in annually changing data of Figure 2(c) is equivalent to  $1/4$  cycle lead/lag, which is point  $(0.25, \pi/2)$  in Figure 1(b). A three-quarter lag is equivalent to a one-quarter lead in quarterly changing data of Figure 2(d), which is point  $(1, \pi)$  in Figure 1(c).

**{Figure 2 about here}**

### 3.2. Overall lead/lag and coincidence statistics

In general, we can regard a phase point above 0 as phase lead and a phase point below 0 as phase lag, the larger the absolute value of the phase, the larger the lead/lag. As observing a cross spectrum requires technicalities in the frequency domain, we provide summary statistics for phase leads/lags. Statistic  $\phi$  is the average of the phase values on the whole spectrum, measuring the overall lead/lag between two time series.



Statistic  $\theta$  is the average of the absolute phase values on the whole spectrum, measuring the degree of departure from a coincident relation between two time series.  $\phi$  is zero when there are equal positive and negative phase values in a cross spectrum, but the two time series may not be coincident. A small  $\theta$  suggests that there are substantial coincident elements in the two time series. A small  $\phi$  with a large  $\theta$  implies that there are no overall leads/lags in the time series but the time series are not coincident either. A large positive (negative)  $\phi$  means large phase leads (lags) in the time series. The maximum value  $\phi$  that can take is  $\pi^-$ , and the minimum value is  $-\pi^+$ . The maximum value  $\theta$  that can take is  $\pi^-$  and the minimum value is 0. We will analyze the empirical results with the above method of interpretations and the two summary statistics in the next Section.

### 3.3. *Frequencies ranges*

For the purpose of inspecting business cycle properties in the short, medium and long terms, we divide the whole spectrum into four sections. For quarterly data, point 1.00 on the spectrum refers to the quarterly frequency that completes a full cycle in two quarters (see Figure 2(a)); point 0.50 represents the semi-annual frequency that completes a full cycle in a year (see Figure 2(b)); and point 0.25 is the annual frequency that completes a full cycle in two years (see Figure 2(c)). Point 0.75 can be regarded a “four-monthly frequency” on a quasi-continuous base. The range from 0.75 up to 1.00 on the spectrum is taken for higher frequencies. The choice is meant to be close to the quarterly frequency and, at the same time, sufficient spectrum components, or energy,

are covered. Then the scope between 0.20 and 0.75 is for the medium frequencies. With 0.20 corresponding to a 1.25-year cycle, this, roughly speaking, represents cycles around annual frequencies. The range from 0.05 to 0.20 represents lower frequencies, kept for the traditional business cycle ranges, or longer cycles over one year<sup>2</sup>, covering both the traditional short cycles of 3-5 years and long cycles of up to 10 years. Finally, the range of the spectrum from 0 to 0.05 is for the long-run trend in business cycles. It includes long cycles over 10 years, so while preserving the long-run features of business cycles, there are sufficient spectrum components in this spectrum range. Using quarterly data and the RATS procedure, a half-cycle lead is one quarter at point 1.00, two quarters at point 0.50, one year at point 0.25, and two years at point 0.125. Therefore, a same phase lead/lag value at different frequency points represents different time lengths, though it means the same fraction of a cycle.

#### **4. Empirical results and discussions**

The data used in this study are quarterly GDP of the US, Japan, Germany, France, the UK, Italy and Canada, starting in the first quarter, 1973 and ending in the fourth quarter, 2011, at constant prices. The period is divided into two sub-periods by the millennium. The choice of the period and its division are influenced by the following

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<sup>2</sup> Our method is substantially different from those using annual data. According to the Nyquist sampling theorem, if a time series is sampled at the frequency of  $2f$ , then all frequency components lower than  $f$  would be reserved and can be recovered. In other words, any frequency components higher than  $f$  would be lost and unrecoverable. If annual data are ever to achieve the same results as quarterly data, one should assume there are no fluctuations that have a frequency higher than two quarters in GDP. This assumption, however, is highly unlikely to hold. So the spectrum based on annual data involves distortions. Refer to one of the books on signals and systems, e.g., Ziemer, R.E., Tranter, W.H. and Fannin, D.R. (1993), *Signals and Systems: Continuous and Discrete* 3<sup>rd</sup> ed, MacMillan Publishing Company, NY.

considerations. The events around 1973 marked a number of strategic changes of the world economically as well as politically. The US President Nixon's announcement on August 15, 1971 of the end to the US commitment to converting US dollars to gold at a fixed price, the signing and withdrawal of the Smithsonian agreement in December 1971 and March 1973 eventually completed the transition to the floating exchange rate regime in 1973. A year ago in 1972, Nixon made a historical visit to the People's Republic (the PR), effectively ending an era featured by two competing camps built on rival ideologies. The strategic alliance of the US and the PR has shifted ever since, which changed the landscape of the world. The divide between the capitalist economy and the socialist planning economy started to thaw, and non-state owned rural and township enterprises thrived in the eastern coast of the PR in the last years of the Mao Zedong era. It can be concluded that these events and strategic changes dominated the world for approximately a whole quarter in the final episode of the 20<sup>th</sup> century, with their impact eventually appearing to fade away around the turn of the century following the 1997 Asian financial crisis. This period witnessed globalization, liked or disliked, reaching every corner of the world. While the former sub-period observed the coming, as well as experienced the nearly going, of the G7, the G7 persisted and the idea for an ambiguous Gx<sup>3</sup> to replace the G7 never materialized in the end. Then the

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<sup>3</sup> The first G7 (six of G7 nations without Canada) Summit was held in November 1975 at Rambouillet, France. In June 1976, Canada joined the group at the San Juan Summit, in Puerto Rico, the United States, marking the birth of the G7. In May 1977 the European Community /European Union joined the group at the London Summit. In July 1989, delegations of 15 developing countries met with the G7 delegations on the eve of the so-called Summit of the Arch, at Paris. Since 1991, limited participation of the USSR and then Russia became more involved gradually and May 1998 witnessed the creation of G8 when Russia developed into a full member at the Birmingham Summit in the UK, though meetings of finance ministers were still mainly confined to the G7. In December 1999, finance ministers and central bank governors of the G20 held their inaugural meeting in Berlin, Germany.

new millennium began with the introduction of the euro<sup>4</sup>, which integrated three of the G7 economies further within the G7 and changed the economic relationship between the G7 and the rest of the world to a certain extent. The G20 become more settled alongside the G7. The latest global financial crisis, and the 2009 London G20 summit that took place less than three months after the 2008 Washington summit, signified that the G20 had become a more relevant economic grouping for the task to “broaden the dialogue on key economic and financial policy issues among systemically significant economies.” (G-20 Meeting of Finance Ministers and Central Bank Governors 1999). It is evident that, to the G7 and with regard to the G7, the world economic landscape has changed considerably since the millennium.

#### 4.1. *First episode*

Prior to progressing to the examination of the results in the frequency domain, time domain features of GDP of these economies are reported in Table 1 for preliminary statistics of the individual countries, and in Table 2 for correlations between the countries. These statistics are based on quarterly changes or growth in respective GDP. Table 1, provided to document relevant figures, is self-evident and needs no further explanations. In Table 2, the correlation between a pair of economies is the usual measure for the closeness between the pair. Seven out of the 21 pairs have a correlation coefficient that is significant at the 1% level, and additional two pairs have a correlation coefficient significant at the 5% level. All correlation coefficients, significant or not, are

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<sup>4</sup> January 1, 1999 marked the official and nominal inception of the euro, when the euro remained a book currency only, notes and coins in circulation were still denominated in the national currencies. It was not until January 1, 2002 that euro notes and coins began to circulate alongside the national currency notes and coins.

positive, indicating G7 economies move in the same direction in general, though their phase differences have to be examined in the frequency domain. It is observed that the correlation of the output growth of the US and that of Canada is the highest, followed by Germany and the UK, and the output growth of the US is least correlated with that of Japan. The highest correlation of the output growth of Germany is with that of France, followed by that of Italy, the UK and the US. All four European economies are highly correlated, among them the UK has the weakest link but the link is still higher than that with the US. Japan has the lowest correlation in output growth with all of the rest. It appears that geography still matters in the modern time.

**{Table 1 about here}**

**{Table 2 about here}**

Cross spectra, coherence and phases in this empirical study are obtained from executing procedures in RATS, letting the time series in concern pass through a tent window in estimation. A tent window has the feature of curved edges to avoid generating spurious frequencies while ensuring consistency. The number of frequencies  $N'$  is not always the same as the number of observations in the time domain  $N$ . The Fourier transform performs most efficiently when the number is in the form of  $2^m$  that is equal to or greater than  $N$ , where  $m$  is an integer. So, in this study, with  $N$  being 107 (27 years' quarterly data, loss of one observation in log difference operations),  $N'$  is chosen as 128 and the window size  $M$  is chosen as 65 ( $N'/2+1$ ) by the procedure using RATS. Following equation (11) through to equation (14), the

significance test statistic is  $\sqrt{\frac{3N}{2M}} = \sqrt{\frac{3 \times 128}{2 \times 65}} = 1.7187$  for  $k \neq 0$ , which is significant for

all the frequency points except the zero frequency<sup>5</sup>. The estimated coherence and phases for the seven economies are exhibited in Figure 3. There are 21 pairs of economies and so are the graphs. Same as in Figure 1, coherence is plotted against the left hand side axis (in blue) with minimum being zero and maximum being one; and phases are plotted against the right hand side axis (in orange) with minimum being -1 (a half cycle lag) and maximum being 1 (a half cycle lead)<sup>6</sup>. While these graphs provide visual observations about the co-movement of economies in terms of closeness and phases on the whole spectrum, some kinds of summaries may be helpful for making overall assessments. Table 3 is provided to report précis of average coherence, overall leads/lags, and coincidence over the whole spectrum. In doing so, Table 3 does not take the full advantage of spectral analysis to examine co-movements with respect to short, medium and long cycle features. It is similar to time domain correlation analysis but included are the contemporary as well as led/lagged correlations, with the phase additionally indicating the degree of lead/lag relations. We further carve up the whole spectrum into long-run trends, lower frequencies, higher frequencies, and middle ranges with the formula discussed in sub-section 2.3, paying specific attention to

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<sup>5</sup> The statistic is  $\sqrt{\frac{3N}{4M}} = \sqrt{\frac{3 \times 128}{4 \times 65}} = 1.2153$  for  $k = 0$  at the zero frequency point. The statistic would have been significant if the window width were chosen as 33. However, a joint consideration of the requirement that a window should not be too narrow to produce imprecise estimates and the fact that the statistic is all significant at all the non-zero frequency points means that a window width of 65 is the most favorable choice. Moreover, we do not evaluate the business cycle features at the zero frequency point alone; instead, we examine a frequency range from 0 to 0.05 for very long cycles and long-run trends. So, the possible defect at the zero frequency point is minimized.

