Basic Engineering

DC Circuits and Electrical Power

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The aim of this package is to provide a short self assessment programme for students who want to understand electrical power in DC circuits.

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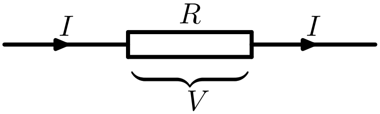
The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Introduction

The basic SI units for electrical systems are:

- **Current**: the **ampere** (symbol: I; unit: A)
- **Potential Difference**: the **volt** (symbol: V; unit: V)
- **Resistance**: the **ohm** (symbol: R; unit: Ω)

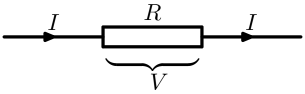
Ohm's law, see the package on **Simple DC Circuits**, states that for the diagram below:



the potential difference across the resistance is related to the current by

$$V = IR$$

Quiz In the diagram



what current flows through a $100\,\Omega$ resistor if a voltage difference of $1,000\,\mathrm{V}$ is applied to it?

(a) $900\,\mathrm{A}$ (b) $10\,\mathrm{A}$ (c) $10^5\,\mathrm{A}$ (d) $0.1\,\mathrm{A}$

Power is the rate of doing work, or, of converting energy from one form to another. Its units are **joules per second**. One joule per second is called a **watt** W (symbol P).

Example 1 If a machine converts 1,000 J of energy in 5 seconds, what is its power?

The power, P, is given by:

$$P = \frac{\text{energy converted}}{\text{time}} = \frac{1,000\,\mathrm{J}}{5\,\mathrm{s}} = 200\,\mathrm{W}\,.$$

2. Electrical Units and Power

When current flows through a wire, the wire gets hot: i.e., power is dissipated. (This heat is why the filament in a light bulb glows.)

This leads to the definition of potential difference:

> when a current of one ampere flows through a resistor, one watt of power is dissipated by the resistor when a potential difference of one volt appears across it.

In general the power, P, voltage and current are related by:

$$P = VI$$

Example 2 If a current of $30\,\mathrm{A}$ flows through a resistor to which a voltage of $100\,\mathrm{V}$ is applied, what power is dissipated in the resistor?

From $P = VI$ and the given data

$$P = 100\,\mathrm{V} \times 30\,\mathrm{A} = 3,000\,\mathrm{W} \quad \text{(or } 3\,\mathrm{kW}.)$$

Quiz If a current of 3 amperes flows along a wire with a potential difference of 4 volts between the ends, how much power is dissipated along the wire?

(a) 0 (b) $7\,\mathrm{W}$ (c) $12\,\mathrm{W}$ (d) $\frac{4}{3}\,\mathrm{W}$

There are other ways of writing the power $P = VI$.

Quiz As well as $P = VI$, which of the equations below also describes the power dissipated by an electrical circuit? (*Hint*: use Ohm's law.)

(a) $P = \frac{I^2}{R}$ (b) $P = I^2 R$ (c) $P = \frac{R}{V^2}$ (d) $P = V^2 R$

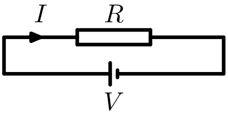
Quiz What is the power consumption of a $100\,\Omega$ resistor if a $50\,\mathrm{mA}$ current flows through it?

(a) $0.25\mathrm{W}$ (b) $2.5 \times 10^6\mathrm{W}$ (c) $2.5 \times 10^4\mathrm{W}$ (d) $5 \times 10^5\mathrm{W}$

From Ohm's law, there are three equivalent expressions for the power dissipation in a circuit:

$$P = VI\,, \qquad P = \frac{V^2}{R}\,, \qquad P = I^2R$$

Exercise 1.



(a) In the circuit if $R = 6\,\Omega$ and $I = 3\,\mathrm{A}$, what is the power?

(b) In the circuit if $V = 8\,\mathrm{V}$ and $R = 2\,\Omega$, what is the power?

