Fault Tolerant Navigation of USV using Fuzzy Multi-sensor Fusion

Wenwen Liu¹, Amit Motwani², Sanjay Sharma², Robert Sutton² and Richard Bucknall¹

¹ Marine Research Group, Mechanical Engineering, University College London, London, UK
² School of Marine Science and Engineering, Plymouth University, Plymouth, UK

{w.liu.11, r.bucknall}@ucl.ac.uk
{amit.motwani, sanjay.sharma, r.sutton}@plymouth.ac.uk

Marine and Industrial Dynamic Analysis
School of Marine Science and Engineering
Plymouth University, PL4 8AA

TECHNICAL REPORT

11 April, 2014
MIDAS.SMSE.2014.TR.010
Abstract

This report presents a fuzzy multi-sensor data fusion process for combining heading estimates from three separate Kalman filters with the aim of constructing a fault tolerant navigation system for the Springer Uninhabited Surface Vehicle (USV). A single gyroscopic unit and three independent compasses are used to acquire data onboard the vessel. The inertial data from the gyroscope is combined in turn with the readings from each compass via a separate Kalman filter (KF). The three ensuing KF estimates of the heading angle of the vehicle are then fused via a fuzzy system designed to produce accurate heading information even in the face of a failure in one of the compasses. A simulation study demonstrates the effectiveness of the proposed fuzzy data fusion process.

1. Introduction

The navigational suite of the Springer Unmanned Surface Vehicle (USV) includes a low cost micro-electro-mechanical (MEMS) gyroscope unit and three digital magnetic compasses for heading determination. The sensorial redundancy may seem wasteful, but in practice, sensor failure is a common occurrence, especially when low cost hardware is involved. By way of example, during recent trials undertaken with the USV, a sporadic communications error between one of the compasses and the main onboard PC impeded data to be sent adequately from the former to the latter. In this case it sufficed to switch to another compass manually, but during an autonomous mission, such a luxury would not exist and a HW failure in the middle of a mission would inevitably lead to the forced abortion of the same.

In this study, a data fusion process based on fuzzy logic is proposed to combine data from three Kalman filters (KF) associated with each of the three compasses. The main goal is to see if the data fusion algorithm can successfully detect and reject poor KF estimates arising from a compass failure, thus providing a more robust navigation system.
This report is structured as follows: Section 2 describes the hardware (gyroscope and compasses) used on the USV for heading determination. Section 3 describes the KF used to combine gyroscope and compass data to obtain a statistically optimal estimate of the heading angle from the said data, whilst also estimating any possible bias in the gyroscope reading. Section 4 describes the fuzzy sensor fusion algorithm used to combine the estimates of the three KFs, and simulation results are shown and discussed in Section 5. Finally the main conclusions are stated in Section 6.

2. Hardware Set-up

For heading information, the navigation system of the Springer USV is equipped with a set of sensors consisting of one gyroscope and three different compasses. The gyroscope is from TinkerKit, which takes analogue raw readings through a microcontroller called Arduino. The three different compasses are TCM2 from PNI America’s premier sensor technology company, HMR3000 from Honeywell and KVH C100 from KVH Industries.

2.1 Gyroscope and Arduino Mega2560 Paring

2.1.1 Arduino

The Arduino Mega 2560 is a microcontroller board based on the Atmega2560. The controller has 54 digital input/output pins (14 of them can be used as PWM outputs),

![Fig. 1: Arduino ATmega 2560 Layout](image-url)
16 analog inputs, 4 UARTs (hardware serial ports), a 16 MHz crystal oscillator, a USB connection, a power jack, an ICSP header, and a reset button. Figure 3-2 presents an overview of the real board. (Arduino, 2014)

The board can connect to the computer using serial communication via the USB Connection. The data could be read by any terminal program like HyperTerminal, PuTTY, Tera Term, etc.

