A Zonal CFD Approach for Fully Nonlinear Simulation of Two vessels in Launch and Recovery Operations

WP3: Coupling fully nonlinear potential method with multiphase models

Shiqiang Yan, Qingwei Ma
& Jinghua Wang
City University London

Project meeting at MMU, 6th June 2016
Coupling Strategies

- Robust QALE-FEM to cover large domain away from floating bodies;
- Self-adaptive wave-maker is applied at outer boundaries of the FNPT domain for wave generation/absorption;
- Strong couple between the FNPT model and the NS model using various technics.

Key:
- FNPT: fully nonlinear potential theory;
- I-NS: incompressible Navier Stokes solver;
- C-NS: compressible Navier Stokes solver.
Objectives & Challenges

**Nonlinear wave absorbing and generation techniques in vessel-fixed coordinate system**

- High sea state: highly nonlinear incident waves
- Large motion of vessels, breaking, aeration, hydro-elasticity: highly nonlinear radiation/diffraction waves due to the vessels
- Wave spectrum is developing and may be changed rapidly due to wave-wave interaction or wind-wave interaction in high sea state
- Evanescent waves near the wave-maker: sufficiently large distance to the vessels

**Numerical techniques to couple the FNPT and multiphase models in space and time**

- FNPT model: unpredictable wave breaking leading to break-down of the single phase model (mesh distortion)
- Significant radiation/diffraction: one-way coupling may not be suitable
- Strong (two-way) coupling:
  - Inviscid in FNPT and viscous turbulent in NS;
  - One-phase in FNPT and two-phases in NS;
  - Velocity potential in FNPT and velocity & pressure in NS
Available Self-adaptive wavemaker theories

- **Time-domain models**

  - $X_t = \frac{\omega}{c_0} (2\eta_i - \eta_m + XD)$
  - Derived for the linear regular wave
  - Approximated coefficients (frequency and transfer function) for irregular waves
  - Easy to be implemented
  - Efficiency significantly reduced as the wave steepness increases due to (1) incorrect implementation of coefficients and (2) mismatch between the wavemaker displacement and velocity

Absorbing efficiency in the cases with different wave frequencies and steepness (Piston self-adaptive wave absorber)
Available time-domain model at City for nonlinear wave absorptions

(1) Replace \( X_t = \frac{\omega}{c_0} (2\eta_i - \eta_m + XD) \)

by

\[
X_t^n = -\frac{\omega c_0 \eta_m^n + D(d\eta_m^n / dt)}{c_0^2 + D^2}
\]

to omit the mismatch of the phases between the wave-maker displacement and velocity.
Available time-domain model at City for nonlinear wave absorptions

- (2) Nonlinear fitting to track the local wave frequency to improve

\[ \text{Initialised } \omega_0^n = \omega_0^{n-1} - 2 \cdot \omega_0^{n-2} \]

Evaluate \( C \) and \( D \) corresponding to \( \omega_k^n \)

Evaluate the wavemaker velocity using governing equation

\[ ka = 0.15, \; fre = 0.6 \]
Objectives & Challenges

Nonlinear wave absorbing and generation techniques in vessel-fixed coordinate system

- High sea state: highly nonlinear incident waves
- Large motion of vessels, breaking, aeration, hydro-elasticity: highly nonlinear radiation/diffraction waves due to the vessels
- Wave spectrum is developing and may be changed rapidly due to wave-wave interaction or wind-wave interaction in high sea state
- Evanescent waves near the wave-maker: sufficiently large distance to the vessels

Numerical techniques to couple the FNPT and multiphase models in space and time

- FNPT model: unpredictable wave breaking leading to break-down of the single phase model (mesh distortion)
- Significant radiation/diffraction: one-way coupling may not be suitable
- Strong (two-way) coupling:
  - Inviscid in FNPT and viscous turbulent in NS;
  - One-phase in FNPT and two-phases in NS;
  - Velocity potential in FNPT and velocity & pressure in NS
New development at City on wave absorption

(1) Evanescent waves near the wave-maker

- Significant interaction between evanescent wave interaction with progressive wave → reduced the absorption efficiency

- Sufficiently large distance to the floating vessels required

- How to optimise the wave-maker geometry to minimise the evanescent waves?