<sup>6</sup> The phase is scaled down by  $\pi$ . So a half cycle lead is 1 as against  $\pi$ , and a half cycle lag is -1 against  $-\pi$ , to make interpretation simpler. It is possible that X leads Y and Y leads Z, but Z leads X instead of lags X. This happens when the sum of the leads (X leads Y and Y leads Z) is greater than 1 and smaller than 2.

shorter cycles, longer cycles, and medium cycles respectively. These curve up summaries are reported in Tables 4 to 7 and will be analyzed next.

**{Figure 3 about here}**

**{Table 3 about here}**

We inspect the results in each of the rows of Table 3, i.e., inspect each economy's co-movements with the rest six in turn. The US economy shares the highest similarity with the Canadian economy, with the largest coherence, smallest phase lead/lag, and highest coincidence. Canada, on the other hand, has the highest coincidence and smallest lead/lag with the US, but shares the largest coherence, a little oddly, with the UK, though its coherence with the US is also fairly large (the second largest). The US appears to lag Canada in business cycle phases but the lag is only about 0.0025 cycles (recall that -1 is a half cycle lag). In Europe, Germany shares the largest coherence with Italy while Germany is in the closest phase as the UK in business cycles, measured by the degrees of lead/lag and coincidence. France is found to share the largest coherence with Italy too and is highly in pace with Germany in business cycle phases. The UK, though oddly shares the largest coherence with Canada, is highly in pace with Germany in business cycle phases. Italy is clearly linked to France and Germany in business cycle co-movements. Finally, Japan shares the least similarity with the rest of G7 economies in all the terms of coherence, phase lead/lag and coincidence. As these measures are blended over the whole range of the spectrum, there are some atypical

results, e.g., the US seems to lag the UK, Japan and Germany, though the lags are very small.

From the viewpoint of economic fundamentals and economies' adjustment to changes in the fundamentals, we would expect that economies share more coherence at lower frequencies, or in longer cycles, on the whole. Table 4 reports these results on the long-run trend and very long cycles over ten years; whereas Table 5 reports the results on lower frequencies covering the traditional business cycle ranges of up to ten years. These results, compared with those in Table 3, are more consistent across the countries in explaining business cycle characteristics in the longer run that is more relevant in our studies.

**{Table 4 about here}**

**{Table 5 about here}**

It is clearly observed in Table 4 and Table 5 that coherences among these economies are considerably higher than those documented in Table 3. That is, there exists higher coherence in business cycle co-movements in longer cycles or at lower frequencies than that in the whole range. Moreover, the US leads all the other economies in business cycle phases to varied degrees in the traditional business cycle ranges, which appears to be sound, considering the dominant role of the US in the world economy and affairs. For example, the phase lead of the US to the UK is 0.1283 that is about a 0.064 cycle lead. For a five-year cycle, it is 0.32 years or a lead slightly longer than one quarter; and for a ten-year cycle, it is 0.64 years or about a three-



quarter lead. The leads of the US to France, Germany and Italy are slightly longer; and the lead/lag between the US and Canada and that between the US and Japan are minimum.

Although the US and the UK share the largest coherence in very long cycles and long-run trends as evident in Table 4, they do not show the same in the traditional business cycle ranges. The largest coherence of the UK in the traditional business cycle ranges, as revealed in Table 5, occurs to be with France, followed by that with Germany, Italy and the US. Indeed, all the four European economies move together in terms of business cycle coherence and phases, though they also enjoy co-movements with the US and Japan to varied degrees in the long run. France has the highest coherence with the rest three European economies and the lowest coherence with Canada overall, and is least in pace with Japan and Canada in business cycle phases. Though Germany shares the largest coherence with France in the long-run and very long cycles, overall it shares the largest coherence, the smallest phase lead/lag and the highest coincidence with Italy, followed by its similarities with Japan in these terms, and has the most discrepancies with Canada. Even in the longer term, Japan has fairly small coherence with the US and appears to enjoy co-movements with the European economy. Canada shares business cycle similarities with the US to the extent greater than that with Europe and Japan. It is difficult to match Canada with Europe and Japan in business cycle patterns, except its somewhat oddly largest coherence with France in the long-run trend and very long cycles.

{Table 6 about here}

{Table 7 about here}

As expected, coherence between economies is considerably smaller at higher frequencies and in middle ranges, than that at lower frequencies or in longer cycles, consistently across all pairs except Japan with the US and the UK. Overall, coherence is slightly smaller at higher frequencies than that in middle ranges. Furthermore, there are no clear patterns of linkage between certain groups of economies in their business cycle co-movements, especially in middle ranges. In short cycles or at higher frequencies, the UK shares the least similarity with the US, with the smallest coherence, the largest phase lag and the lowest coincidence. Germany appears to bear much similarity with the UK, with the largest coherence and the highest coincidence, so does France with Italy. But Germany and France have the smallest coherence between them at higher frequencies or in shorter cycles. Some of these results, if mistaken as the whole story, would appear to be controversial. Nevertheless, the analysis indicates that the focus of international business cycle co-movements is not on short cycle features<sup>7</sup>.

One of the implications that coherence is the highest and considerably higher at lower frequencies implies that economies tend to move together in longer cycles or at lower frequencies, even if they do not behave so in the short to medium terms. It in turn implies support for, and extension of, real business cycle theory with technology shocks or real shocks impacting economies across borders. This is due to the facts that

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<sup>7</sup> This does not mean that one can simply use lower frequency data, such as annual data. As footnote 2 implies that the spectrum using annual GDP would be distorted and be different from the spectrum at annual and lower frequencies using quarterly GDP.

there exist clearer patterns in international co-movements of national economies at lower frequencies or in longer cycles and that there are considerably weaker linkages and no identifiable patterns of linkages between certain groups of economies in their business cycle co-movements in middle ranges and shorter cycles.

#### 4.2. *Second episode*

Similar to the analysis of the first episode, time domain features of GDP of these economies are presented first in Table 8 and Table 9. Growth in GDP across G7 economies is lower than that in the first sub-period, due to the latest financial crisis. The mean of quarter on quarter growth in this sub-period is about two thirds of that in the first sub-period for the G7 as a whole, whereas the volatility in output remains largely unchanged. Yet, G7 economies have become increasingly integrated in this sub-period. Correlation coefficients are highly significant at the 1% level between all pairs of economies in output growth. Moreover, the correlation coefficient is greater than 0.5 between all pairs of economies, and is greater than 0.65 in nearly two thirds cases. This is in utter contrast to the observed correlation in the first episode, where less than one fifth pairs have a correlation of 0.3 or higher. The correlation between the three euro economies is the highest, surpassing the correlation between the US and Canada in output growth. The correlation between the US and Canada becomes even lower than that between the US and the UK and between the US and France. There are noticeable developments in the landscape of the G7 since the new millennium, which will be inspected in detail in the frequency domain in the following.

As with the first episode, the number for the Fourier transform  $N'$  is in the form of  $2^m$  that is equal to or greater than the number of time-domain observations  $N$ . There are 48 quarterly data in the second sub-period (last quarter in 1999 is included for log difference operations, so this sub-period is continuous from the previous sub-period), so  $N'$  is then chosen as 64 and the window size  $M$  is chosen as 33 ( $N'/2+1$ ) by the procedure using RATS. The significance test statistic is  $\sqrt{\frac{3N}{2M}} = \sqrt{\frac{3 \times 64}{2 \times 33}} = 1.7056$  for  $k \neq 0$ . The estimated coherence and phases in episode 2 for the G7 are plotted in Figure 4. Compared with Figure 3, it can be observed that G7 economies have become synchronized considerably in this sub-period, with stronger coherence, reduced phase leads/lags and increased coincidence, especially in lower frequencies. Coherence is as high as 0.9 in lower frequencies for over half of the total cases; and the phase line is rather smooth around 0, except that in higher frequencies. These are in illustrious contrast to episode 1 where coherence can only reach 0.9 in few pairs in very long cycles or long-run trends, and the phase line fluctuated widely over the whole spectrum.