2.1.2 Gyroscope (TinkerKit)

Gyroscope measures the rotation of the USV. The TinkerKit gyroscope is based on LPR5150AL from ST Microelectronics. It is a two-axis gyroscope that outputs 0V to 5V from the signal pins when Springer rotates. (TinkerKit, 2014)

![Fig. 2: TinkerKit Gyroscope Layout](image)

After connected to the Arduino, this 10-bit resolution device can output digits range from 0 to 1023 ($2^{10} - 1$). The following equation shows how to transfer the digits to the analogue voltage.

$$ Volts = ADC \times \frac{V_{REF}}{1023} $$  \hspace{1cm} (1)

where $V_{REF}$ is the reference voltage, which equals to Vcc (3.3V).

Then the angular velocity (gyroRate) measured by the Gyro can be obtained from the following equations:
\[ \text{gyroVal} = \text{gyroReading} - \text{gyroZero} \]  
\[ \text{gyroRate} = \frac{\text{gyroVal}}{\text{Sensitivity}} \]  

where gyroZero is the voltage value obtained while there is no rotation and Sensitivity can be found from the Gyro’s data sheet.

### 2.1.3 Gyroscope and Arduino Connection

The signal pins are connected to the analogue pins on the Arduino. The two + pins are the Vcc pins which should be connected to the 3.3V pin on the Arduino and the two – pins are connected to the ground.

![Fig. 3 Schematic Drawing of the Arduino/Gyro Connection](image)

### 2.2 Compasses

Compasses measure the heading of USV. Three different compasses TCM2, HMR3000 and KVH C100 are assembled on the Springer USV.
The TCM2 compass is based on the magneto-inductive effect. It combines a two-axis inclinometer to measure the tilt and roll. (PNI, 2014)

HMR3000 uses the AMR effect; it includes three perpendicular sensors and a fluidic tilt sensor to provide a tilt-compensated heading. (Honeywell, 2014)

KVH C100 is a flux-gate compass that offers modules incorporating both rate gyros that compensate for error from acceleration, as well as inclinometers that provide accurate readings of heading, pitch, and roll. (KVH, 2014)

The TCM2 compass has simple design with low operating power, however it is very sensitive to the electrical and environmental disturbances. The flux-gate compass KVH C100 can output accurate heading although it has greater power consumption. Among these three compasses, the HMR 3000 is the most accurate with disturbance resistant capability. All the compasses output specific NMEA 0183 Standard sentences. The following table illustrates the technical details.

<table>
<thead>
<tr>
<th>Compass Model</th>
<th>TCM2</th>
<th>HMR3000</th>
<th>KVH C100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension (mm)</td>
<td>73.5<em>50.8</em>32.75</td>
<td>114.0<em>46.0</em>28.0</td>
<td>74.9<em>30.5</em>25.0</td>
</tr>
<tr>
<td>Weight (oz)</td>
<td>1.6</td>
<td>0.75</td>
<td>2.25</td>
</tr>
<tr>
<td>Baud Rate</td>
<td>300-38400</td>
<td>300-9600</td>
<td>1200-38400</td>
</tr>
<tr>
<td>Supply Voltage</td>
<td>+5</td>
<td>+12</td>
<td>+5</td>
</tr>
<tr>
<td>Current (mA)</td>
<td>15-20</td>
<td>&lt;40</td>
<td>35</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>\approx10</td>
<td>\approx10</td>
<td>\approx10</td>
</tr>
<tr>
<td>Temperature</td>
<td>-20~70</td>
<td>-40~65</td>
<td>-20~70</td>
</tr>
<tr>
<td>Tilt Range</td>
<td>\pm50</td>
<td>\pm80</td>
<td>\pm40</td>
</tr>
<tr>
<td>Output</td>
<td>Digital NMEA 0183/Analogue</td>
<td>Digital NMEA 0183/Analogue</td>
<td>Digital NMEA 0183/Analogue</td>
</tr>
</tbody>
</table>

