Adaptive and interval Kalman filtering techniques in autonomous surface vehicle navigation: a survey

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Abstract

This paper reviews the Kalman Filter (KF) as a tool for navigation systems, with focus on uninhabited surface vehicle (USV) navigation. The most common types of sensors being used on such vessels are identified, the KF being at the centre of the data-fusion algorithm. However, some of the limitations of the KF are noted, such as its performance degradation when inaccurate noise statistics are assumed, therefore justifying the use of adaptive techniques to enhance its robustness. The use of such adaptive mechanisms applied to the KF is evidenced in ongoing USV navigation research. A review of the theoretical background of the interval Kalman filter (IKF) is given, and its potential advantage to overcome the limitations posed by the ordinary KF in the face of incomplete knowledge of system dynamics and noise models is discussed. Though the IKF is proposed as a solution to overcome these shortcomings of the ordinary KF, its lack of usage in actual navigation systems is accounted for by the inherent difficulties of its practical implementation. These difficulties are outlined and focus the direction of the research that is required for its successful implementation. USVs are rationalised to constitute ideal platforms on which to develop, test, and prove the effectiveness of IKF-based navigation systems.

1. Introduction

Unmanned surface vehicles (USVs) are finding ever increasing applications in today’s world: though initially their development was primarily a military exclusivity, the last decade has seen a dramatic shift toward smaller and more affordable USV platforms that has prompted civilian, commercial and research interests in these vehicles. The number of uses has multiplied manifold, and potential applications seem to be bounded only by imagination. With the availability of low-cost sensors and microprocessors, the demand to incorporate into USVs ever increasing levels of autonomy at reduced costs is ever increasing (Motwani 2012).

In autonomous navigation, the location and attitude of the vehicle form the basis of the state vector, which is used by a control system, namely the autopilot, to work out the necessary actuator adjustments, such as motor torque or rudder angle, to return the vehicle to the desired course. Navigational sensors for surface vessels typically include compasses, inertial units, global positioning satellite (GPS) fixes, and speed logs. The state vector may be obtained directly through measurement, or predicted through dead-reckoning or even from the known actuator inputs using the system model. However, both measurements as well as system models always contain a certain degree of uncertainty: the former are corrupted by noise, whereas the latter are never completely known with total accuracy, and are additionally subject to unmeasurable and uncontrollable inputs.

The Kalman Filter (KF) estimates the state vector by combining the measured data with predictions made from the system model. It serves both as an observer of states which are not directly measureable, as well as an estimator that minimises, in a statistical sense, the uncertainty of the valuation.

This survey starts by providing a quick review of the basics of Kalman filtering, and some of its shortcomings, which serve to justify the use of adaptive mechanisms to increase robustness of the filter. Finally, the interval Kalman filter (IKF) is proposed, the work undertaken using the IKF in the field of autonomous navigation reviewed, and future undertakings justly propositioned.

2. The Kalman Filter and its use in Autonomous Navigation

Since its inception in 1960, the KF (Kalman, 1960) has been used extensively in innumerable applications: although initially used in spacecraft navigation (eg Smith et al, 1961; Schmidt, 1981), it has since been applied in numerous other fields. Tracking problems, such as ballistic missile tracking (Siouris et al, 1997), computer-vision and pattern recognition (eg Ondel et al, 2007), neural network design (Haykin, 2001), change detection systems (Severo and Gama, 2006), manufacturing (Sorenson, 1985), and economics (Bouyé, 2009), illustrate but a few. Also, knowledge of the state-vector is an important aspect of system control (eg Crain, 2002), and the KF is a key part of the optimal linear-quadratic-Gaussian (LQG) control problem (eg, Mäkilä, 2004).

Basically the KF estimates the true values of the state vector of a dynamic system described by a stochastic-deterministic model, by combining predictions with sensor measurements. The degrees of uncertainty of both the prediction and the measurement are used to compute a weighted average that provides an unbiased estimate of the state vector that minimises the mean square error between it and the true state vector. Although at each measurement update, the KF takes into account all past measurements, the procedure can be made recursive so that data relating to only the previous time-step need be stored, making the KF ideally suited to computer implementation. A detailed description of the algorithm is given in the appendix.