![Graph showing transfer function and efficiency vs. frequency ratio](image)
New development at City on wave absorption

- Optimise the wave-maker geometry to minimise the evanescent waves
New development at City on wave absorption

(2) How to deal with rapid change of the wave spectrum for random extreme sea?

- Available Local wave frequency tracking may lead to instability when subjected rapid change of local wave frequency;
- Partially due to limit of on dealing with regular waves

\[ X_t^n = \frac{-\omega c_0 \eta_m^n + D(d\eta_m^n/dt)}{c_0^2 + D^2} \]
Available Frequency-domain models

- **Elevation feedback** (Brorsen & Frigarrd, 1992, 1995; Schaffer et al, 2000)

  \[
  X(t) = \int_{-\infty}^{\infty} X(i\omega) e^{i\omega t} d\omega
  \]

  Wave elevations

  \[2\eta_i(t) - \eta_m(t) = \int_{-\infty}^{\infty} \tilde{I}(i\omega) e^{i\omega t} d\omega\]

  Transfer Function

  \[
  \tilde{H}(i\omega) = \frac{\tilde{X}(i\omega)}{\tilde{I}(i\omega)} = \frac{1}{iC_o(\omega) - D(\omega)}
  \]

  Convolution

  \[
  X(t) = \int_{-\infty}^{\infty} [2\eta_i(t-\tau) - \eta_m(t-\tau)] h(\tau) d\tau
  \]

- **Force feedback** (Spinneken and Swan, 2009; Chatry et al., 1998; Naito, 2006)

  **wavemaker velocity**

  \[
  X_t(t) = \int_{-\infty}^{\infty} X_t(i\omega) e^{i\omega t} d\omega
  \]

  Force/moment on wavemaker

  \[
  F(t) = \int_{-\infty}^{\infty} F(i\omega) e^{i\omega t} d\omega
  \]

  Transfer Function

  \[
  \tilde{H}(i\omega) = \frac{\tilde{X}_t(i\omega)}{\tilde{F}(i\omega)} = \frac{1}{iN(\omega) - i\omega M(\omega)}
  \]

  Convolution

  \[
  X_t(t) = \int_{-\infty}^{\infty} F(t-\tau) h(\tau) d\tau
  \]
Available Frequency-domain models

- Transfer function and the impulse response are anti-casual;

\[ X_t(t) = \int_{-\infty}^{\infty} \tilde{X}_t(i\omega)e^{i\omega t} d\omega \]

Force/moment on wavemaker

\[ F(t) = \int_{-\infty}^{\infty} \tilde{F}(i\omega)e^{i\omega t} d\omega \]

Transfer Function

\[ \tilde{H}(i\omega) = \frac{\tilde{X}_t(i\omega)}{\tilde{F}(i\omega)} = \frac{1}{N(\omega) - i\omega M(\omega)} \]

Convolution

\[ X_t(t) = \int_{-\infty}^{\infty} F(t-\tau)h(\tau)d\tau \]
Available Frequency-domain models

- Transfer function and the impulse response are anti-casual;
- Available techniques to deal with causality problems
  - Naito (2006): Bode relations
  - Chatry etc (1998): Kramers-Kronig relations

<table>
<thead>
<tr>
<th>Force feedback (Spinneken and Swan, 2009; Chatry et al., 1998; Naito, 2006)</th>
<th><img src="image-url" alt="Graph" /></th>
</tr>
</thead>
</table>

| ![Graph](image-url) | |
Available Frequency-domain models

- Transfer function and the impulse response are anti-casual;
- Available techniques to deal with causality problems
  - Nioto (2006): Bode relations
  - Chatry etc (1998): Kramers-Kronig relations

✓ Force feedback (Spinneken and Swan, 2009; Chatry et al., 1998; Naito, 2006)

Convolution

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) \delta(t) dt \, dt \, dt \]

Transfer function and the impulse response are anti-casual.