Table 1: Compasses Specifications
### Table 2: Compasses Output Format

<table>
<thead>
<tr>
<th>TCM2</th>
<th>C98.4P4.7R5.9*33</th>
</tr>
</thead>
<tbody>
<tr>
<td>$: the beginning of the sentence</td>
<td></td>
</tr>
<tr>
<td>C98.4: heading angle with respect to the magnetic North in degree ($0^\circ \sim 360^\circ$)</td>
<td></td>
</tr>
<tr>
<td>P4.7: pitch angle ($\pm 20^\circ$)</td>
<td></td>
</tr>
<tr>
<td>R5.9: roll angle ($\pm 20^\circ$)</td>
<td></td>
</tr>
<tr>
<td>*33: check sum for the string</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HMR3000</th>
<th>PTNTHPR,62.7,N,2.9,N,1.3,N*2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PTNTHPR$: the beginning of the sentence</td>
<td></td>
</tr>
<tr>
<td>62.7: true heading angle ($0^\circ \sim 360^\circ$)</td>
<td></td>
</tr>
<tr>
<td>2.9: pitch angle ($\pm 45^\circ$)</td>
<td></td>
</tr>
<tr>
<td>1.3: roll angle ($\pm 45^\circ$)</td>
<td></td>
</tr>
<tr>
<td>N: indicates the sensor operate in normal situation</td>
<td></td>
</tr>
<tr>
<td>*2D: checksum</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>KVH C100</th>
<th>SHCHDT,155.7,T*2C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SHCHDT$: the beginning of the sentence</td>
<td></td>
</tr>
<tr>
<td>155.7,T: true heading</td>
<td></td>
</tr>
<tr>
<td>*2C: checksum</td>
<td></td>
</tr>
</tbody>
</table>

### 3. Kalman Filter

In order to estimate the heading angle of the vehicle at each time instant, a Kalman filter (KF) is used to fuse gyro and compass data. The gyro is used to create a predictive model for the estimate of the heading angle, whilst the compass reading is employed to correct the *a priori* estimate, as described next.

Let $\Omega_t$ represent the actual turning rate of the vehicle in deg/s, and $\Omega_0$ the gyroscope reading (also in deg/s). This reading in general can be considered to be the sum of three components: the actual turning rate, a bias (low cost MEMS gyros are typically subject to null drift due to various reasons (Shiau et al, 2012)), and a measurement noise (Equation 4, Fig. 4),

$$\Omega_0 = \Omega_t + b + \omega$$  \(4\)
The heading angle of the vehicle can be obtained by discrete integration of the turning rate, i.e.

$$\theta(k + 1) = \theta(k) + T_s \times \Omega_i(k)$$

(5)

where $T_s$ is the sampling time. In terms of the gyro reading, this is equivalent to:

$$(k + 1) = \theta(k) + T_s \times \left[ \Omega_0(k) - b(k) - \omega(k) \right]$$

(6)

The above constitutes a predictive model of the USV heading angle, where $\Omega_0(k)$ is a known input, $b(k)$ an unknown parameter that needs to be estimated, and $\omega(k)$ a random variable but with known pdf (given by the gyro characteristics). Equation (Eq.6) can be viewed as a state equation with $\theta$ being the state of the system, the object of the estimation problem. However, since the bias term is unknown, it can be included in the description as a state to be estimated. Defining the state vector $x$ as

$$x = [\theta \quad b]^T$$

(7)

then the following state equation includes both Equation (Eq.6), which describes the propagation of $\theta$, as well as a second equation that reflects the unchanging nature of $b$.

$$
\begin{bmatrix}
\theta(k+1) \\
b(k+1)
\end{bmatrix} =
\begin{bmatrix}
1 & -T_s \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\theta(k) \\
b(k)
\end{bmatrix} +
\begin{bmatrix}
T_s \\
0
\end{bmatrix} \Omega_0(k) +
\begin{bmatrix}
- T_s \\
0
\end{bmatrix} \omega(k)
$$

(8)