**{Figure 4 about here}**

**{Table 10 about here}**

Table 10 reports coherence, overall leads/lags, and coincidence for the whole spectrum. The US economy no longer shares the highest similarity with the Canadian economy; it is the UK in terms of coherence, Japan in terms of phase leads and Germany in terms of coincidence. On the other hand, Canada still shares the largest

coherence with the US. These new pictures, however, should be viewed in the context that all G7 economies become synchronized in episode 2. The coherence between the US and Canadian economies has also increased, but the coherence between the US and the UK and that between the US and France have increased even more. The US still appears to lag Canada in business cycle phases but the lag is less than 0.05 cycles, whereas the US leads the rest of the G7. Canada is the best performing economy since the new millennium after all – its growth in GDP is the highest amongst the G7 and it is least affected by the global financial crisis. In Europe, Germany still shares the largest coherence with Italy and moreover, is in the closest phase measured by the degree of phase differences. On the other hand, France is found to share the largest coherence with Germany and is highly in pace with Germany in business cycle phases in terms of coincidence. The UK is highly in pace with Germany in business cycle phases, though it shares the largest coherence with the US. Italy has further integrated into Germany in business cycle co-movements, with the largest coherence, smallest phase difference and highest coincidence. Japan has also become more integrated with the rest of the G7.

**{Table 11 about here}**

**{Table 12 about here}**

The results on the long-run trend and very long cycles are presented in Table 11, while the results on lower frequencies covering the traditional business cycle ranges are presented in Table 12. Similar to episode 1, the coherence among G7 economies in episode 2 is considerably higher in business cycle co-movements in long-run trends

and longer cycles than that in the whole range; and the coherence in episode 2 is considerably higher than that in episode 1. In particular, G7 economies have synchronized most in longer cycles or lower frequencies, comparing figures in Table 12 with those in Table 5. The UK shares the largest coherence in very long cycles and traditional business cycles with the US as well as France, indicating G7 economies have become increasingly integrated across continents. The largest coherence of the UK in very long cycles is with France, followed by that with the US and Japan; while its largest coherence in the traditional business cycle ranges is with the US, followed by that with Japan and France. The three euro land economies, as well as the UK economy, move even closer, against a background that all G7 economies have also become more synchronized during this sub-period. Then, the results on higher frequencies and middle ranges are reported in Table 13 and Table 14 respectively. On the one hand, the coherence between G7 economies in this sub-period is considerably and consistently smaller at higher frequencies and in middle ranges, than that at lower frequencies, across all pairs with no exceptions as in episode 1. On the other hand, the coherence in these ranges is higher in this episode than that in episode 1 across all pairs of G7 economies. The results confirm that the focus of international business cycle co-movements is not on short cycle features. Meanwhile, integration in G7 economies, though being dominant at lower frequencies, has taken place over the whole spectrum.

**{Table 13 about here}**

**{Table 14 about here}**

## 5. Conclusions

In this paper, business cycle coherence and phases in output growth amongst the G7 have been inspected in the frequency domain. The present study has contributed to the existing literature by extending the decomposition of the trend component and the cycle component to the decomposition of all frequency components in a transformed domain on the one hand, and enriching empirical research on international business cycle on the other hand. The empirical investigation in the study has been reinforced by the frequency domain analysis method, which is more effective in presentation when cycles and phases are concerned. The paper has identified patterns in international business cycle co-movements among the G7, offering a general outlook of international business cycle co-movements and detailing the lower frequency, higher frequency and middle range characteristics of international linkages of output growth and fluctuations. It opens up a new channel of research in our continuing search for knowledge and understanding about international business cycles.

Due to these distinctive features, this study is able to examine business cycles more effectively with regard to international linkages of economic activity and international transmission of output fluctuations, compared with previous research. The results and findings of this study can be summarized as follows. Firstly, there exist co-movements among G7 economies in output growth and fluctuations overall, in terms of coherence; and the co-movements are not in the same pace, exhibited by phase leads/lags and phase differences. Secondly, co-movements among G7 economies are

considerably stronger at lower frequencies or in longer cycles. There are also clearer patterns of linkages of international output growth and fluctuations in longer business cycles. Thirdly, co-movements between G7 economies are not only considerably weaker in shorter cycles and in middle ranges than in longer cycles, but also exhibit no clear patterns of linkage between groups of economies in international output fluctuations.

To engage in the debate of whether output growth and fluctuations have become more closely linked across economies over time, the present study has investigated G7 economies in two episodes. The fourth major finding of this study is that G7 economies have become considerably more synchronized in episode 2. The three euro economies become more integrated since the millennium, against a background that all G7 economies have also become more synchronized. In episode 1, Japan has fairly small coherence with the US even in longer cycles. This has changed in episode 2 too as Japan shares fairly high coherence with the US, as well as the rest of the G7. As suggested earlier, to the G7 and with regard to the G7, the world economic landscape has changed considerably since the millennium.

The above results and findings render two non-trivial implications. Firstly, as economies tend to move together in longer cycles or at lower frequencies, even if they do not behave so in the short to medium terms, analysis in different ranges of cycles is technically viable to identify useful patterns in international linkages of output growth and fluctuations. Secondly, the results and findings are in support of real business cycle



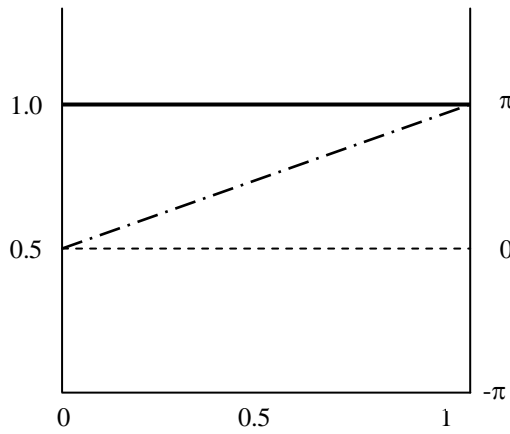
theory being extended to an international arena, with long effect real shocks impacting economies across borders, which also implies that shocks of short term nature, such as monetary shocks, play little role in inducing international co-movements of business cycles.

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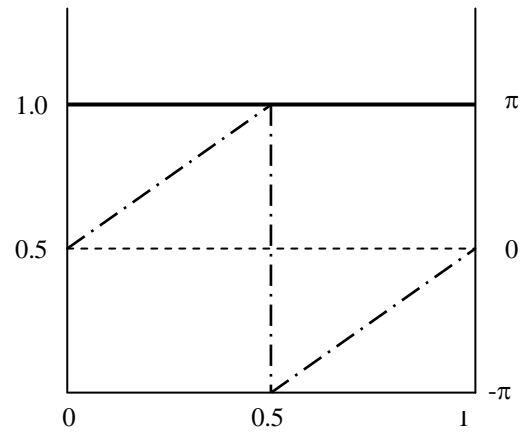
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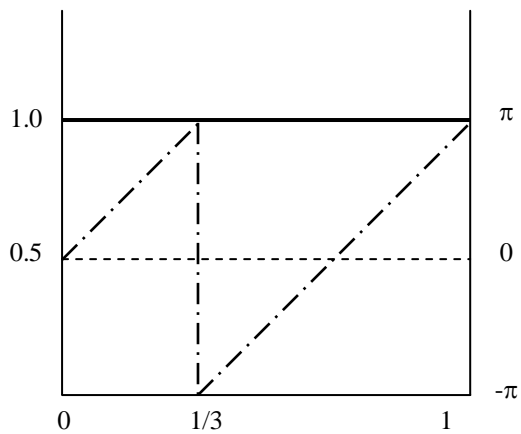
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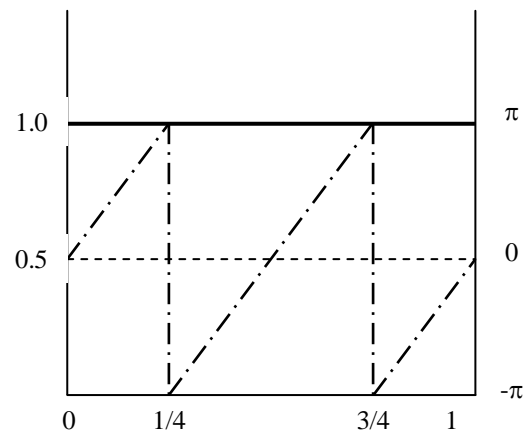
(a)  
 Perfect coherence  
 one phase lag  
 Coherence: LHS  
 Phase: RHS



(b)  
 Perfect coherence  
 two phase lags  
 Coherence: LHS  
 Phase: RHS



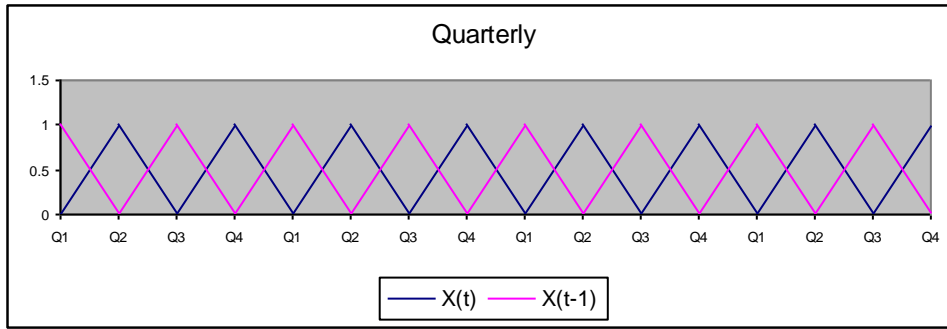
(c)  
 Perfect coherence  
 Three phase lags  
 Coherence: LHS  
 Phase: RHS