The KF algorithm's inherent structure allows it to naturally combine measurements from various sensors, taking into account the accuracy of each sensor. The data-fusion from various sensors provides a more reliable estimate than can be obtained from each individual sensor alone: this has prompted the use of the KF as a tool for combining low-cost sensors to synergistically create highly-reliable estimates that would otherwise require more precise sensors. Whereas in spacecraft attitude estimations, KF algorithms are developed for highly accurate inertial sensors, multi-sensor data-fusion (MSDF) has been prominent in the case of surface-vehicle navigation. Particularly for unmanned ground vehicles (UGVs), and since the availability of GPS and low-cost and low-power solid-state inertial navigation systems (INS), the KF has been used to perform INS/GPS integration (eg Wolf et al 1997; Tan et al 2007).

This preference is due to several factors. On the one hand, standard GPS receivers are unable to provide the rate or precision required when used on a small vessel such as an USV. On the other, implementation of inertial measurement units (IMUs) in surface/ground systems is more difficult than in airborne systems due to the high noise to signal ratio introduced by interaction of the vehicle with the surface and its relatively slow and vibration-clad movement (Lamon, 2008). The main limitation of low-cost INS systems, in that it can only provide accurate estimates of position and attitude for a short time span before integration drift becomes significant, can be overcome by incorporating a GPS fix at a the lower GPS sampling rate; thus, this strategy combines the short-term accuracy of INS with the long-term stability of GPS, providing a relatively low-cost and reliable solution for surface navigation.

Although they have been used extensively in UGVs, INS/GPS navigation systems are being successfully implanted in autonomous surface vessels (ASVs) as well. An example can be found in the USV being developed at Virginia Tech University (VaCAS, 2011), which uses differential GPS (DGPS) together with a puck sized micro electro-mechanical system (MEMS) based technology inertial sensor offered by MicroStrain Inc that is popularly used in mobile robotic applications (MicroStrain 2012). Technological advances of INS/GPS systems are reviewed in Schmidt (2003), while a comparison of low-cost IMUs for autonomous navigation can be found in Chao et al (2010). A review of MEMS systems may be perused in Barbour et al (2010), and a comparison of several MEMS-based IMUs is presented in De Agostino et al (2010).

Others have used KFs to combine GPS, IMU, as well as magnetic compass sensor readings. For example, Zhang et al (2005) describe the use of an unscented KF (UKF), a non-linear version of the KF, to combine low-cost IMU, GPS and digital compass using a sophisticated dynamical model of the vehicle. Even others have successfully implemented KF-based ASV navigation without IMUs altogether. Sutton (2007) describes the navigational sensor suite of the ASV *Springer*, comprised of a GPS receiver, three low-cost digital magnetic sensors, and a speed and depth sensor. Data from the sensors are combined using various data-fusion architectures based on KFs (Xu et al, 2007). The use of redundant data (by using three separate compasses simultaneously) allows for the construction of fault-tolerant navigation systems. Successful sea trials have demonstrated the effectiveness of the navigation systems of *Springer*. Another example is the ASV *Charlie*, equipped solely with GPS and magnetic compass, which uses an extended KF (EKF), basically, a KF for non-linear systems which linearises the model around the estimated state (Caccia et al, 2008). The reader wishing to know more about the workings of the UKF and EKF may consult, for example, Aich & Madhumita (2010).

3. Adaptive Kalman Filtering

The basic KF scheme yields an optimal estimate only for linear processes with stochastic process and measurement noise that are white and Gaussian, and when these are known precisely, along with knowledge of the initial state estimate and estimate error covariance. In practice, process dynamics models are always an approximation of the true dynamics, initial estimates might be incorrect, and the assumed process and measurement noise covariances inaccurate, affecting the effectiveness of the KF. There is no general theory that guarantees a statistically optimal estimate when knowledge of system dynamics and noise statistics are incomplete. Under these situations, it is usual in practice to assume some completely specified linear model for the system as well as process and measurement noise covariance matrices. However, this can incur in divergence of the predicted mean square state estimation error (which typically remains bounded) from the actual mean square error (which may actually diverge) (Price, 1968).

As an example that illustrates the performance degradation of the KF under incorrect modelling assumptions, consider the simplified model of a tracking system (Chui and Chen, 2008) given by the discrete-time mixed (deterministic-stochastic) system of difference equations:

$$\begin{cases} x_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_k + \omega_k \\ z_k = [1 \quad 0\,] x_k + \nu_k \end{cases} \tag{1}$$

where T is the sampling time, x_k the state vector representing position and velocity at the sampling instant kT, z_k the measurement of the position at time instant kT, and ω_k and ν_k white and Gaussian random sequences with covariances Q and R respectively. Figure 1 shows the simulated system state $x_k(1)$ using the following values: $T = 0.01$, $Q = 1$, $R = 0.1$, and initial state $x_0(1) = x_0(2) = 1$. Also shown is the KF estimate under ideal conditions, that is, using the same equations and parameter values used to simulate the real state, along with the KF estimate obtained when the system noise covariance Q is underestimated by two orders of magnitude. The deterioration in the quality of the estimate in this last case is clearly appreciable.