(c) Finally, what is the power if $V = 8\,\mathrm{V}$ and $I = 0.25\,\mathrm{A}$?

Quiz If a current of $3\,\mathrm{A}$ flows along a wire with a potential difference of $4\,\mathrm{V}$ for one hour, how much energy is dissipated?

(a) 12J (b) 720J (c) 4,320J (d) 43,200J

In the **electricity supply** industry the SI units of watts and joules are too small. Instead the units used are:

power unit: kilowatt ($1\,\mathrm{kW} = 10^3\,\mathrm{W}$)
energy unit: kilowatt-hour ($1\,\mathrm{kW\,h} = 10^3 \times 60 \times 60\,\mathrm{J}$)

Example 3 The **unit of electricity** familiar from household bills is one kilowatt-hour (i.e., it is an amount of energy consumption). What is it in joules?

$$1\ \mathrm{kW\,h} = 1,000 \times 60 \times 60 = 36 \times 10^5 = 3.6 \times 10^6\,\mathrm{J}\,.$$

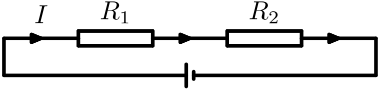
This is such a large number that it is easy to understand why your electricity is not sold in joules!

Quiz If a household electricity metre changes from 5732 to 5786 units, how much electrical energy has been dissipated in the house?

(a) $2 \times 10^8\,\mathrm{J}$ (b) $2 \times 10^{10}\,\mathrm{J}$ (c) $2 \times 10^6\,\mathrm{J}$ (d) $5.4 \times 10^3\,\mathrm{J}$

3. Series and Parallel Circuits

In a **series circuit**:



the same current flows through each resistor. Hence in the diagram the power dissipated in them are

$$P_1 = I^2 R_1\,, \quad \text{and } P_2 = I^2 R_2\,,$$

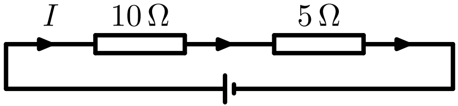
respectively and the total power dissipated is

$$P_T = I^2(R_1 + R_2)\,,$$

By Ohm's law the voltage source is $V = I(R_1 + R_2)$, the power can also be written as $P_T = \dfrac{V^2}{R_1 + R_2}$.

> **Note**: the equations show that power dissipation in resistors connected in series is directly proportional to their resistance.

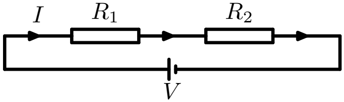
Example 4 In the series circuit



since $10\,\Omega$ is twice as big as $5\,\Omega$ the power dissipated in the $10\,\Omega$ resistor will be twice that dissipated in the $5\,\Omega$ resistor.

If $I = 2\,\text{A}$ the power dissipation, $P = I^2R$, will be $2^2 \times 10 = 40\,\text{W}$ in the $10\,\Omega$ resistor and $2^2 \times 5 = 20\,\text{W}$ in the $5\,\Omega$ resistor.

Exercise 2.

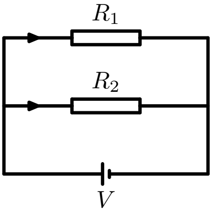


(a) If above $R_1 = 5\,\Omega$ and $R_2 = 15\,\Omega$, how much more power is used in the $15\,\Omega$ resistor?

(b) If $I = 0.8\,\text{A}$, calculate the power dissipation in each resistor.

(c) How much energy is dissipated over 30 minutes?

Example 5 If two resistors are connected in **parallel**, the effective resistance is less than either of the two individual resistors. (This is because there are more ways for the current to flow.)



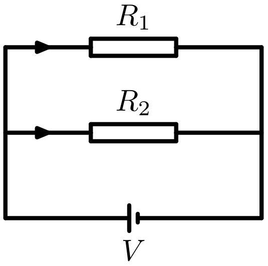
The potential difference across the two parallel resistors is the same, V. Hence the total power in the **resistors in parallel** is

$$P_T = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \frac{V^2(R_1 + R_2)}{R_1 R_2} \,.$$

This should be compared with the result for **resistors in series**:

$$P_T = \frac{V^2}{R_1 + R_2} \,.$$

Exercise 3. Consider a $10\,\Omega$ and a $5\,\Omega$ resistor connected in parallel across a $2\,\mathrm{V}$ source.