The compass reading, $z(k)$, on the other hand provides a direct measurement of the heading angle of the vehicle, and can be modelled as
\[ z(k) = \theta(k) + \nu(k) \]  \hspace{1cm} (9)

where \( \nu(k) \) is the noise in the compass reading. In terms of the state vector, this can be written as

\[ z(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta(k) \\ b(k) \end{bmatrix} + \nu(k) \]  \hspace{1cm} (10)

Equations (Eq.8) and (Eq.10) constitute a predictive and measurement model pair described as a standard stochastic-deterministic state-space set of equations:

\[
\begin{align*}
    x(k + 1) &= A x(k) + B u(k) + B_\omega \omega(k) \\
    z(k) &= C x(k) + \nu(k)
\end{align*}\]  \hspace{1cm} (11)

where

\[
A = \begin{bmatrix} 1 & -T_s \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} T_s \\ 0 \end{bmatrix}, B_\omega = \begin{bmatrix} -T_s \\ 0 \end{bmatrix}, C = [1 \ 0]
\]  \hspace{1cm} (12)

the known input \( u(k) \) is the gyroscope reading at sampling time \( k \), \( C x(k) \) represents the actual heading of the vehicle at time \( k \), \( z(k) \) is the compass reading at time \( k \), and \( \omega(k) \) and \( \nu(k) \) are random variables which represent the gyroscope and compass measurement noise respectively.

Assuming that \( \omega(k) \) and \( \nu(k) \) are white noise sequences, Normally distributed with zero mean and standard deviations given by \( q_z^2 \) and \( r_z^2 \), denoting \( Q = \text{cov}(B_\omega \omega) = \begin{bmatrix} T_s^2 q & 0 \\ 0 & 0 \end{bmatrix} \), \( R = r \), then given some initial state estimate \( \hat{x}(0) \) and some initial confidence about this estimate, \( P(0) = \text{cov}\{(\hat{x}(0) - x(0))(\hat{x}(0) - x(0))^T\} \), and assuming as well that \( E(\omega(l)\nu(k)) = 0 \ \forall l, k \), \( E(x(0)\omega(k)) = 0 \), \( E(x(0)\nu(k)) = 0 \ \forall k \), then the MMSE estimate of the state vector \( x(k) \) can be estimated according to the recursive KF algorithm (Equations Eq.13 to Eq.17):
The KF is a popular technique applied to navigation algorithms as an optimal estimator for linear stochastic system (Hu et al, 2003). In this example, the KF is used to estimate both the heading angle of the USV as well as the gyroscope bias. The estimated heading angle is optimal in a statistical sense, with a lower MSE than that of the raw compass readings or inferred heading from gyro readings alone. However, the assumptions of the KF must hold for this to be the case. In this study the possible failure of the compass is contemplated. In such a case, the KF heading estimate will no longer yield an accurate result, as will be shown in simulations in Section 5.

\[ \hat{x}(k + 1 \mid k) = A \hat{x}(k \mid k) + B u(k) \]  \hspace{1cm} (13)

\[ P(k + 1 \mid k) = A P(k \mid k) A^T + Q \]  \hspace{1cm} (14)

**Kalman gain:**

\[ K(k) = P(k + 1 \mid k) C^T [C P(k + 1 \mid k) C^T + R]^{-1} \]  \hspace{1cm} (15)

**correction:**

\[ \hat{x}(k + 1 \mid k + 1) = \hat{x}(k + 1 \mid k) + K(k) [z(k) - C \hat{x}(k + 1 \mid k)] \]  \hspace{1cm} (16)

\[ P(k + 1 \mid k + 1) = (I - K(k) C) P(k + 1 \mid k) \]  \hspace{1cm} (17)

4. Fuzzy Multi-sensor Fusion

The *Springer* USV is, as detailed in Section 2, equipped with three digital magnetic compasses. A KF can be built to fuse data between the gyro and each of the compasses as described in the previous section, resulting in three distinct KFs that are identical in their predictive models but with different heading measurement noise covariance. However, if a compass fails (either permanently or intermittently), the corresponding KF estimate will be erroneous. There should henceforth exist some mechanism by which a faulty KF estimate should be rejected for use in the vehicle’s navigation system.