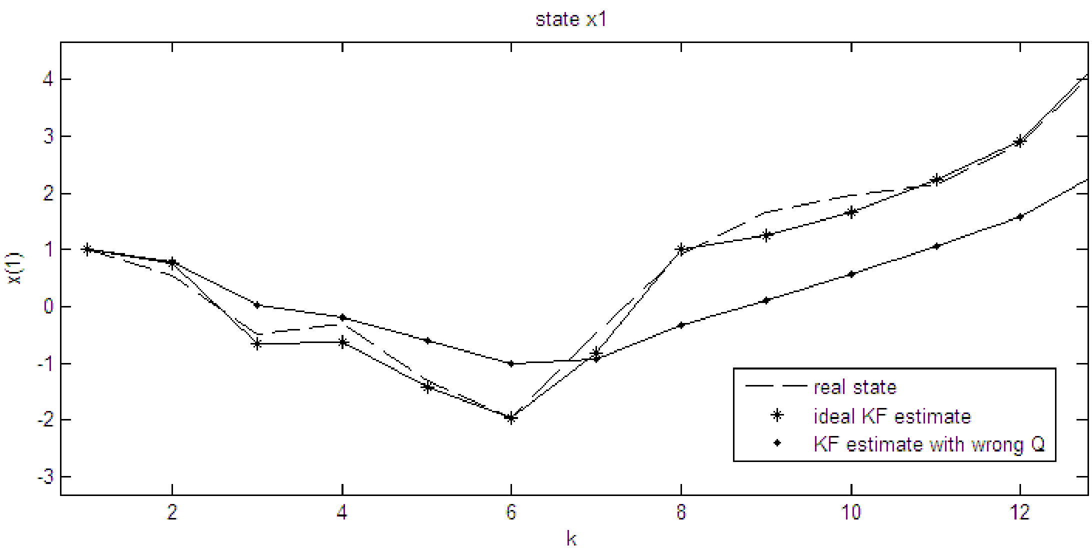

Figure 1: simulation of real state and KF estimates under correct and incorrect assumptions.

Not only are accurate a priori noise statistics often crucial, but moreover, the sensor accuracy may be changing in time - for example, GPS accuracy is affected by the positions of the satellites, interference of the radio signal, physical barriers to the signal like mountains or those due to atmospheric conditions. In this scenario, the measurement noise covariance R must be continuously adapted so as to accommodate for changes in GPS accuracy in order to achieve good performance from the KF. Likewise, in USV navigation, the sea conditions are continuously varying, and should ideally be reflected in a varying process noise covariance Q that most accurately reflects the conditions at each moment.

To enhance robustness, adaptive capabilities are sought. Strategies for adapting measurement and process noise covariance matrices using innovation-based estimation have been applied for integrating IMU measurements with GPS data (Mohamed and Schwarz, 1999; Loebis et al, 2004). Methods based on covariance scaling and multiple model adaptive estimation have also been explored (Hide et al, 2003).

To provide adaptive capabilities, KFs are often used in combination with artificial intelligence (AI) techniques, in particular those based on fuzzy logic. For example, fuzzy logic has been used to discriminate, through selective weighting, KFs working in parallel (Hsiao, 1999), and in decentralised, cascaded, and federated architectures (Lendek et al, 2008; Escamilla-Ambrosio and Mort, 2002; Xu, 2007). Fuzzy logic has also been used to tune the parameters of a KF. For example, through analysis of the innovations sequence, the divergence of the filter can be monitored and corrective action carried out through the tuning of the process noise and measurement noise covariances using fuzzy rules (Abdelnour et al, 1993). This kind of approach has been used in INS-GPS navigation systems, for instance, fuzzy rules based on covariance matching of the actual and theoretical measurement noise covariance (Xu et al, 2006; Loebis et al, 2004b). More examples of similar techniques can be found, such as the adjustment of covariance factors for vehicle position estimation by fusing GPS and yaw-rate gyro and speed sensor based information (Kobayashi et al, 1998).