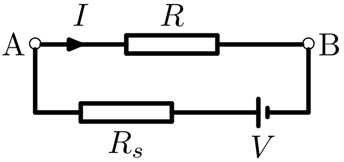


(a) What is the power dissipated in the $10\,\Omega$ resistor?

(b) What is the power dissipated in the $5\,\Omega$ resistor?

(c) How does the total power dissipated differ from the case if the same resistors were connected in series?

Example 7 The series circuit below represents a power source with an internal resistor R_s. If a *load resistor* R is connected across the terminals A and B, how does the power to load, P_L, depend upon R?



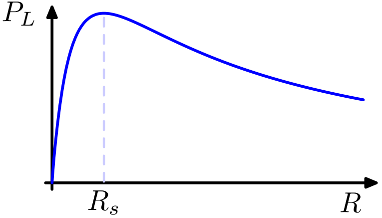
The current I is given by

$$V = I(R_s + R)\,, \quad \Rightarrow\ I = \frac{V}{R_s + R}\,.$$

Using $P_L = I^2 R$, the power to load is thus

$$P_L = \frac{V^2 R}{(R_s + R)^2}\,.$$

The curve of this is shown on the next page.



Some important cases for the **power to load** are:

Short Circuit: if there is no resistance between the terminals, $R = 0$, the power to load is

$$P_L = \frac{V^2 \times 0}{(R_s + 0)^2} = \frac{0}{R_s} = 0 \, .$$

No power can be extracted from a short circuit: there *must be a resistance to extract power*.

Open Circuit: if the terminals are disconnected then there is an infinite resistance, $R \to \infty$, and no current flows. Again the power to load vanishes: *a current must flow to extract power*.

Maximum Power: in the curve above it is shown that the maximum power across the load resistance is when $R = R_s$, i.e., when the load resistance is equal to the internal resistance of the source (perhaps a battery or generator). This is called **resistance matching**.

EXERCISE 4. The power to load is given by

$$P_L = \frac{V^2 R}{(R_s + R)^2} \,.$$

(a) What is P_L when $R = R_s$?

(b) What is P_L when $R = 100R_s$??

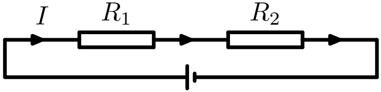
(c) What is P_L when $R = 0.001R_s$??

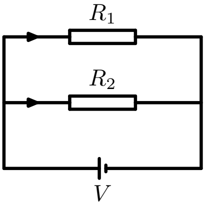
(d) The maximum power will be given when

$$\frac{dP_L}{dR} = 0 \,,$$

use the quotient rule of differentiation to show that the maximum is at $R = R_s$.

4. Final Quiz





Begin Quiz

1. In the series circuit above: what is the total power if $I = 0.1\,\mathrm{A}$, $R_1 = 3\,\Omega$ and $R_2 = 2\,\Omega$?

(a) $5\,\mathrm{W}$ (b) $50\,\mathrm{mW}$ (c) $0.5\,\mathrm{W}$ (d) $2.5\,\mathrm{W}$

2. In the parallel circuit above: what is the total power if $V = 3\,\mathrm{V}$, $R_1 = 3\,\Omega$ and $R_2 = 2\,\Omega$?

(a) $1.8\,\mathrm{W}$ (b) $2.7\,\mathrm{W}$ (c) $750\,\mathrm{mW}$ (d) $7.5\,\mathrm{W}$

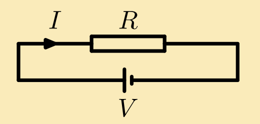
3. If the parallel circuit runs for a day, how much energy is used?