In this study, a fuzzy multi-sensor fusion algorithm is proposed to overcome compass failure. This algorithm is compared to a crisp decision-making algorithm, both of which attempt to fuse data from the three KFs in such a way as to disregard erroneous
estimates caused by faulty compass readings. This is accomplished by assigning a weight to each of the KF state estimates, as shown in Fig. 5.

![Diagram of KF fusion process](image)

The fused state estimate is then computed as:

\[ \hat{x}(k) = \sum_{i=1}^{3} w_i(k) \hat{x}_{KF_i}(k) \]  \hspace{1cm} (18)

The principles on which the weighting decision is based is the same for both algorithms, and is based on observation of the innovations sequence of each KF. The innovations sequence of a KF is defined as:

\[ \{inn(k)\} = \{z(k) - C \hat{x}(k+1 | k)\} \]  \hspace{1cm} (19)

which is, at each time-step \( k \), the difference between the compass measurement and the predicted heading angle at the said time-step. It is well established that under an ideal scenario, the innovations sequence should be comprised of a zero-mean, white noise sequence (Subramanian et al, 2009, Bijker et al, 2008). Thus this sequence could be monitored to detect a failure in the correct estimation by one of the KFs.

In order to monitor the innovations sequence, which in general is a random process and thus whose individual values are meaningless, a simple moving average (SMA) of the innovations sequence of each KF is computed:

\[ SMA(k) = \frac{inn(k) + inn(k-1) + \cdots + inn(k-K+1)}{K} \]  \hspace{1cm} (20)
where $K$ is the number of samples considered in the moving average. Since the SMA is, in the ideal case, a sum of zero-mean independent random variables, it is in itself a zero-mean random variable, tending to be normally distributed by the Central Limit Theorem. However, its variance is $K$ times smaller than that of the innovations random variable. Thus, sporadic high values of the SMA are more improbable than for the innovations, and will almost only occur when the innovations stops being a white sequence. Hence it is this value that is chosen to indicate a compass fault in the KF estimate.

### 4.1 Crisp decision algorithm

The crisp decision algorithm updates the SMA of each KF at each sampling instant and then accepts or rejects the filter by assigning it a weight of 1 or 0 according whether the SMA lies within a predefined minimum and maximum threshold value:

\[
\begin{align*}
\text{IF} & \quad (SMA(k) < SMA_{\min} \text{ OR } SMA(k) > SMA_{\max}) \\
& \quad w_i(k) = 0 \\
\text{ELSE} & \quad w_i(k) = 1 \\
\text{END}
\end{align*}
\]

after which the weights are normalised so that their sum equals one.

### 4.2 Fuzzy sensor-fusion

The problem with the crisp decision algorithm is the choice of the threshold values, and the sudden change in the fused estimate that occurs when a change of decision is made regarding the inclusion or exclusion of some filter. In order to obtain a smoother decision process, the following fuzzy membership functions are defined (Figures 6 and 7):
Input membership functions:

Negative function: \( \mu_N = \begin{cases} 
1 & \text{if } SMA < SMAN \\
\frac{SMA}{SMAN} & \text{if } SMAN \leq SMA < 0 \\
0 & \text{if } SMA \geq 0
\end{cases} \)

Zero function: \( \mu_z = \begin{cases} 
1 - \frac{SMA}{SMAN} & \text{if } SMAN \leq SMA < 0 \\
1 - \frac{SMA}{SMA} & \text{if } 0 \leq SMA \leq SMAP
\end{cases} \)

Positive function: \( \mu_P = \begin{cases} 
0 & \text{if } SMA < 0 \\
\frac{SMA}{SMA} & \text{if } 0 \leq SMA < SMAP \\
1 & \text{if } SMA \geq SMAP
\end{cases} \)

Fig. 6: input membership functions

Fig. 7 output membership functions
As indicated by the output fuzzy membership functions, the output to the fuzzy logic inference system is chosen to be a change in the weight of the filter, \( \Delta w \), rather than the weight itself. This is to avoid brusque transitions in the overall estimate.