However, one of the problems with using fuzzy logic techniques has been on how to determine adequate membership functions of the fuzzy sets. The *a priori* approach is a heuristic one, based on experience and observation, but is rarely optimal. To this end, genetic algorithms have been used to optimize the fuzzy membership functions, as Loebis et al (2004a) have done in an autonomous underwater vehicle (AUV) navigation application. Also, neuro-fuzzy techniques have been used extensively in mobile robotic applications (eg, Godjevac and Steele, 1999), and for navigation systems in particular: for instance, an adaptive neuro-fuzzy inference system has been developed for tuning the fuzzy membership functions which endow a INS-GPS navigation system with learning capabilities (Tiano et al, 2001).

As mentioned, the KF's optimal performance is conditioned not only upon accurate knowledge of stochastic model statistics (Q and R), but also upon accurate modelling of the system dynamics. When there is insufficient knowledge of process and measurement noise statistics, the aforementioned techniques based on AI enhancement effectively try to infer such information from the filter's performance, often through learning mechanisms. However, it is to be noted that in the face of deficient system modelling, these strategies basically search for an adequate noise statistics model not just to reflect the true process and measurement disturbances, but also to compensate for the unknown parameters of system dynamics model. This is clearly a compromise, where all uncertainty is lumped into stochastic variables, so that they neither reflect the true stochastic processes that drive the system, nor the modelling uncertainty of the otherwise deterministic process dynamics, but somehow manage to compensate the lack of modelling of one by incorporating it into the other. While this approach has been used somewhat successfully, it does so at the cost of using highly sophisticated AI methods that must arbiter this otherwise procedure otherwise guided only by heuristics, and it raises the question of finding alternative means to describe the uncertainty in the modelling of the system dynamics.

One of the methods suggested in the literature is to use the EKF to perform on-line system identification when Q and R are known but the dynamic process model is unknown or changing. The incertitude is modelled as statistical uncertainty of model parameters, and the process equations rearranged to include these parameters as states, whence the EKF can be applied to estimate this augmented state vector (which contains the uncertain process parameters) (Chui and Chen, 2008).

Another is to use interval arithmetic to describe the uncertainty of the system's dynamic model, which as will be discussed, has been scarcely used to date.

4. Interval Kalman Filtering

As has been mentioned, in practice, process dynamics are only known with some degree of certainty. The basic approach to handling this inconvenience has traditionally been to appeal to a probabilistic description of this uncertainty, via the inclusion of a process and a measurement noise, and apply a statistically optimal filter such as the KF. This approach carries with it the necessity to introduce experimentally some distribution law describing the process and measurement noise (first and second order moments, if Gaussian noise is assumed, as is the case of the KF) (Kolev, 1993).

An alternative approach to treating processes with uncertain data is to apply the notions and methods of interval analysis. The idea of using interval arithmetic to describe the uncertainty in the system model was proposed in the late 90's (Chen et al, 1997). Although Xie at al (2004) had proposed the introduction of a bounded uncertain parameter in both the state and output matrices, whereby it was shown that a robust KF filter could be devised for which the error covariance would be guaranteed to be bounded too, Chen et al (1997) proposed incorporating bounded uncertainty in the system model directly as intervals, that is, describing the system matrices as matrices whose components are intervals. This type of uncertainty can easily arise in practice. For example, when modelling from first principles, the values of certain physical parameters may not be known exactly, but known to lie within certain bounded limits with absolute certainty. Or when using system identification techniques to model a dynamic system, several models that differ only in the values of the matrix coefficients may be obtained under slightly different conditions, and these may all be contained in an interval.

With such a description, the IKF equations (figure 2) have been obtained using the same derivation as the regular KF but applied to the interval system (Chen et al, 1997). Suppose some elements of the matrices A, B, Γ, and H are uncertain within some definite bounds. The process can then be described as:

$$\begin{cases} x_{k+1} = A_k^I x_k + B_k^I u_k + \Gamma_k^I \varepsilon_k \\ z_k = H_k^I x_k + \eta_k \end{cases} \tag{2}$$

where $M_k^I = M_k \pm \Delta M_k = [M_k - |\Delta M_k|,\ M_k + |\Delta M_k|]$ for $M \in \{A, B, \Gamma, H\}$, that is, the elements of each matrix may be made up of finite intervals rather than point values.