(a) $648\,\mathrm{kW\,h}$ (b) $10.8\,\mathrm{kW\,h}$ (c) $0.18\,\mathrm{kW\,h}$ (d) $0.45\,\mathrm{kW\,h}$

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)



Given the circuit drawn above with resistance $R = 6\,\Omega$ and current $I = 3\,\mathrm{A}$, the power is

$$\begin{aligned} P &= I^2 R \\ &= 9\,A^2 \times 6\,\Omega = 54\,\mathrm{W}\,. \end{aligned}$$

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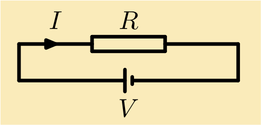
Exercise 1(b)

For the circuit drawn below



with resistance $R = 2\,\Omega$ and voltage $V = 8\,\text{V}$, the power dissipation is

$$\begin{aligned} P &= \frac{V^2}{R} \\ &= \frac{64\,\text{V}^2}{2\,\Omega} = 32\,\text{W}\,. \end{aligned}$$

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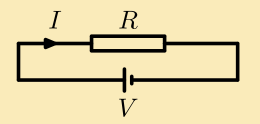
□

Exercise 1(c)

In the circuit drawn below



with the electric current $I = 0.25\,\mathrm{A}$ and voltage $V = 8\,\mathrm{V}$, the power is given by

$$\begin{aligned} P &= VI \\ &= 8\,\mathrm{V} \times 0.25\,\mathrm{A} = 2\,\mathrm{W}\,. \end{aligned}$$

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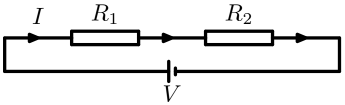
Exercise 2(a)



If two resistors $R_1 = 5\,\Omega$ and $R_2 = 15\,\Omega$ are added in series, as shown above, the electric current flow I is the same through each and the power dissipation is proportional to their resistance $P_1 = I^2 R_1$ and $P_2 = I^2 R_2$. Therefore

$$\frac{P_2}{P_1} = \frac{I^2 R_2}{I^2 R_1} = \frac{R_2}{R_1} = 3\,.$$

The power dissipated in R_2 is three times more than that dissipated in R_1.

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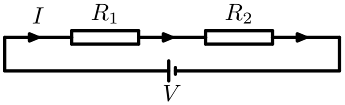
□

Exercise 2(b)

If two resistors $R_1 = 5\,\Omega$ and $R_2 = 15\,\Omega$ are added in series, as shown below



the electric current flow $I = 0.8\,\mathrm{A}$, through each resistor is the same. Therefore the power dissipation in each resistor is given by

$$P_1 = I^2 R_1 = (0.8\,\mathrm{A})^2 \times 5\,\Omega = 3.2\mathrm{W}\,,$$

and

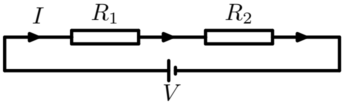
$$P_2 = I^2 R_2 = (0.8\,\mathrm{A})^2 \times 15\,\Omega = 9.6\mathrm{W}\,.$$

Click on the **green** square to return

□

Exercise 2(c)

If two resistors $R_1 = 5\,\Omega$ and $R_2 = 15\,\Omega$ are added in series



with current $I = 0.8\,\mathrm{A}$, the equivalent total resistance R_T is:

$$R_\mathrm{T} = R_1 + R_2 = (5 + 15)\,\Omega = 20\,\Omega\,.$$

and the total power, i.e. the energy dissipated per second, is given by

$$P_T = I^2 R_T = (0.8\,\mathrm{A})^2 \times 20\,\Omega = 12.8\,\mathrm{W}\,.$$

Therefore the energy dissipated over 30 minutes is

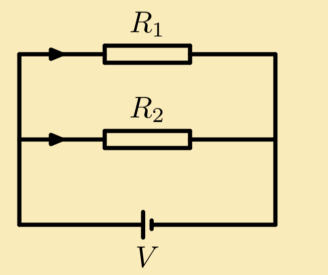
$$P_T \times 30 \times 60\,\mathrm{s} = 12.8\,\mathrm{W} \times 1800\,\mathrm{s} = 22,040\,\mathrm{J}\,.$$