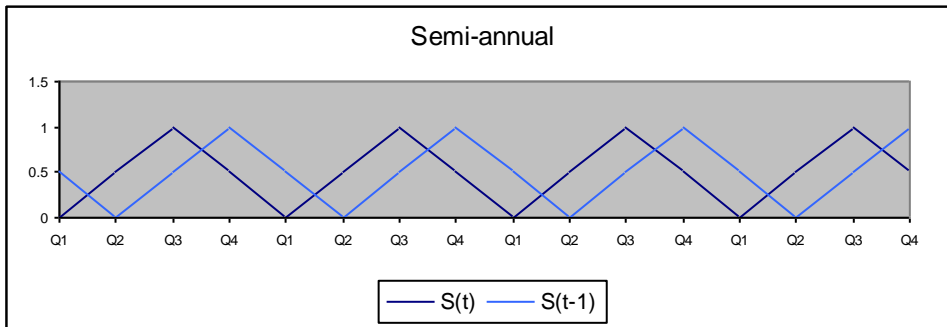
(d)  
 Perfect coherence  
 Four phase lags  
 Coherence: LHS  
 Phase: RHS

———— Coherence    - · - · - Phase

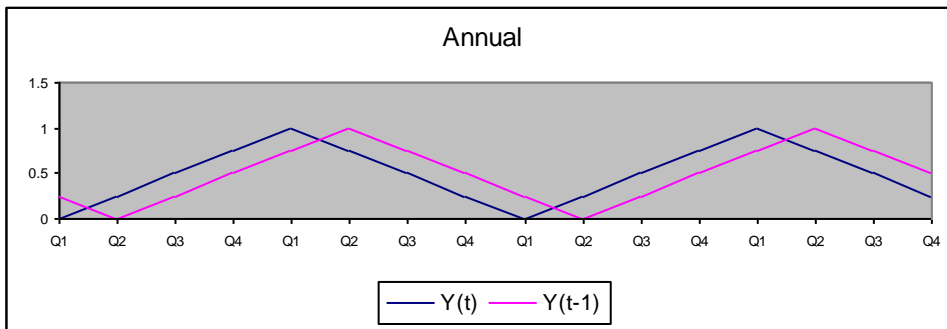
**Figure 1. Coherence and phase**



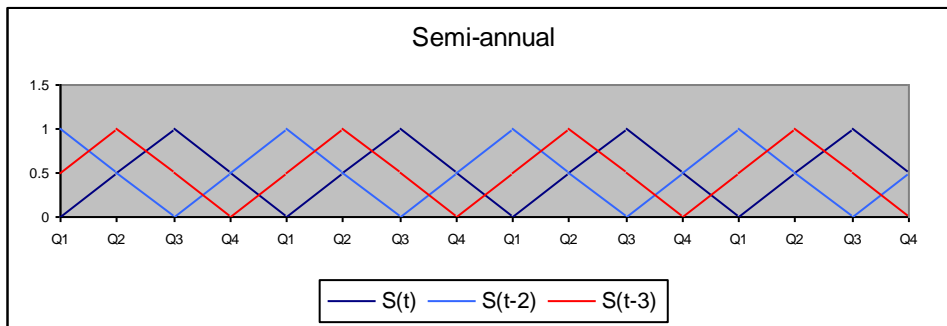
(a) lag in one quarter is half cycle



(b) lag in one quarter is 1/4 cycle

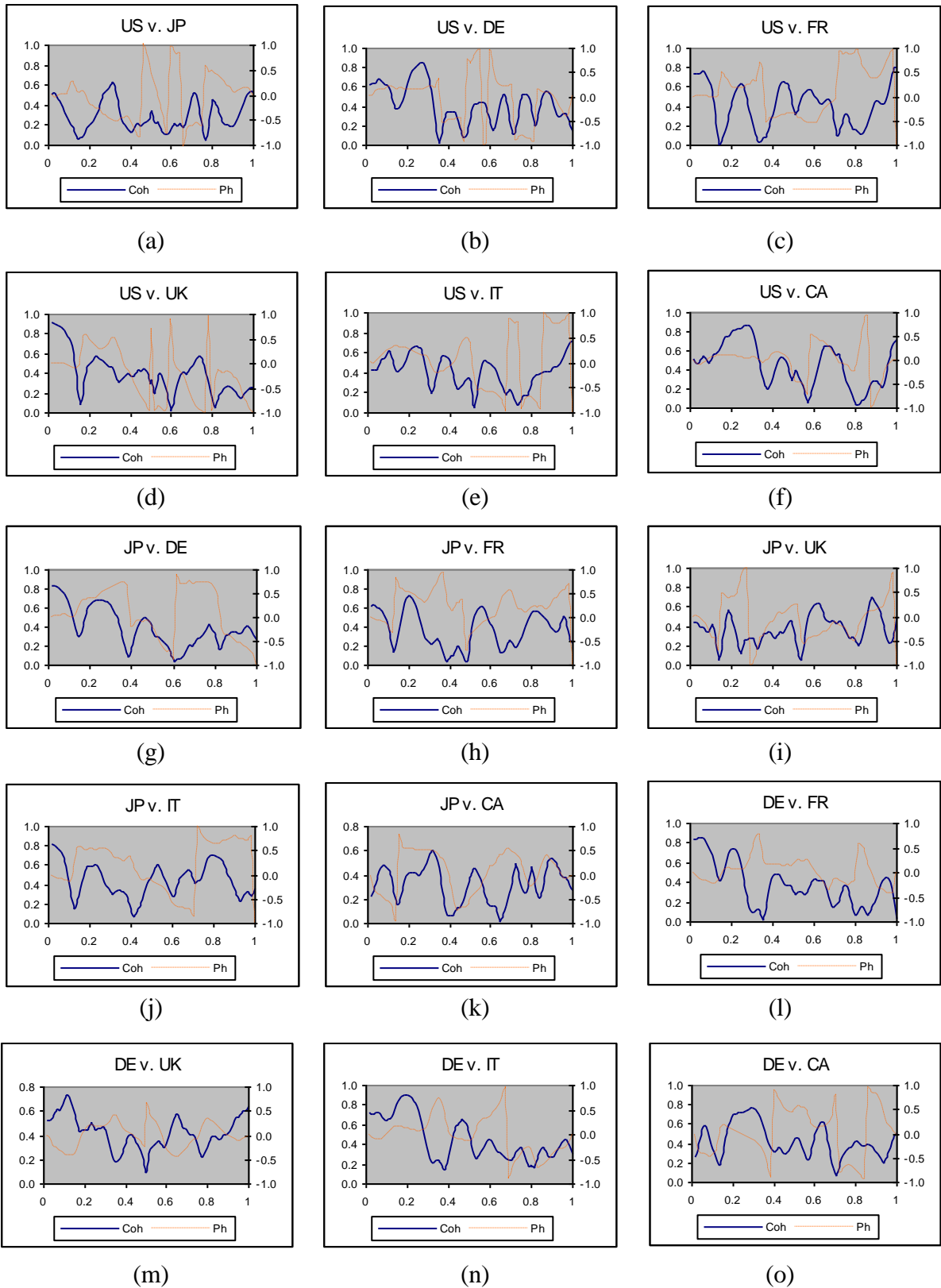


(c) lag in one quarter is 1/8 cycle



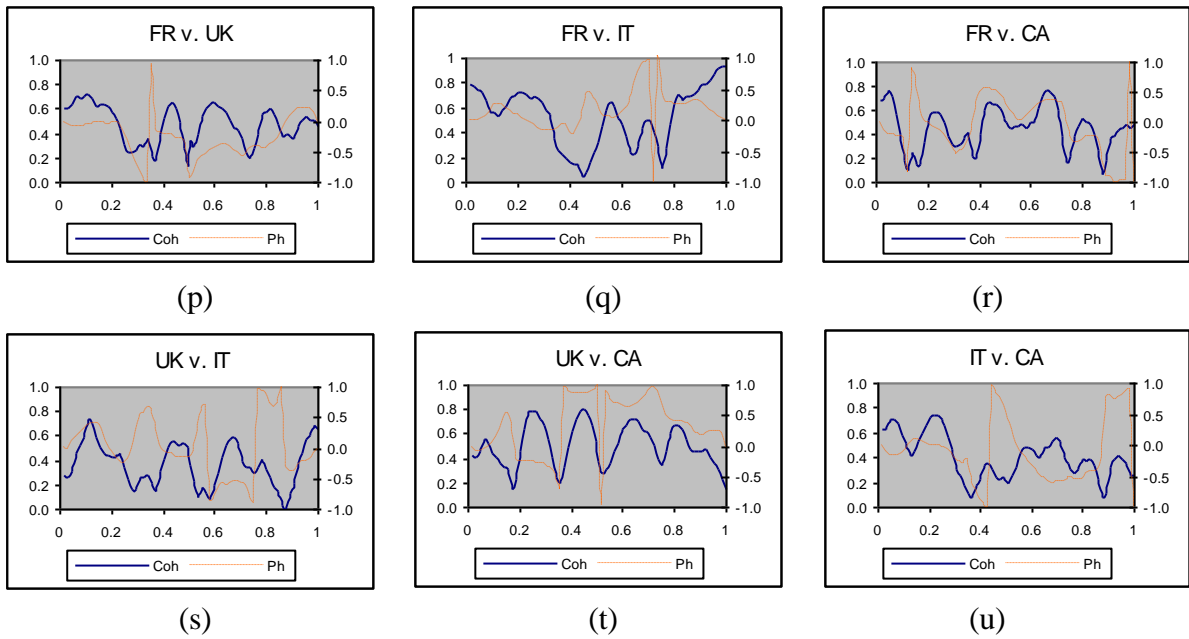
(d) a three-quarter lag is equivalent to a one-quarter lead in a four-quarter cycle