The IKF algorithm yields the state estimate as an interval vector $\hat{x}_k^I$ for each time step k (figure 3), and can be summarised by the following equations, which mimic those of the ordinary KF but are described in terms of intervals (Chen et al, 1997):

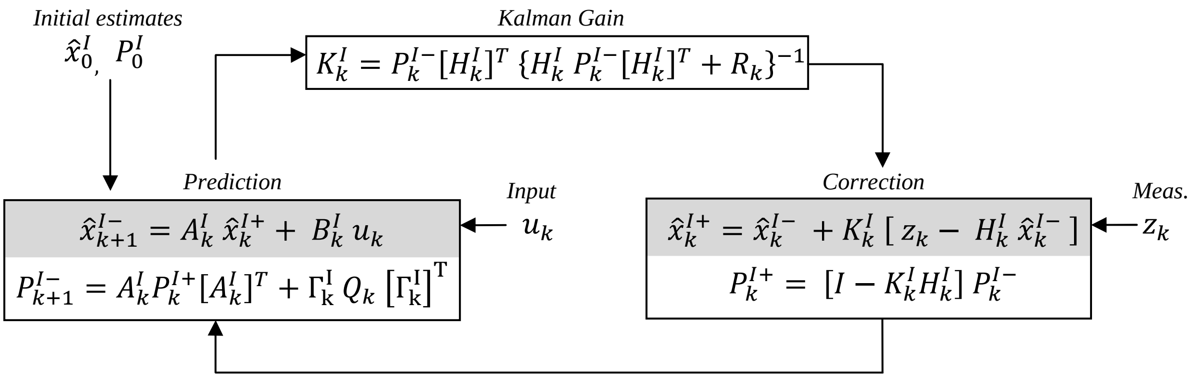

Figure 2: IKF recursive formulation

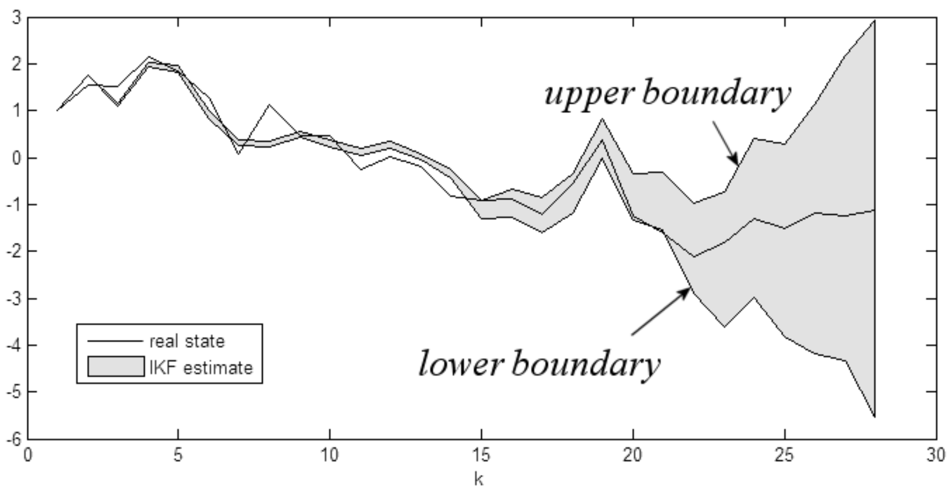

Figure 3: IKF estimate depicting its upper and lower boundaries

Having being derived upon the same principles, the IKF is statistically optimal in the same sense as the standard KF, and it maintains the same recursive formulation. However, the result of the IKF is a sequence of interval estimates rather than point estimates, the fundamental advantage of which being that this guarantees infallible bounds on the ordinary KF estimate that would be obtained based on the true system contained in the interval system. Also, using machine interval arithmetic means that computer round-off errors are automatically accounted for when implemented on computers (Kolev, 1993).

One of the difficulties of implementing the IKF lies in the computation of the inverse interval matrix for the Kalman gain. Singularity problems arise when there are intervals containing zero. In the field of interval mathematics, the problem of inverting interval matrices has been given less attention than that of solving systems of linear interval equations. Owing to this, several workaround strategies to by-pass this difficulty have been proposed. For instance, Chen et al (1997) proposed a “suboptimal IKF” in which the inverse interval matrix is replaced by its worst-case inversion, which is an ordinary (non-interval) matrix.

Another practical difficulty encountered in the implementation of the IKF is that the interval estimate trajectory diverges due to the conservative nature of interval modelling. The range of interval values is often excessively wide due to the inclusion of all possible estimates produced by the interval system. The rapid expansion of the boundaries makes the IKF unsuitable to be implemented on a computer as the digital representation limits would soon be surpassed.