Based on the aforedescribed membership functions, the following fuzzy rules are established:

- **Rule 1**: If SMA negative then \( \Delta w \) is negative
- **Rule 2**: If SMA is zero then \( \Delta w \) is positive
- **Rule 3**: If SMA is positive then \( \Delta w \) is negative

Then, at each sampling time \( k \), depending upon the value of the SMA, \( \Delta w \) is computed as follows:

- **Case 1**: SMA < SMAN
  
  Rule 1 applies and \( \Delta w \) is given by the horizontal projection of the centroid of the negative output membership function, i.e. \( \Delta w = \frac{DWN}{2} \).

- **Case 2**: SMAN < SMA \( \leq 0 \)
  
  Both Rule 1 and Rule 2 apply. Let \( \mu_N \) represent the degree of membership of the input to the Negative input membership function (Rule 1), and \( \mu_Z \) its degree of membership to the Zero input membership function (Rule 2). Then \( \Delta w \) is computed as the horizontal projection of the centroid of the area comprising the portions of the Negative and Positive output membership functions below the values \( \mu_N \) and \( \mu_Z \) respectively (Figure 5):

  \[
  \Delta w = \frac{-\frac{1}{2} DWN^2 \times \mu_N + \frac{1}{2} DWP^2 \times \mu_Z}{-DWN \times \mu_N + DWP \times \mu_Z}
  \]  

  (21)

- **Case 3**: 0 < SMA < SMAP
  
  Both Rule 3 and Rule 3 apply. Let \( \mu_Z \) represent the degree of membership of the input to the Zero input membership function (Rule 2), and \( \mu_P \) its degree of membership to the Positive input membership function (Rule 3). Then \( \Delta w \)
is computed as the horizontal projection of the centroid of the area comprising
the portions of the Positive and Negative output membership functions below
the values $\mu_Z$ and $\mu_P$ respectively:

$$
\Delta w = \frac{-\frac{1}{2} DWN^2 \times \mu_P + \frac{1}{2} DWP^2 \times \mu_Z}{-DWN \times \mu_P + DWP \times \mu_Z}
$$

(22)

- **Case 4**: $\text{SMAP} \leq \text{SMA}$

  Rule 3 solely applies, and $\Delta w$ is given by the horizontal projection of the
centroid of the negative output membership function, i.e. $\Delta w = \frac{DWN}{2}$.

![Fig. 8: calculation of the output $\Delta w$ for Case 2 (SMAN < SMA ≤ 0)](image)

Once the $\Delta w$ has been calculated at time step $k$ for each KF ($\Delta w_i(k), i = 1,2,3$),
these values are normalised so that their sum equals zero to ensure that the sum of the
weights themselves will remain equal to one,

$$
\Delta w_{i,n}(k) = \Delta w_i(k) - \frac{1}{3} \sum_{j=1}^{3} \Delta w_j(k), \quad i = 1,2,3
$$

(23)

resulting in the updated weights of each filter given by

$$
w_i(k) = w_i(k - 1) + \Delta w_{i,n}(k), \quad i = 1,2,3
$$

(24)

However, direct application of Equation 24 might result in updated values of the
weights not restricted to the interval $[0, 1]$. To restrict the values of the weights to this
interval, the following procedure is carried out. Instead of directly updating all the weights according to Equation 24, these are tentatively updated in some auxiliary variables:

\[ w_i^{\text{new}} = w_i(k - 1) + \Delta w_{i,n}(k), \quad i = 1,2,3 \] (25)

Three possibilities exist:

- If all \( w_i^{\text{new}} \) are between 0 and 1, then these are taken directly as the updated weights \( w_i(k) \).
- If (only) one of the \( w_i^{\text{new}} \) is less than zero, e.g. \( w_j^{\text{new}} < 0 \), then \( \Delta w_j^* \) is defined as \( \Delta w_j^* = -w_j(k - 1) \), i.e. the part of \( \Delta w_j(k) \) that is actually used to make the corresponding updated weight equal to zero. Then, \(-\Delta w_j^* \) is distributed amongst the two remaining weight increments, \( \Delta w_{i,n}(k), l = 1,2,3 & l \neq j \), proportionally to their original values: \( \Delta w_i^* = -\Delta w_j^* \times \frac{\Delta w_{i,n}}{1-\Delta w_{j,n}}, \quad l = 1,2,3 & l \neq j \). Finally, the updated weights are obtained as \( w_i(k) = w_i(k - 1) + \Delta w_i^*, \quad i = 1,2,3. \)
- If two of the \( w_i^{\text{new}} \) obtained using Equation 25 are negative, e.g. \( w_i^{\text{new}} < 0 \) and \( w_m^{\text{new}} < 0 \), then that implies that the third weight, \( w_n^{\text{new}} \), will be larger than one, since the sum of the three is always equal to unity. Therefore it suffices to take \( w_i(k) = 0, w_m(k) = 0, w_n(k) = 1. \)

For both the crisp and fuzzy data fusion algorithms, the initial weights are assumed equal (\( w_i = \frac{1}{3}, i = 1,2,3 \)) and they are not modified until time instant K has been reached, which is the number of samples required to compute the SMA.

5. Results and Discussion

Both data fusion systems were implemented in a simulation study using simulated gyroscope and compass readings, based on a prescribed turning rate of the vehicle. The turning rate of the vehicle, in deg/s, was prescribed according to:
\[
\Omega_i(k) = \sin(k) + \sin\left(\frac{k}{10}\right) + \sin\left(\frac{k}{100}\right)
\]  

(26)

to allow excitation of different frequencies. The gyro was simulated based on this turning rate according to Equation 4, with a constant bias of 3 deg/s, and a white, normally distributed random measurement noise with variance \( q = \left(0.5 \text{ deg}^2/s\right)^2 \). The actual heading angle of the USV is calculated from integration of \( \Omega_i(k) \), based on which three different compass readings are simulated according to Eq.9, using three different measurement noise sequences \( \nu_i \) with variances \( R_1 = (1.5 \text{ deg})^2 \), \( R_2 = (5.5 \text{ deg})^2 \), \( R_3 = (9.5 \text{ deg})^2 \) for each one, respectively. A KF is then simulated for each gyro-compass pair, as described in Section 3. The KF state vectors are initialised with the correct initial vehicle heading, but with zero gyro-bias estimates. The initial error covariance is chosen as

\[
P(0) = \begin{bmatrix}
0.1 & 0 \\
0 & 0.1
\end{bmatrix}
\]  

(27)

At each sampling instant the SMA is calculated with \( K = 30 \), and threshold values for the crisp decision rules and fuzzy membership functions are given in Table 3:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMA_max</td>
<td>5</td>
</tr>
<tr>
<td>SMA_min</td>
<td>-5</td>
</tr>
<tr>
<td>SMAN</td>
<td>-10</td>
</tr>
<tr>
<td>SMAP</td>
<td>10</td>
</tr>
<tr>
<td>DWN</td>
<td>-0.1</td>
</tr>
<tr>
<td>DWP</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 3. Threshold values for crisp decision rules and parameters of fuzzy membership functions.

The simulation is run for 1000 time steps. After one third of the total simulation time, Compass 2 \( (R_2 = (5.5 \text{ deg})^2) \) is made to fail so that the readings remain stuck at the last value before failure. The simulation results are shown in Figure 9.
(a) actual USV change in heading rate $\Omega_i(k)$ and gyroscope output $\Omega_a(k)$

(b) actual and KF estimates of the heading, compass measurements, and crisp and fuzzy data fusion estimates

(c) actual and KF estimates of the gyroscope bias
Table 4 summarises the mean square errors of the various estimates with respect to the true heading of the vehicle.