**Figure 2. Exhibits of phase lags/leads**

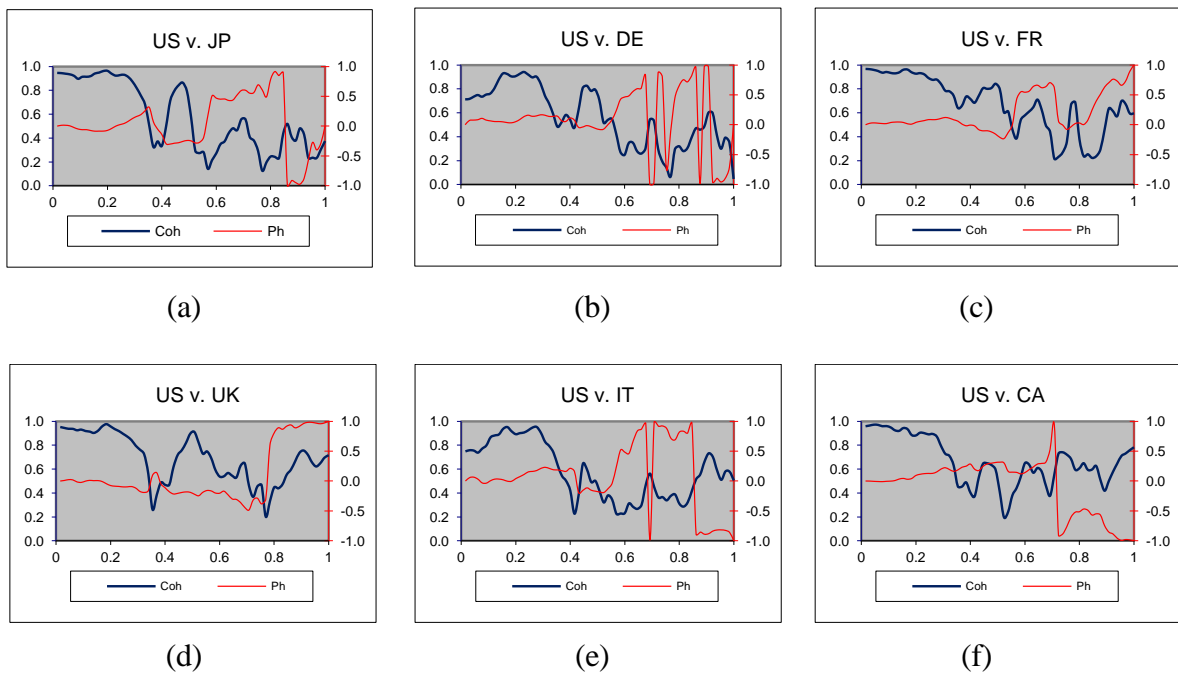


**Figure 3. Coherence and phase lags/leads between G7 economies – first episode**

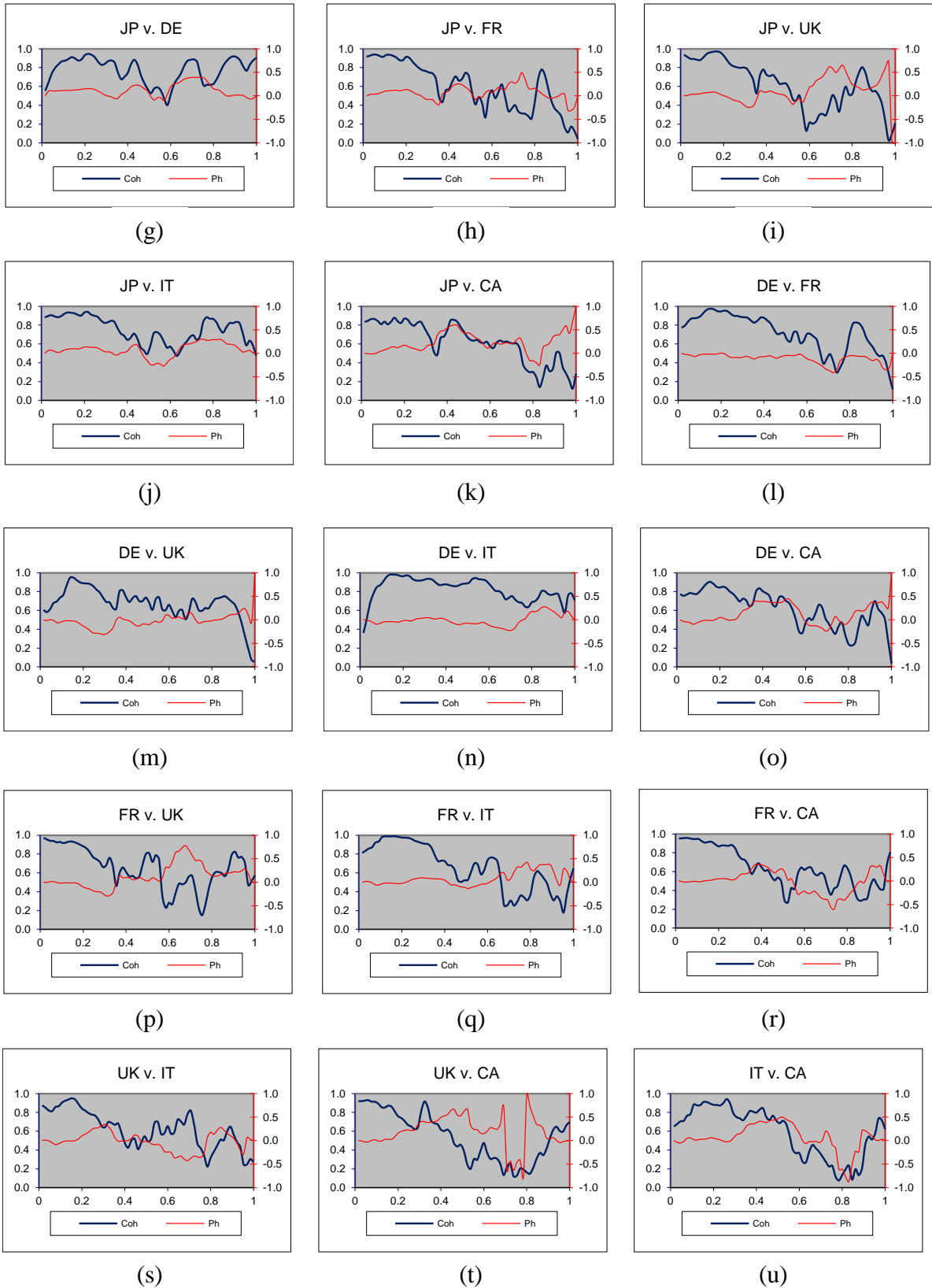
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**Figure 3. Coherence and phase lags/leads between G7 economies – first episode**  
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**Figure 4. Coherence and phase lags/leads between G7 economies – second episode**  
(continued next page)



**Figure 4. Coherence and phase lags/leads between G7 economies – second episode**

(continued from previous page)



**Table 1. Preliminary descriptive statistics – episode 1**

	US	JP	DE	FR	UK	IT	CA
Mean	0.7404e-2	0.6895e-2	0.4689e-2	0.5568e-2	0.4888e-2	0.5766e-2	0.6402e-2
Max	0.3780e-1	0.2582e-1	0.3395e-1	0.2431e-1	0.3549e-1	0.3337e-1	0.4059e-1
Min	-0.2060e-1	-0.3497e-1	-0.2275e-1	-0.1964e-1	-0.2765e-1	-0.2379e-1	-0.2930e-1
Median	0.8180e-2	0.7200e-2	0.4060e-2	0.6440e-2	0.5950e-2	0.4660e-2	0.6290e-2
Std	0.8666e-2	0.9384e-2	0.9576e-2	0.7564e-2	0.9239e-2	0.8553e-2	0.1050e-1

US: the United States; JP: Japan; DE: Germany; FR: France; UK: the United Kingdom; IT: Italy; CA: Canada.  
Apply to all the tables and figures.

**Table 2. Correlation statistics – episode 1**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.0570	0.2292*	0.1765	0.2200*	0.1574	0.4303 <sup>†</sup>
JP	0.0570	1.0000	0.1297	0.1760	0.1892	0.0585	-0.0141
DE	0.2292	0.1297	1.0000	0.3261 <sup>†</sup>	0.2732 <sup>†</sup>	0.3570 <sup>†</sup>	0.1713
FR	0.1765	0.1760	0.3261	1.0000	0.2966 <sup>†</sup>	0.4247 <sup>†</sup>	0.1853
UK	0.2200	0.1892	0.2732	0.2966	1.0000	0.1312	0.0564
IT	0.1574	0.0585	0.3570	0.4247	0.1312	1.0000	0.3661 <sup>†</sup>
CA	0.4303	-0.0141	0.1713	0.1853	0.0564	0.3661	1.0000

\* significant at the 5% level; <sup>†</sup> significant at the 1% level.