Available techniques to deal with causality problems

- Nioto (2006): Bode relations
- Chatry et al. (1998): Kramers-Kronig relations

Force feedback (Spinneken and Swan, 2009; Chatry et al., 1998; Naito, 2006)
Available Frequency-domain models

- Transfer function and the impulse response are anti-casual;
- Available techniques to deal with causality problems:
  - Nioto (2006): Bode relations
  - Chatry etc (1998): Kramers-Kronig relations
- IIR filter (Spinneken etc, 2009) may be used to avoid the causality issue:
  - Most importantly, Finding force in the FNPT model requires solving additional Laplace equation

\[ X_t(t) = \int_{-\infty}^{\infty} \tilde{X}_t(i\omega)e^{i\omega t} d\omega \]

\[ F(t) = \int_{-\infty}^{\infty} \tilde{F}(i\omega)e^{i\omega t} d\omega \]

\[ \tilde{H}(i\omega) = \frac{\tilde{X}_t(i\omega)}{\tilde{F}(i\omega)} = \frac{1}{N(\omega) - i\omega M(\omega)} \]

\[ X_t(t) = \int_{-\infty}^{\infty} F(t-\tau)h(\tau)d\tau \]

\[ \text{Force feedback (Spinneken and Swan, 2009; Chatry et al., 1998; Naito, 2006)} \]
Available Frequency-domain models

✓ Elevation feedback (Brorsen & Frigarrd, 1992, 1995; Schaffer et al, 2000)

wavemaker displacement

\[ X(t) = \int_{-\infty}^{\infty} \tilde{X}(i\omega) e^{i\omega t} d\omega \]

Wave elevations

\[ 2\eta_i(t) - \eta_m(t) = \int_{-\infty}^{\infty} \tilde{I}(i\omega) e^{i\omega t} d\omega \]

Transfer Function

\[ \tilde{H}(i\omega) = \frac{\tilde{X}(i\omega)}{\tilde{I}(i\omega)} = \frac{1}{iC_o(\omega) - D(\omega)} \]

Convolution

\[ X(t) = \int_{-\infty}^{\infty} [2\eta_i(t-\tau) - \eta_m(t-\tau)] h(\tau) d\tau \]

✓ Does not require to solving additional Laplace equation in the FNPT;

✓ Causality issue exists;

✓ IIR (Schaffer) or FIR (Frigarrd etc, 1995) filters can be used to deal with the issue
New development at City on wave absorption

- Elevation feedback (Brorsen & Frigarrd, 1992, 1995; Schaffer et al, 2000)
- Transfer wave elevation to wavemaker velocity

Exist approach
New approach

- Transfer function in the existing models becomes infinite at zero frequency

Convolution

\[ X_i(t) = \int_{-\infty}^{\infty} [2\eta_i(t-\tau) - \eta_m(t-\tau)]h(\tau)\,d\tau \]
New development at City on wave absorption

- Preliminary Test (Linear random wave absorption)

Random wave spectrum
Frequency range (0~5)

Linear wave theory → generate the wavemaker motion → progressive wave + evanescent waves

FIR Filter

\[ X^{n+1} = \sum_{j=-M}^{M} h^{j} (2\eta_{I} - \eta_{m})^{n+1-M+j} \]

\[ X_{t}^{n+1} = \sum_{j=-M}^{M} h^{j} (2\eta_{I} - \eta_{m})^{n+1-M+j} \]

IIR Filter

\[ X^{n} = \sum_{k=0}^{M} a_{k} \tilde{\eta}^{n-k} + \sum_{k=1}^{N} b_{k} X^{n-k} \]

\[ X_{t}^{n} = \sum_{k=0}^{M} a_{k} \tilde{\eta}^{n-k} + \sum_{k=1}^{N} b_{k} X_{t}^{n-k} \]

\[ \tilde{H}(i\omega) = \frac{a_{0} + \sum_{k=1}^{M} a_{k} e^{-ik\omega\Delta t}}{1 - \sum_{k=1}^{N} b_{k} e^{-ik\omega\Delta t}} \]
New development at City on wave absorption

- Preliminary Test (Linear random wave absorption)

![Graphs showing wave absorption](image)

**Exist approach**

**New approach**

Impulse response function
New development at City on wave absorption

- Preliminary Test (FIR filter)

\[ \sum_{j=-M}^{M} h^j (2\eta_I - \eta_m)^{n+1-M+j} \]

FIR filter suffers from a phase shift
New development at City on wave absorption

- Preliminary Test (FIR filter)

- By taking the wave elevations at a distance in front of the wave paddle, the problem may be partially resolved.