A few theoretical proposals have been put forward towards resolving this difficulty. Chui and Chen (2008) have proposed a weighted-average IKF to obtain a point-value estimate from the interval estimate. However, the problem of selecting which point-estimates to compute an average from, their relative weights, and how this affects the estimated error covariance (which is also obtained as interval estimates) remains to be researched.

In another attempt to overcome this practical difficulty, Weng et al (2000) have described how to use evolutionary programming techniques on an IKF to firstly, reduce the interval estimate at each iteration to an “optimal interval”, and in a second stage find a nominal value within that interval which can be taken as the estimate for practical purposes. Two simple interval systems are simulated, showing how the optimal intervals are relatively smaller as compared to the original intervals provided by the IKF, indicating that this method yields a less conservative interval estimate.

However, the methods proposed have been applied to simulation studies alone. The systems involved typically contain a single scalar measurement, so that the need to invert the matrix of intervals is bypassed, as it is reduced to a scalar interval. Practical issues, such as real-time feasibility, have also to be looked at. In the case of ASV inertial-based navigation, incoming data needs to be processed quickly to make the most of the high-rate inertial sensors involved.

More recently however, Ahn et al (2012) have proposed an alternative IKF (AIKF) formulation in which the interval uncertainties are incorporated into the process and measurement noise covariances, and the KF reformulated taking these into account. Because the estimates are no longer intervals, the filter is not afflicted by the problem of expanding boundaries.

Additionally, Siouris et al (1997) extended the use of the IKF to nonlinear systems, via the extended IKF (EIKF), in a parallel manner to how the EKF extends the KF to nonlinear systems by linearization around the current estimated state. This filter was proposed by He & Vik (1999) for an integrated INS/GPS navigation system with uncertain model parameters, and a simulation study carried out.

Fuzzy techniques have also been used with the IKF to tune its stochastic parameters (error covariances) in the same manner as with ordinary KFs (Tiano et al, 2005). However, on the whole the use of the IKF thus far has been restricted to the handful of case-studies mentioned in this review, and their potentiality demonstrated via simulation only. Nevertheless, despite the inherent computational difficulties arising from the interval mathematics, the advantages of interval techniques have attracted attention in recent times and have started to be applied in a wider setting. In 1998, Kieffer et al presented a state estimation technique in which process and measurement uncertainty are described as bounded intervals to model uncertainty sources (rather than the traditional stochastic modelling of uncertainty used in the Kalman and particle filters). This new technique, based on interval analysis and the notion of set inversion, returns as estimate at each time step a set guaranteed to enclose all values of the state that are consistent with the information available so far, given a set containing the initial value of the state. Prediction and correction phases are alternated in a way reminiscent of Kalman filtering (Kieffer et al, 1998).

The strategies presented by Kieffer and co-workers have been used in practical applications, such as robot localisation (Kieffer et al, 2000), and vehicle tracking (Kieffer et al, 2004). In recent years similar strategies based on interval analysis have been applied to localisation and navigation problems in the field of mobile robotics. For example, Ashokaraj et al (2004) use an interval-based adaptive mechanism for correcting biases and other defects that arise in the estimates of an UKF that fuses INS and other sensor data for robot localisation. Whilst Soares et al (2008) use bounded intervals to model uncertainty sources in a target tracking problem. The results are compared with those obtained using a particle filter (which, like a KF, is based on modelling noise in a probabilistic way). Even though it was found that the particle filter converges generally in less time, the interval based approach clearly outperformed the particle filter in terms of precision and robustness. It is worth noting that the interval based strategy included the implementation of a set inverter via interval analysis (SIVIA) algorithm (this algorithm is used to search the preimage of a given set under a given function). It is suggested that inserting an effort parameter in the SIVIA could increase the interval-based tracker's performance to as to outperform all other conventional tracking algorithms.

5. Discussion

The ARTEMIS, the first prototype ASV produced at MIT Sea Grant to perform automated bathymetric data-collection, was equipped with a low cost fluxgate compass for heading information, speed log, and DGPS receiver for position information. The compass and speed measurements were used to compute, via dead-reckoning (DR), the position of the vessel every 0.2 seconds. The DGPS position fix would be obtained every second, and be used simply to reset the DR position (Rodriguez-Ortiz, 1996). However, with this approach, no prediction is made, and accuracy relies completely on the accuracy of measured data. In more recent USV navigation systems, the compass and GPS measurements have been combined with model-based prediction in an optimal manner with the KF – such autonomous navigation systems have been implemented on ASVs such as *Charlie* and *Springer* (section 2) and shown to be capable of providing accurate estimates with relatively inaccurate sensors.