**Table 3. Coherence and phases – episode 1**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	<u>0.2957</u>	0.4245	0.4095	0.3987	0.4130	<b>0.4659</b>
	0.0000	-0.0418	-0.0365	0.1175	<u>-0.2195</u>	0.0433	<b>-0.0051</b>
	0.0000	0.3947	0.3857	0.4211	0.4621	<u>0.4767</u>	<b>0.2298</b>
JP	<u>0.2957</u>	1.0000	0.3967	0.3736	0.3729	<b>0.4473</b>	0.3251
	<b>0.0418</b>	0.0000	0.1104	<u>0.2200</u>	-0.0712	0.1977	0.0444
	0.3947	0.0000	0.4641	<b>0.3529</b>	0.3542	<u>0.4770</u>	0.3702
DE	0.4245	0.3967	1.0000	<u>0.3768</u>	0.4223	<b>0.4562</b>	0.4269
	0.0365	<u>-0.1104</u>	0.0000	0.0259	<b>-0.0062</b>	-0.0098	0.0352
	0.3857	<u>0.4641</u>	0.0000	0.2148	<b>0.2108</b>	0.3022	0.4220
FR	0.4095	<u>0.3736</u>	0.3768	1.0000	0.4763	<b>0.5310</b>	0.4530
	-0.1175	-0.2200	<b>-0.0259</b>	0.0000	<u>-0.2580</u>	0.1613	-0.0637
	<u>0.4211</u>	0.3529	<b>0.2148</b>	0.0000	0.3252	0.2408	0.3853
UK	0.3987	<u>0.3729</u>	0.4223	0.4763	1.0000	0.3732	<b>0.5044</b>
	0.2195	0.0712	<b>0.0062</b>	0.2580	0.0000	0.0259	<u>0.3380</u>
	0.4621	0.3542	<b>0.2108</b>	0.3252	0.0000	0.3889	<u>0.4961</u>
IT	0.4130	0.4473	0.4562	<b>0.5310</b>	<u>0.3732</u>	1.0000	0.4118
	-0.0433	<u>-0.1977</u>	<b>0.0098</b>	-0.1613	-0.0259	0.0000	-0.1072
	0.4767	<u>0.4770</u>	0.3022	<b>0.2408</b>	0.3889	0.0000	0.4125
CA	0.4659	<u>0.3251</u>	0.4269	0.4530	<b>0.5044</b>	0.4118	1.0000
	<b>0.0051</b>	-0.0444	-0.0352	0.0637	<u>-0.3380</u>	0.1072	0.0000
	<b>0.2298</b>	0.3702	0.4220	0.3853	<u>0.4961</u>	0.4125	0.0000

First row: coherence; second row: overall lags  $\varphi$  (economies in column titles lead economies in row titles, negative figures mean lags); third row: coincidence  $\theta$ .

The largest coherence in each row is in bold, the smallest is underlined.

The smallest overall lag/lead in each row is in bold, the largest is underlined.

The highest coincidence in each row is in bold, the lowest is underlined.

The significance test statistic is  $\sqrt{\frac{3N}{2M}} = \sqrt{\frac{3 \times 128}{2 \times 65}} = 1.7187$  for  $k \neq 0$ , which is significant for all the frequency points except the zero frequency.

**Table 4. Coherence and phases (very long cycles and long-run trends) – episode 1**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.4954	0.6367	0.7362	<b>0.9012</b>	<u>0.4330</u>	0.4917
	0.0000	-0.0175	0.0449	0.0077	0.0067	<b>0.0052</b>	<u>-0.0639</u>
	0.0000	0.0175	0.0449	0.0077	<b>0.0067</b>	0.0214	<u>0.0639</u>
JP	0.4954	1.0000	<b>0.8309</b>	0.6186	0.4272	0.7988	<u>0.2967</u>
	0.0175	0.0000	0.0212	-0.0250	<b>0.0007</b>	-0.0374	<u>-0.2147</u>
	<b>0.0175</b>	0.0000	0.0212	0.0250	0.0203	0.0374	<u>0.2147</u>
DE	0.6367	0.8309	1.0000	<b>0.8499</b>	0.5363	0.7215	<u>0.3823</u>
	-0.0449	<b>-0.0212</b>	0.0000	-0.0618	-0.0868	-0.0541	<u>-0.2429</u>
	0.0449	<b>0.0212</b>	0.0000	0.0618	0.0868	0.0541	<u>0.2429</u>
FR	0.7362	0.6186	<b>0.8499</b>	1.0000	<u>0.6163</u>	0.7648	0.7181
	-0.0077	0.0250	0.0618	0.0000	-0.0410	<b>-0.0045</b>	<u>-0.1128</u>
	0.0077	0.0250	0.0618	0.0000	0.0410	<b>0.0054</b>	<u>0.1128</u>
UK	<b>0.9012</b>	0.4272	0.5363	0.6163	1.0000	<u>0.2872</u>	0.4292
	-0.0067	<b>-0.0007</b>	<u>0.0868</u>	0.0410	0.0000	0.0353	-0.0596
	<b>0.0067</b>	0.0203	<u>0.0868</u>	0.0410	0.0000	0.0502	0.0596
IT	0.4330	<b>0.7988</b>	0.7215	0.7648	<u>0.2872</u>	1.0000	0.6558
	-0.0052	0.0374	0.0541	<b>0.0045</b>	-0.0353	0.0000	<u>-0.0853</u>
	0.0214	0.0374	0.0541	<b>0.0054</b>	0.0502	0.0000	<u>0.0853</u>
CA	0.4917	<u>0.2967</u>	0.3823	<b>0.7181</b>	0.4292	0.6558	1.0000
	0.0639	0.2147	<u>0.2429</u>	0.1128	<b>0.0596</b>	0.0853	0.0000
	0.0639	0.2147	<u>0.2429</u>	0.1128	<b>0.0596</b>	0.0853	0.0000

See notes to Table 3.

**Table 5. Coherence and phases (lower frequencies) – episode 1**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	<u>0.2016</u>	0.5414	0.3948	0.5423	0.5201	<b>0.6215</b>
	0.0000	0.0577	0.1670	0.1523	0.1283	<u>0.2565</u>	<b>0.0550</b>
	0.0000	0.0994	0.1670	0.1561	0.1916	<u>0.2565</u>	<b>0.0749</b>
JP	<u>0.2016</u>	1.0000	<b>0.5502</b>	0.4675	0.3547	0.4592	0.3588
	-0.0577	0.0000	0.1656	0.2210	<b>-0.0352</b>	<u>0.2261</u>	-0.1172
	<b>0.0994</b>	0.0000	0.1659	0.3990	0.3747	0.3252	<u>0.6009</u>
DE	0.5414	0.5502	1.0000	0.6487	0.5864	<b>0.7669</b>	<u>0.4422</u>
	-0.1670	-0.1656	0.0000	-0.0960	<u>-0.2665</u>	<b>0.0675</b>	-0.1432
	0.1670	0.1659	0.0000	0.1390	<u>0.2851</u>	<b>0.1081</b>	0.2722
FR	0.3948	0.4675	0.6487	1.0000	<b>0.6615</b>	0.6322	<u>0.3362</u>
	-0.1523	<u>-0.2210</u>	0.0960	0.0000	-0.0483	0.1518	<b>0.0069</b>
	0.1561	<u>0.3990</u>	0.1390	0.0000	<b>0.0483</b>	0.1518	0.3572
UK	0.5423	<u>0.3547</u>	0.5864	<b>0.6615</b>	1.0000	0.5461	0.3779
	-0.1283	<b>0.0352</b>	<u>0.2665</u>	0.0483	0.0000	0.2557	0.1353
	0.1916	<u>0.3747</u>	0.2851	<b>0.0483</b>	0.0000	0.2641	0.2398
IT	0.5201	<u>0.4592</u>	<b>0.7669</b>	0.6322	0.5461	1.0000	0.5866
	<u>-0.2565</u>	-0.2261	-0.0675	-0.1518	-0.2557	0.0000	<b>-0.0225</b>
	0.2565	<u>0.3252</u>	0.1081	0.1518	0.2641	0.0000	<b>0.0800</b>
CA	<b>0.6215</b>	0.3588	0.4422	<u>0.3362</u>	0.3779	0.5866	1.0000
	-0.0550	0.1172	<u>0.1432</u>	<b>-0.0069</b>	-0.1353	0.0225	0.0000
	<b>0.0749</b>	<u>0.6009</u>	0.2722	0.3572	0.2398	0.0800	0.0000

See notes to Table 3.

**Table 6. Coherence and phases (higher frequencies) – episode 1**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.3211	0.3771	0.3816	<u>0.2215</u>	<b>0.4413</b>	0.2785
	0.0000	0.2479	-0.2735	<u>0.5693</u>	-0.4208	0.1372	<b>-0.1236</b>
	0.0000	<b>0.2479</b>	0.3383	0.6943	0.5428	<u>0.8386</u>	0.5428
JP	<u>0.3211</u>	1.0000	0.3282	0.4422	0.4046	<b>0.4725</b>	0.3715
	-0.2479	0.0000	-0.2556	0.2387	<b>0.0004</b>	<u>0.6062</u>	0.0513
	0.2479	0.0000	0.5618	0.3637	0.3381	<u>0.7312</u>	<b>0.1627</b>
DE	0.3771	0.3282	1.0000	<u>0.2199</u>	<b>0.4370</b>	0.3101	0.3610
	0.2735	0.2556	0.0000	0.0259	0.0835	<u>-0.3777</u>	<b>0.0238</b>
	0.3383	0.5618	0.0000	0.2148	<b>0.1304</b>	0.3777	<u>0.6045</u>
FR	0.3816	0.4422	<u>0.2199</u>	1.0000	0.4773	<b>0.7295</b>	0.3963
	<u>-0.5693</u>	-0.2387	<b>-0.0259</b>	0.0000	-0.0738	0.2117	-0.5122
	<u>0.6943</u>	0.3637	0.2148	0.0000	0.2295	<b>0.2117</b>	0.6359
UK	<u>0.2215</u>	0.4046	0.4370	<b>0.4773</b>	1.0000	0.3382	0.4605
	<u>0.4208</u>	<b>-0.0004</b>	-0.0835	0.0738	0.0000	0.2442	0.2922
	<u>0.5428</u>	0.3381	<b>0.1304</b>	0.2295	0.0000	0.4936	0.2922
IT	0.4413	0.4725	0.3101	<b>0.7295</b>	0.3382	1.0000	<u>0.3042</u>
	-0.1372	<u>-0.6062</u>	0.3777	-0.2117	-0.2442	0.0000	<b>0.0737</b>
	<u>0.8386</u>	<u>0.7312</u>	0.3777	<b>0.2117</b>	0.4936	0.0000	0.6383
CA	<u>0.2785</u>	0.3715	0.3610	0.3963	<b>0.4605</b>	0.3042	1.0000
	0.1236	-0.0513	<b>-0.0238</b>	<u>0.5122</u>	-0.2922	-0.0737	0.0000
	0.5428	<b>0.1627</b>	0.6045	0.6359	0.2922	<u>0.6383</u>	0.0000

See notes to Table 3.