- Sensitive to the frequency range used to find the impulse response function → difficult to handle rapid change of spectrum.
New development at City on wave absorption

- Preliminary Test (IIR filter)

Exist approach

New approach
New development at City on wave absorption

- Preliminary Test (IIR filter: stability test)

- The existing approach is more sensitive to the frequency range, frequency resolution, M and N → difficult to handle rapid change of spectrum
New development at City on wave absorption

(2) How to deal with rapid change of the wave spectrum for random extreme sea?

- Introduce wavelet analysis, which can capture the change of the wave spectrum
- Extend the developed technique to 3D problems in vessel fixed coordinate system
WP3: Coupling fully nonlinear potential method with multiphase models

Yan, Ma and Wang, City University London, 6th June, 2016, MMU

Numerical techniques to couple the FNPT and multiphase models in space and time

- FNPT model: unpredictable wave breaking leading to break-down of the single phase model (mesh distortion)
- Significant radiation/diffraction invalid the one-way coupling
- Strong (two-way) coupling:
  - Inviscid in FNPT and viscous turbulent in NS;
  - One-phase in FNPT and two-phases in NS;
  - Velocity potential in FNPT and velocity & pressure in NS

- Using empirical formula to artificially remove wave energy?
- Coupling with I-NS or C-NS developed in WP1?
- Develop robust approach based on potential theory?
FNPT model: Dealing with wave breaking

- Mesh moves conforming to the motion of body
- ALE form of governing equations
- Mesh distortion and requiring local remeshing
- Fixed background mesh
- Local mesh moves together with the motion of bodies
- Interpolation in the overlap area
- Available in CFD software
- Fixed Euler mesh
- Rigid/solid bodies are treated as one phase
- Requiring technique to identify the body surface, e.g. VOF, level set
- No need to deal with mesh problem
FSI Models

✓ Fluid momentum equation in $\Omega_t$
\[
\frac{\partial \rho \mathbf{u}}{\partial t} - \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) - \nabla \cdot \mathbf{\sigma}(\mathbf{u}) = \rho \mathbf{g}
\]

✓ Solid momentum equation in $\Omega_0$
\[
\frac{\partial \rho_0 \mathbf{U}}{\partial t} - \nabla_0 \cdot \mathbf{P}(\mathbf{F}) = \rho_0 \mathbf{g}
\]

✓ Rigid body momentum balance equation
\[
M \frac{d \mathbf{u}_c}{dt} = \mathbf{f}_c \quad J \frac{d \mathbf{\omega}}{dt} + \mathbf{\omega} \times J \mathbf{\omega} = \mathbf{t}_c
\]

One-Fluid IBM @ City

\[
\frac{\partial \rho \mathbf{u}}{\partial t} - \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) - \nabla \cdot \mathbf{\mu} \nabla \mathbf{u} + \nabla p = \rho \mathbf{g}
\]

➢ Continuity equation

\[
\frac{\partial \rho \mathbf{u}}{\partial t} - \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) - \nabla \cdot \mathbf{\sigma}(\mathbf{F}) = \rho \mathbf{g}
\]

➢ Conservation of deformation gradient
\[
\frac{\partial \rho \mathbf{u}}{\partial t} = \nabla \mathbf{u} \mathbf{F} \quad \text{or} \quad \frac{\partial (\mathbf{F}^{-1})}{\partial t} + \nabla \mathbf{u} \mathbf{F}^{-1} = 0
\]

➢ Under constraint $\mathbf{u} = \mathbf{P}(\mathbf{u})$

$\mathbf{P}(\mathbf{u})$ is a vector projection operator, projecting any vector field into a rigid body space.
One-Fluid IBM: Unified framework