Since its creation, the KF has become one of the most widely used estimator algorithms: it is well suited to computational implementation, and a wide range of real-life systems can be modelled as linear systems with white noise uncertainty. There are KF versions for nonlinear systems, and systems with correlated noise processes that are not strictly white may be modelled as white with the addition of an adequate shaping filter (Grewal & Andrews, 2008).

In recent years, however, there has been a surge of new estimation methods based on interval mathematics. Though the theoretical stage for the IKF has been developed for quite some time now, implementation has been scarce and limited to simulations. This is due to some of the practical problems that have been discussed, such as the complexities associated with interval computation. However, with the introduction of fast and efficient software for interval arithmetic, such as the MATLAB toolbox INTLAB, an increased popularity of the use of interval analysis has ensued (Rump, 1999).

It is thought that Kalman filtering can have much to gain in terms of performance and robustness if it can make the most of the advantages that interval techniques have to offer. The IKF is unique in the sense that while it maintains the probabilistic description of uncertainty which has rendered such success for the KF, it can also include process dynamic uncertainty as bounded intervals in a seamless manner by simply allowing for the process to be described as an interval system. Being able to exploit the capability of working with intervals rather than point-estimates should clearly be regarded as an advantage, as it dispels some of the limitations of the ordinary KF discussed earlier, making it inherently more robust in its ability to handle model uncertainty without the need to resort to noise covariance tuning, but at the same time without discarding the possibility of being able to combine it with AI techniques to further increase performance. It seems clear that the possibilities of the IKF need to be exploited.

Efficient computational schemes for interval matrix inversion can be investigated, with work having recently been carried out in this field (Nirmala et al, 2011). An important research topic is the devising of methods to limit the expansion of the intervals. While a few solutions have been proposed in theory, the IKF has not yet been implemented in real systems, so actual performance and real-time implementation feasibility still needs to be assessed.

As the demands for ASVs to carry out ever increasingly complex tasks utilising low-cost architectures increases, there is much research being carried out on designing more robust KF-based strategies for ASV navigation, in particular, based on implementing adaptive mechanisms to overcome uncertainties in the models used, or even the time-varying nature of these, as maritime conditions are prone to change during the operation of the vehicle. These changing conditions may be unpredictable but bounded, so that dynamic models of the ASV could be naturally described as time-varying systems within certain bounds. The IKF is a natural way to extend the ordinary and much used KF to such interval systems, and thus may be used advantageously for USV navigation. Also, with the aid of modern day interval arithmetic software packages, the IKF may be implemented easily so that conversely, USVs make ideal test-beds for such implementations as their typically flexible payloads allow them to be equipped with full-scale computers on which computationally intensive algorithms can be executed in real time, as well as their ability to host a variety of sensors.

6. Conclusions

As demands for autonomy and mission complexity increase, robust navigation systems for USVs become all the more crucial. While most USVs have typically been remotely operated, a new generation of USVs with true autonomous navigation capabilities (including obstacle avoidance systems) are being developed. With the large number of emerging USV platforms and the wide use of the KF in navigation and sensor fusion in general, research is currently being undertaken in developing adaptive navigation techniques for ASVs. The IKF is a natural extension of the KF for interval systems, and may prove a robust solution for ASV navigation – however, there is no evidence of any actual ASVs using IKF-based navigation systems.

While the superior qualities of the IKF in terms of robustness have been spelt out in theory and simulation studies, inherent difficulties in its implementation have as yet not been overcome, and may account for its absence in USV navigation technology. However, the versatile payload of many USVs allow for full-scale computers to be used onboard, unlike most mobile robotic applications, and with available interval-computation packages, it is thought that these overcoming these difficulties and demonstrating its viability and effectiveness as a navigation system for USVs would provide the incentive for its widespread use.

Appendix

Basically, the Kalman filter (KF) is an estimator. The need for such an estimator arises when it is necessary to appraise the value of some quantity that cannot be measured without some degree of uncertainty. In nature, all processes contain some degree of noise or disturbance that cannot be accounted for in a deterministic fashion, and this includes measuring devices. For example, micro electro-mechanical accelerometers are subject to electronic noise from ambient electromagnetic fields that bear upon the circuitry that converts motion into a voltage signal, as well as mechanical noise, such as thermo-mechanical noise that results from molecular agitation which affects the small moving parts of the sensor (Mohd-Yasin et al, 2010).