**Table 7. Coherence and phases (middle ranges) – episode 1**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	<u>0.2938</u>	0.3954	0.3987	0.3957	0.3690	<b>0.4876</b>
	0.0000	-0.2002	<b>0.0055</b>	-0.0838	<u>-0.2454</u>	-0.0545	0.0358
	0.0000	<u>0.5735</u>	0.4959	0.4077	0.5393	0.4150	<b>0.2038</b>
JP	<u>0.2938</u>	1.0000	0.3483	0.2965	0.3593	<b>0.4034</b>	0.2975
	0.2002	0.0000	<u>0.2652</u>	0.2318	-0.1191	<b>0.0278</b>	0.1077
	<u>0.5735</u>	0.0000	0.5404	<b>0.3625</b>	0.3835	0.4427	0.4113
DE	0.3954	0.3483	1.0000	<u>0.3316</u>	0.3607	0.4127	<b>0.4556</b>
	<b>-0.0055</b>	<u>-0.2652</u>	0.0000	0.1109	0.0330	0.1360	0.1130
	0.4959	<u>0.5404</u>	0.0000	<b>0.2115</b>	0.2363	0.3432	0.3975
FR	0.3987	<u>0.2965</u>	0.3316	1.0000	0.4127	0.3952	<b>0.4885</b>
	<b>0.0838</b>	-0.2318	-0.1109	0.0000	<u>-0.4163</u>	0.1555	0.1201
	0.4077	0.3625	<b>0.2115</b>	0.0000	<u>0.4684</u>	0.2981	0.3044
UK	0.3957	0.3593	0.3607	0.4127	1.0000	<u>0.3478</u>	<b>0.5652</b>
	0.2454	0.1191	<b>-0.0330</b>	0.4163	0.0000	-0.1358	<u>0.4478</u>
	0.5393	0.3835	<b>0.2363</b>	0.4684	0.0000	0.4053	<u>0.6943</u>
IT	0.3690	0.4034	<b>0.4127</b>	0.3952	<u>0.3478</u>	1.0000	0.3908
	0.0545	<b>-0.0278</b>	-0.1360	-0.1555	0.1358	0.0000	<u>-0.2129</u>
	0.4150	<u>0.4427</u>	0.3432	<b>0.2981</b>	0.4053	0.0000	0.4318
CA	0.4876	<u>0.2975</u>	0.4556	<b>0.4885</b>	0.5652	0.3908	1.0000
	<b>-0.0358</b>	-0.1077	-0.1130	0.1201	<u>-0.4478</u>	0.2129	0.0000
	<b>0.2038</b>	0.4113	0.3975	0.3044	<u>0.6943</u>	0.4318	0.0000

See notes to Table 3.

**Table 8. Preliminary descriptive statistics – episode 2**

	US	JP	DE	FR	UK	IT	CA
Mean	0.4166e-2	0.1940e-2	0.29382e-2	0.3090e-2	0.4265e-2	0.1265e-2	0.5138e-2
Max	0.1932e-1	0.1920e-1	0.1929e-1	0.1116e-1	0.1397e-1	0.1387e-1	0.1359e-1
Min	-0.2328e-1	-0.4021e-1	-0.4092e-1	-0.1701e-1	-0.2107e-1	-0.3590e-1	-0.2068e-1
Median	0.5425e-2	0.2701e-2	0.3344e-2	0.4035e-2	0.5913e-2	0.2502e-2	0.5972e-2
Std	0.6920e-2	0.1154e-1	0.9454e-2	0.5555e-2	0.7631e-2	0.7958e-2	0.6464e-2

**Table 9. Correlation statistics – episode 2**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.5569 <sup>†</sup>	0.5172 <sup>†</sup>	0.6679 <sup>†</sup>	0.6721 <sup>†</sup>	0.5637 <sup>†</sup>	0.6017 <sup>†</sup>
JP	0.5569	1.0000	0.6857 <sup>†</sup>	0.6448 <sup>†</sup>	0.6474 <sup>†</sup>	0.7136 <sup>†</sup>	0.5443 <sup>†</sup>
DE	0.5172	0.6857	1.0000	0.7713 <sup>†</sup>	0.6250 <sup>†</sup>	0.8316 <sup>†</sup>	0.5943 <sup>†</sup>
FR	0.6679	0.6448	0.7713	1.0000	0.7252 <sup>†</sup>	0.8088 <sup>†</sup>	0.6950 <sup>†</sup>
UK	0.6721	0.6474	0.6250	0.7252	1.0000	0.6938 <sup>†</sup>	0.5601 <sup>†</sup>
IT	0.5637	0.7136	0.8316	0.8088	0.6938	1.0000	0.6766 <sup>†</sup>
CA	0.6017	0.5443	0.5943	0.6950	0.5601	0.6766	1.0000

\* significant at the 5% level; † significant at the 1% level.

**Table 10. Coherence and phases – episode 2**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.5747	<u>0.5638</u>	0.6736	<b>0.6897</b>	0.5786	0.6703
	0.0000	<b>0.0398</b>	0.0946	<u>0.2053</u>	0.0980	0.0797	-0.0925
	0.0000	0.3387	<b>0.0358</b>	0.2452	0.3314	<u>0.4205</u>	0.3467
JP	<u>0.5747</u>	1.0000	<b>0.7703</b>	0.5901	0.6124	0.7503	0.6152
	<b>-0.0398</b>	0.0000	0.1151	0.0706	0.0989	0.0725	<u>0.2343</u>
	<u>0.3387</u>	0.0000	0.1331	<b>0.1236</b>	0.2207	0.1311	0.2589
DE	<u>0.5638</u>	0.7703	1.0000	0.7115	0.6720	<b>0.8224</b>	0.6208
	-0.0946	-0.1151	0.0000	-0.1064	-0.0156	<b>-0.0068</b>	<u>0.1364</u>
	<b>0.0358</b>	0.1331	0.0000	0.1066	0.1027	0.0891	<u>0.1941</u>
FR	0.6736	<u>0.5901</u>	<b>0.7115</b>	1.0000	0.6523	0.6587	0.6374
	<u>-0.2053</u>	-0.0706	0.1064	0.0000	0.1263	0.0790	<b>-0.0142</b>
	<u>0.2452</u>	0.1236	<b>0.1066</b>	0.0000	0.1925	0.1180	0.1805
UK	<b>0.6897</b>	0.6124	0.6720	0.6523	1.0000	0.6217	<u>0.5425</u>
	-0.0980	-0.0989	<b>0.0156</b>	-0.1263	0.0000	-0.0201	<u>0.1892</u>
	<u>0.3314</u>	0.2207	<b>0.1027</b>	0.1925	0.0000	0.1498	0.2976
IT	<u>0.5786</u>	0.7503	<b>0.8224</b>	0.6587	0.6217	1.0000	0.5900
	<u>-0.0797</u>	-0.0725	<b>0.0068</b>	-0.0790	0.0201	0.0000	0.0630
	<u>0.4205</u>	0.1311	<b>0.0891</b>	0.1180	0.1498	0.0000	0.1984
CA	<b>0.6703</b>	0.6152	0.6208	0.6374	<u>0.5425</u>	0.5900	1.0000
	0.0925	<u>-0.2343</u>	-0.1364	<b>0.0142</b>	-0.1892	-0.0630	0.0000
	<u>0.3467</u>	0.2589	0.1941	<b>0.1805</b>	0.2976	0.1984	0.0000

First row: coherence; second row: overall lags  $\varphi$  (economies in column titles lead economies in row titles, negative figures mean lags); third row: coincidence  $\theta$ .

The largest coherence in each row is in bold, the smallest is underlined.

The smallest overall lag/lead in each row is in bold, the largest is underlined.

The highest coincidence in each row is in bold, the lowest is underlined.

The significance test statistic is  $\sqrt{\frac{3N}{2M}} = \sqrt{\frac{3 \times 64}{2 \times 33}} = 1.7056$  for  $k \neq 0$ , which is significant for all the frequency points except the zero frequency.