- Incompressible flow: computational domain $\Omega$
  - Momentum equation $\rho \left[ \frac{\partial u}{\partial t} + (\nabla u)u \right] + \nabla p - \nabla \cdot \sigma' - \rho g = 0$
    - Fluid phase: $\nabla \cdot \sigma' = \nabla \cdot 2\mu(D(u))$
    - Solid phase: $\nabla \cdot \sigma' = f(F)$ depending on material properties
    - Rigid body: $\nabla \cdot \sigma' = \rho \left[ \frac{\partial P(u)}{\partial t} + (\nabla P(u))P(u) \right] + \nabla p - \rho g$

- Continuity equation $\nabla \cdot u = 0$

- Level-set functions $\frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = 0$

- Rigid body Heaviside function $H_r(x, t) = H_r(X)$

- Fractional step method for velocity-pressure coupling
One-Fluid IBM: Benchmark Tests

Fluid structure interaction: Dam Breaking with elastic obstacle

A water column of width $a = 14.6 \text{ cm}$ and height $2a$ is placed in the left corner of a rectangular tank of size $4a \times 2.5a$. A rectangular incompressible obstacle in the shape of a column of width $b = 1.2 \text{ cm}$, height $20/3b$, Young’s modulus $E = 107 \text{ g/cm/s}^2$ and $s = 2.5 \text{ g/cm}^2$ is fixed at the centre bottom of the tank.
One-Fluid IBM: Water Entry Problems

Free dropping of wedges or ship sections

<table>
<thead>
<tr>
<th>Case No</th>
<th>Model</th>
<th>Drop height (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wedge-01</td>
<td>Wedge tilting angle 0°</td>
<td>500</td>
</tr>
<tr>
<td>Wedge-05</td>
<td>Wedge tilting angle 20°</td>
<td>500</td>
</tr>
<tr>
<td>SS-09</td>
<td>Ship Section Model III</td>
<td>170</td>
</tr>
<tr>
<td>SS-11</td>
<td>Ship Section Model III</td>
<td>300</td>
</tr>
</tbody>
</table>

Table 1. Summary of the test cases
Note: the dead-rise angle of the wedge model is 30°

Experimental data from Wave Induced Loads on Ships (WILS) Joint Industry Project used in the ISOPE comparative study on water entry problems
Wedge Dropping (wedge velocity)

Computational mesh near the wedge used by the IBM (uniform Euler grid)

Comparison of the wedge velocities

- Motion of the wedge is well predicted, validating the unified equation for rigid body motion
Wedge Dropping (free surface elevation)

- Good agreement between the numerical results by IBM and the experiment
- The OpenFOAM gives worse results

Snapshot of the volume fraction, velocity fields near the dropping wedge at $t \approx 0.35s$ (Case Wedge-01; OpenFOAM: incompressible with $d_{s,min} = 0.3125\text{mm}$; IBM: $d_s = 1\text{mm}$)
Wedge Dropping: Pressure and Force

- Both IBM and OpenFOAM lead to larger peak pressure/force compared to experiments, consistent with the results by other numerical methods.
Similar “over-estimation” are found in Case Wedge-05
Such inconsistencies have been reported by other numerical methods.
Ship-Section Dropping (free surface elevation)

Computational mesh near the wedge used by the IBM (uniform Euler grid)

Snapshot of the volume fraction, velocity fields near the dropping ship section (Case SS-11; OpenFOAM: incompressible with $d_{min} = 0.3125\text{mm}$; IBM: $d_s = 1\text{mm}$)

- IBM results agree well with experimental data
- Entrapped air bubbles are well predicted by the IBM but are not resolved in the OpenFOAM
Ship-Section Dropping (pressure & force)

Time histories of pressures and forces on the dropping ship section (Case SS-09, Ship section model III; OpenFOAM: incompressible $d_{s_{min}} = 0.3125$mm; IBM: $d_s = 1$mm)

- IBM results reasonably agree with the experimental data, better than the incompressible OpenFOAM
FNPT model: Dealing with wave breaking

- Testing the idea of extending the one-fluid IBM to deal with local wave breaking
- Or single-phase MLPG_R?

- Introduce wavelet analysis, which can capture the change of the wave spectrum
- Extend the developed technique to 3D problems in vessel fixed coordinate system

Planned work in the next 6 months
Thank You