This is $P_T = 22,040\,/3.6 \times 10^6 \approx 0.006\,\mathrm{kWh}\,.$

Click on the **green** square to return □

Exercise 3(a)

If two resistors $R_1 = 10\,\Omega$ and $R_2 = 5\,\Omega$ are connected in parallel across a $V = 2\,\text{V}$ source, the current flow through the first resistor R_1 is by Ohm's law



$$I_1 = \frac{V}{R_1} = \frac{2\,\text{V}}{10\,\Omega} = 0.2\,\text{A}\,,$$

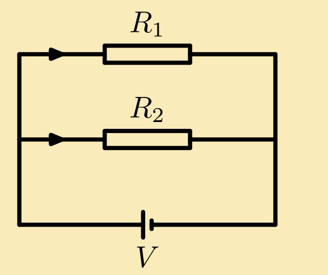
Therefore the power dissipated in the R_1 resistor is

$$P_1 = I_1 V = 0.2\,\text{A} \times 2\,\text{V} = 0.4\,\text{W}\,,$$

Click on the **green** square to return

Exercise 3(b)

If two resistors $R_1 = 10\,\Omega$ and $R_2 = 5\,\Omega$ are connected in parallel across a $V = 2\,\text{V}$ source, the current flow through the second resistor R_2 is



$$I_2 = \frac{V}{R_2} = \frac{2\,\text{V}}{5\,\Omega} = 0.4\,\text{A}\,,$$

Therefore the power dissipated in the R_2 resistor is

$$P_2 = I_2 V = 0.4\,\text{A} \times 2\,\text{V} = 0.8\,\text{W}\,,$$

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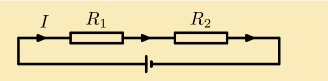
□

Exercise 3(c)

If now the same resistors $R_1 = 10\,\Omega$ and $R_2 = 5\,\Omega$ are connected in series across a $V = 2\,\mathrm{V}$ source,



the total power dissipation is given by

$$P_T^{\text{series}} = \frac{V^2}{R_1 + R_2} = \frac{4\,\mathrm{V}^2}{(10+5)\,\Omega} \approx 0.3\,\mathrm{W}\,,$$

while the total power dissipated in the parallel circuit is

$$P_T^{\text{parallel}} = \frac{V^2}{R_1} + \frac{V^2}{R_2} = (0.4 + 0.8)\,\mathrm{W} = 1.2\,\mathrm{W}\,.$$

The power dissipated in the parallel circuit is four times more than in the series circuit. This is why lights are generally fitted in parallel and not in series.

Click on the **green** square to return □

Exercise 4(a)

The power to load is given by

$$P_L = \frac{V^2 R}{(R_s + R)^2}\,,$$

therefore when $R = R_s$ this gives

$$P_L = \frac{V^2 R_s}{(R_s + R_s)^2} = \frac{V^2 R_s}{4R_s^2} = \frac{V^2}{4R_s}\,.$$

Click on the **green** square to return

□

Exercise 4(b)

The power to load is given by

$$P_L = \frac{V^2 R}{(R_s + R)^2}\,,$$

therefore plugging in $R = 100R_s$ gives

$$\begin{aligned} P_L &= \frac{V^2 \times 100R_s}{(R_s + 100R_s)^2} = \frac{V^2 \times 100R_s}{(100+1)^2 R_s^2} \\ &= \frac{V^2 \times 100}{101^2 R_s} \approx 10^{-2} \times \frac{V^2}{R_s}\,. \end{aligned}$$

This is much less than when $R = R_s$ (see the curve on p.14).