<table>
<thead>
<tr>
<th>method</th>
<th>MSE (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KF1</td>
<td>0.71</td>
</tr>
<tr>
<td>KF2</td>
<td>9.995</td>
</tr>
<tr>
<td>KF3</td>
<td>17.67</td>
</tr>
<tr>
<td>Compass 1</td>
<td>2.02</td>
</tr>
<tr>
<td>Compass 2</td>
<td>9.688</td>
</tr>
<tr>
<td>Compass 3</td>
<td>99.17</td>
</tr>
<tr>
<td>Crisp decision fusion</td>
<td>502.18</td>
</tr>
<tr>
<td>Fuzzy decision fusion</td>
<td>11.62</td>
</tr>
</tbody>
</table>

Table 4. MSE results for the simulation of 1000 time-steps.
It is observed how each KF estimate improves upon the corresponding raw compass estimate, at least for the two KFs that operate under correct hypotheses. However, the KF associated with the failed compass cannot provide a reliable estimate. From Fig. 9(c) it is also observed how this KF’s estimate of the gyroscope bias also starts deviating from the true bias once the compass has failed.

It is evident from Fig 9(b) that whilst both the crisp and the fuzzy logic fusion of the compass data are able to reject the KF associated with the failed compass, the crisp estimates immediately reincorporates this KF when the SMA falls back within the threshold limits, due purely to the change in turning rate, resulting in an incorrect estimate. The fuzzy logic fusion is more cautious, and does not restore confidence to the failed compass KF so readily.

Although from Table 4 the MSE of the fuzzy logic fused data seems to be considerably larger than that of the best KF (KF1), this is purely because of the initial transient period (bear in mind that the fuzzy fusion algorithm doesn’t start changing the weights until enough samples are obtained so that the SMA can be calculated, and furthermore, the changes in the weights are gradual. In fact, if the simulation time is increased, then the MSE of the fuzzy algorithm estimate tends to that of the best KF, as can be seen in the results of Table 5, which corresponds to a simulation with a total time of 5000 time-steps.

<table>
<thead>
<tr>
<th>method</th>
<th>MSE (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KF1</td>
<td>0.73</td>
</tr>
<tr>
<td>KF2</td>
<td>5,709</td>
</tr>
<tr>
<td>KF3</td>
<td>5.6</td>
</tr>
<tr>
<td>Compass 1</td>
<td>2.23</td>
</tr>
<tr>
<td>Compass 2</td>
<td>5,755</td>
</tr>
<tr>
<td>Compass 3</td>
<td>99.17</td>
</tr>
<tr>
<td>Crisp decision fusion</td>
<td>91.4</td>
</tr>
<tr>
<td>Fuzzy decision fusion</td>
<td>1.19</td>
</tr>
</tbody>
</table>

Table 5. MSE results for the simulation of 5000 time-steps.
6. Conclusions

A fuzzy multi-sensor data fusion algorithm is presented in this technical report. In the course of this analysis, hardware set-up, applied Kalman filter theory and designed fuzzy system are demonstrated as well as the simulation results with one compass failed in the middle of the operation. This designed system can distribute the weights of the estimations from each KF automatically by analysing the innovation sequences and produce continuous final optimal estimation for the USV heading. It is a sufficient algorithm for practical operations since the failure of the navigation instrument cannot be predicted at that time. This simulations illustrate how this fuzzy MSDF algorithm intelligently discards the estimations from the KF associated with the failed compass thus providing robustness to the navigation system of the USV.

References:


American Society of Agricultural and Biological Engineers, Vol. 52(5), pp. 1411-1422.


Nomenclature / List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>KF</td>
<td>Kalman Filter</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>MSDF</td>
<td>Multi-Sensor Data Fusion</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>SMA</td>
<td>Simple Moving Average</td>
</tr>
<tr>
<td>USV</td>
<td>Unmanned Surface Vessel</td>
</tr>
</tbody>
</table>