Not only do all processes have inherent noise, but also, process models only describe real processes with a certain degree of accuracy. Real processes may also be affected by unmeasurable external disturbances that are not accounted for: for example, the velocity of a vehicle is perturbed by gusts of wind that cannot be foreseen as deterministic inputs to the velocity model of the vehicle.

To account for these uncertainties, stochastic descriptions of processes must be used. The Kalman filter was originally designed to estimate the state vector of a system described by the following linear deterministic-stochastic process:

$$\begin{cases} x_{k+1} = A_k x_k + B_k u_k + \Gamma_k \varepsilon_k \\ z_k = H_k x_k + \eta_k \end{cases} \tag{3}$$

where x_k, u_k, and v_k are, respectively, the state vector, the (deterministic) process input, and the measurement vector at time-step k, and ξ_k and η_k are the process noise and measurement noise sequences, which account for the uncertainty in the process and measurements respectively.

If it is assumed that ξ_k and η_k are white noise sequences with zero mean Gaussian distributions with known (or assumed) covariances $var(\xi_k) = Q_k$ and $var(\eta_k) = R_k$, and that $E(\xi_l \eta_k^T) = 0 \; \forall l, k$, $E(x_0 \xi_k^T) = 0$ and $E(x_0 \eta_k^T) = 0 \; \forall k$, then the Kalman filter equations provide a statistically optimal estimate of the state vector, and can be written as a set of recursive equations (figure 4) (see, for example, Chui and Chen, 2008).

The KF equations allow one to recursively obtain the state estimate at each time step from the previous estimate and new observed data, assuming initial estimates for x_0 and P_0. The advantage of the recursive formulation is that only the previous estimate needs to be stored, as the necessary information of all previous data is contained in it, which makes the computation at each time step feasible. Under the stated hypothesis, the Kalman filter provides an unbiased and minimum error variance estimate of the process state vector.

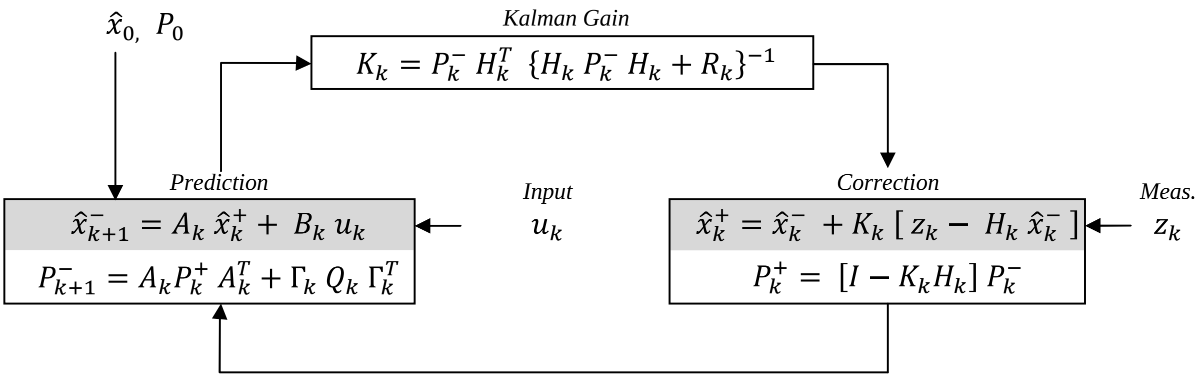

Figure 4: Kalman filter recursive formulation

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Nomenclature / List of Acronyms

AI	Artificial Intelligence
ASV	Autonomous Surface Vehicles
AUV	Autonomous Underwater Vehicle
DGPS	Differential GPS
DR	Dead Reckoning
EIKF	Extended Interval Kalman Filter
EKF	Extended Kalman Filter
GPS	Global Positioning System
IKF	Interval Kalman Filter
INS	Inertial Navigation System
KF	Kalman Filter
LQG	Linear Quadratic Gaussian
MEMS	Micro-Electro-Mechanical Systems
MSDF	Multi-Sensor Data Fusion
UGV	Unmanned Ground Vehicle
UKF	Unscented Kalman Filter
USV	Uninhabited Surface Vehicle / Unmanned Surface Vessel
Q	Process Noise Covariance
R	Measurement Noise Covariance