Click on the **green** square to return □

Exercise 4(c)

The power to load is given by

$$P_L = \frac{V^2 R}{(R_s + R)^2}\,,$$

therefore plugging in $R = 0.001R_s$ gives

$$\begin{aligned} P_L &= \frac{V^2 \times 0.001R_s}{(R_s + 0.001R_s)^2} = \frac{V^2 \times 0.001R_s}{(1 + 0.001)^2 R_s^2} \\ &= \frac{V^2 \times 10^{-3}}{1.001^2 R_s} \approx 10^{-3} \times \frac{V^2}{R_s}\,. \end{aligned}$$

Again this is much less than when $R = R_s$ (see the curve on p.14).

Click on the **green** square to return □

Exercise 4(d)

Using the quotient rule of differentiation we have

$$\begin{aligned}\frac{dP_L}{dR} &= \frac{d}{dR}\left(\frac{V^2R}{(R_s+R)^2}\right)\\ &= \frac{V^2\times(R_s+R)^2 - V^2R\times 2(R_s+R)}{(R_s+R)^4}\\ &= V^2(R_s+R)\times\frac{(R_s+R)-2R}{(R_s+R)^4}\\ &= V^2\frac{R_s-R}{(R_s+R)^3}\,.\end{aligned}$$

Therefore $\frac{dP_L}{dR} = 0$ when $R_s = R$. This agrees with the curve on p.14.

Click on the **green** square to return

Solutions to Quizzes

Solution to Quiz:

If a potential difference of $1,000\,V$ is applied to a wire with resistance $R = 100\,\Omega$, the measured electric current I is by Ohm's law given by

$$I = \frac{V}{R} = \frac{1,000\,V}{100\,\Omega} = 10\,A\,.$$

End Quiz

Solution to Quiz:

If a current of $3\,A$ flows along a wire with a potential difference of 4 volts between the ends, the power P dissipated along the the wire is given by

$$P = VI = 4\,V \times 3\,A = 12\,\mathrm{W}\,.$$

In this calculation the power is calculated in units of **watts**

$$\text{watts} = \text{volts} \times \text{amperes}\,.$$

End Quiz

Solution to Quiz:

From the equation expressing the power P dissipated by a circuit in terms of current I and voltage V

$$P = VI$$

and using Ohm's law

$$V = IR\,,$$

we can write power also as

$$P = VI = IR \times I = I^2R\,.$$

End Quiz

Solution to Quiz:

If a $50\,\mathrm{mA} = 50 \times 10^{-3}\mathrm{A} = 5 \times 10^{-2}\,\mathrm{A}$ current flows through a $100\,\Omega = 10^2\,\Omega$ resistor the power consumption P is given by

$$\begin{aligned} P &= I^2 R \\ &= \left(5 \times 10^{-2}\,A\right)^2 \times\ 10^2\,\Omega \\ &= 25 \times 10^{-4} \times 10^2\,\mathrm{W} \\ &= = 25 \times 10^{-2}\,\mathrm{W} = 0.25\,\mathrm{W}\,. \end{aligned}$$

End Quiz

Solution to Quiz:

When a current of 3 A flows along a wire with a potential difference of 4 V, the power

$$P = VI = 4\,V \times 3\,A = 12\mathrm{W}\,,$$

gives the value of dissipated energy per second. Therefore this electric flow during one hour $1\mathrm{h} = 60 \times 60\,\mathrm{s} = 3600\,\mathrm{s}$ gives

$$E_{\mathrm{dissipated}} = 12\mathrm{W} \times 3600\,\mathrm{s} = 43,200\,\frac{\mathrm{J}}{\mathrm{s}} \times \mathrm{s} = 43,200\mathrm{J}\,.$$

This is a large number of joules. For this reason the joule is not used as a unit of energy in electricity supply. End Quiz

Solution to Quiz:

When a household electricity metre changes from 5732 to 5786 units, it means that the energy consumption in kilowatt-hours is

$$5786 - 5732 = 54\text{kWh}\,.$$

Using the relation

$$1\ \text{kW h} = 3.6 \times 10^6\text{J}$$

the energy value in joules is

$$54 \times 3.6 \times 10^6\text{J} \approx 2 \times 10^8\text{J}\,.$$

End Quiz