**Table 11. Coherence and phases (very long cycles and long-run trends) – episode 2**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.9432	<u>0.7208</u>	0.9634	0.9442	0.7533	<b>0.9647</b>
	0.0000	0.0091	<u>0.0495</u>	0.0185	0.0109	0.0404	<b>-0.0023</b>
	0.0000	0.0091	<u>0.0495</u>	0.0185	0.0109	0.0404	<b>0.0023</b>
JP	<b>0.9432</b>	1.0000	<u>0.6443</u>	0.9266	0.9142	0.8936	0.8478
	-0.0091	0.0000	<u>0.0682</u>	0.0210	0.0112	0.0408	<b>-0.0073</b>
	0.0091	0.0000	<u>0.0682</u>	0.0210	0.0112	0.0408	<b>0.0073</b>
DE	0.7208	0.6443	1.0000	<b>0.8010</b>	0.6008	<u>0.5134</u>	0.7656
	-0.0495	<u>-0.0682</u>	0.0000	-0.0164	<b>-0.0045</b>	-0.0107	-0.0269
	0.0495	<u>0.0682</u>	0.0000	0.0164	<b>0.0061</b>	0.0107	<u>0.0269</u>
FR	<b>0.9634</b>	0.9266	<u>0.8010</u>	1.0000	0.9559	0.8350	0.9548
	-0.0185	<u>-0.0210</u>	-0.0164	0.0000	<b>0.0027</b>	0.0094	-0.0125
	0.0185	<u>0.0210</u>	0.0164	0.0000	<b>0.0038</b>	0.0094	0.0125
UK	0.9442	0.9142	<u>0.6008</u>	<b>0.9559</b>	1.0000	0.8474	0.9214
	-0.0109	-0.0112	0.0045	-0.0027	0.0000	<b>0.0017</b>	<u>-0.0119</u>
	0.0109	0.0112	0.0061	<b>0.0038</b>	0.0000	0.0053	<u>0.0119</u>
IT	0.7533	<b>0.8936</b>	<u>0.5134</u>	0.8350	0.8474	1.0000	0.6771
	-0.0404	<u>-0.0408</u>	0.0107	-0.0094	<b>-0.0017</b>	0.0000	-0.0280
	0.0404	<u>0.0408</u>	0.0107	0.0094	<b>0.0053</b>	0.0000	0.0280
CA	<b>0.9647</b>	0.8478	0.7656	0.9548	0.9214	<u>0.6771</u>	1.0000
	<b>0.0023</b>	0.0073	0.0269	0.0125	0.0119	<u>0.0280</u>	0.0000
	<b>0.0023</b>	0.0073	0.0269	0.0125	0.0119	<u>0.0280</u>	0.0000

See notes to Table 10.

**Table 12. Coherence and phases (lower frequencies) – episode 2**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.9313	<u>0.8343</u>	<b>0.9414</b>	0.9335	0.8639	0.9323
	0.0000	-0.0586	<u>0.0608</u>	0.0297	-0.0128	<b>0.0085</b>	0.0222
	0.0000	0.0596	<u>0.0608</u>	0.0297	<b>0.0212</b>	0.0235	0.0273
JP	<b>0.9313</b>	1.0000	0.8762	0.9149	0.9308	0.9124	<u>0.8404</u>
	0.0586	0.0000	<u>0.1213</u>	0.0981	<b>0.0500</b>	0.0783	0.0934
	0.0596	0.0000	<u>0.1213</u>	0.0981	<b>0.0500</b>	0.0783	<u>0.0953</u>
DE	<u>0.8343</u>	0.8762	1.0000	<b>0.9264</b>	0.8346	0.9222	0.8355
	-0.0608	<u>-0.1213</u>	0.0000	-0.0267	-0.0476	-0.0500	<b>-0.0234</b>
	0.0608	<u>0.1213</u>	0.0000	<b>0.0279</b>	0.0476	0.0500	0.0367
FR	0.9414	<u>0.9149</u>	0.9264	1.0000	0.9193	<b>0.9612</b>	0.9212
	-0.0297	<u>-0.0981</u>	0.0267	0.0000	-0.0238	-0.0285	<b>0.0056</b>
	0.0297	<u>0.0981</u>	0.0279	0.0000	0.0266	0.0285	<b>0.0188</b>
UK	<b>0.9335</b>	0.9308	<u>0.8346</u>	0.9193	1.0000	0.8914	0.8657
	0.0128	<u>-0.0500</u>	0.0476	0.0238	0.0000	<b>-0.0115</b>	0.0291
	<b>0.0212</b>	<u>0.0500</u>	0.0476	0.0266	0.0000	0.0396	0.0381
IT	0.8639	0.9124	0.9222	<b>0.9612</b>	0.8914	1.0000	<u>0.8482</u>
	<b>-0.0085</b>	<u>-0.0783</u>	0.0500	0.0285	0.0115	0.0000	0.0328
	<b>0.0235</b>	<u>0.0783</u>	0.0500	0.0285	0.0396	0.0000	0.0364
CA	<b>0.9323</b>	0.8404	<u>0.8355</u>	0.9212	0.8657	0.8482	1.0000
	-0.0222	<u>-0.0934</u>	0.0234	<b>-0.0056</b>	-0.0291	-0.0328	0.0000
	0.0273	<u>0.0953</u>	0.0367	<b>0.0188</b>	0.0381	0.0364	0.0000

See notes to Table 10.

**Table 13. Coherence and phases (higher frequencies) – episode 2**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	<u>0.3867</u>	0.4077	0.5236	<b>0.6100</b>	0.4296	0.5746
	0.0000	0.0830	0.1039	<u>0.3429</u>	0.2251	<b>0.0813</b>	-0.2560
	0.0000	0.5279	0.5754	<b>0.4022</b>	0.5330	<u>0.6727</u>	0.5352
JP	<u>0.3867</u>	1.0000	0.7231	0.4243	0.4383	<b>0.6750</b>	0.4767
	-0.0830	0.0000	0.1313	0.0824	0.1879	<b>0.0733</b>	<u>0.2647</u>
	<u>0.5279</u>	0.0000	0.1562	<b>0.1451</b>	0.3320	<u>0.1757</u>	0.3075
DE	<u>0.4077</u>	0.7231	1.0000	0.5729	0.5946	<b>0.7827</b>	0.4891
	-0.1039	-0.1313	0.0000	-0.1495	0.0531	<b>0.0011</b>	<u>0.1572</u>
	<u>0.5754</u>	0.1562	0.0000	0.1495	<b>0.1000</b>	0.1319	0.2398
FR	0.5236	<u>0.4243</u>	<b>0.5729</b>	1.0000	0.5459	0.4825	0.4941
	<u>-0.3429</u>	<b>-0.0824</b>	0.1495	0.0000	0.2720	0.1302	-0.1061
	<u>0.4022</u>	<b>0.1451</b>	0.1495	0.0000	0.2720	0.1777	0.2397
UK	<b>0.6100</b>	0.4383	0.5946	0.5459	1.0000	0.5154	<u>0.3546</u>
	-0.2251	-0.1879	<b>-0.0531</b>	<u>-0.2720</u>	0.0000	-0.1015	0.1670
	<u>0.5330</u>	0.3320	<b>0.1000</b>	0.2720	0.0000	0.1894	0.3582
IT	0.4296	0.6750	<b>0.7827</b>	0.4825	0.5154	1.0000	<u>0.4098</u>
	-0.0813	-0.0733	<b>-0.0011</b>	<u>-0.1302</u>	0.1015	0.0000	0.0207
	<u>0.6727</u>	<b>0.1757</b>	0.1319	0.1777	0.1894	0.0000	0.2566
CA	<b>0.5746</b>	0.4767	0.4891	0.4941	<u>0.3546</u>	0.4098	1.0000
	0.2560	<u>-0.2647</u>	-0.1572	0.1061	-0.1670	<b>-0.0207</b>	0.0000
	<u>0.5352</u>	0.3075	0.2398	<b>0.2397</b>	0.3582	0.2566	0.0000

See notes to Table 10.

**Table 14. Coherence and phases (middle ranges) – episode 2**

	US	JP	DE	FR	UK	IT	CA
US	1.0000	0.5780	0.5870	<b>0.6730</b>	0.6639	<u>0.5343</u>	0.5982
	0.0000	<b>0.1134</b>	0.1507	0.1598	-0.1824	<u>0.2221</u>	0.1752
	0.0000	0.2775	0.2787	0.2255	<b>0.1955</b>	<u>0.3509</u>	0.2750
JP	0.5780	1.0000	<b>0.7473</b>	0.5900	<u>0.5683</u>	0.6967	0.6862
	-0.1134	0.0000	0.1381	0.1002	0.0799	<b>0.0272</b>	<u>0.2844</u>
	0.2775	0.0000	0.1616	0.1405	0.1889	<b>0.1332</b>	<u>0.2844</u>
DE	<u>0.5870</u>	0.7473	1.0000	0.7296	0.6997	<b>0.8719</b>	0.6490
	<u>-0.1507</u>	-0.1381	0.0000	-0.1204	-0.0702	<b>-0.0642</b>	0.1467
	<u>0.2787</u>	0.1616	0.0000	0.1208	0.1124	<b>0.0799</b>	0.2173
FR	0.6730	<u>0.5900</u>	<b>0.7296</b>	1.0000	0.5948	0.6792	0.6109
	<u>-0.1598</u>	-0.1002	0.1204	0.0000	0.1426	<b>0.0312</b>	-0.0081
	0.2255	0.1405	0.1208	0.0000	<u>0.2538</u>	<b>0.0836</b>	0.2220
UK	0.6639	0.5683	<b>0.6997</b>	0.5948	1.0000	0.6351	<u>0.4936</u>
	0.1824	-0.0799	0.0702	-0.1426	0.0000	<b>-0.0390</b>	<u>0.2957</u>
	0.1955	0.3835	<b>0.1124</b>	0.2538	0.0000	0.1878	<u>0.3909</u>
IT	<u>0.5343</u>	0.6967	<b>0.8719</b>	0.6792	0.6351	1.0000	0.6373
	<u>-0.2221</u>	<b>-0.0272</b>	0.0642	-0.0312	0.0390	0.0000	0.2017
	<u>0.3509</u>	0.1332	<b>0.0799</b>	0.0836	0.1878	0.0000	0.2140
CA	0.5982	<b>0.6862</b>	0.6490	0.6109	<u>0.4936</u>	0.6373	1.0000
	-0.1752	-0.2844	-0.1467	<b>0.0081</b>	<u>-0.2957</u>	-0.2017	0.0000
	0.2750	0.2844	0.2173	0.2220	<u>0.3909</u>	<b>0.2140</b>	0.0000

See notes to Table 10.