

Changing Units

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The aim of this package is to provide a short self assessment programme for students who wish to learn how to convert between different units.

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The full range of these packages and some instructions, should they be required, can be obtained from our web page [Mathematics Support Materials](#).

1. Introduction

The use of different units in science and everyday life makes it important to be able to **convert between different units**.

Example 1 One kilometre is roughly $\frac{5}{8}$ of a mile. What is a mile in kilometres?

$$1 \text{ km} = \frac{5}{8} \text{ mile} \quad (\textit{multiply by 8})$$

$$8 \text{ km} = 5 \text{ miles} \quad (\textit{divide by 5})$$

$$\frac{8}{5} \text{ km} = 1 \text{ mile}$$

Therefore one mile is roughly $\frac{8}{5}$ of a kilometre.

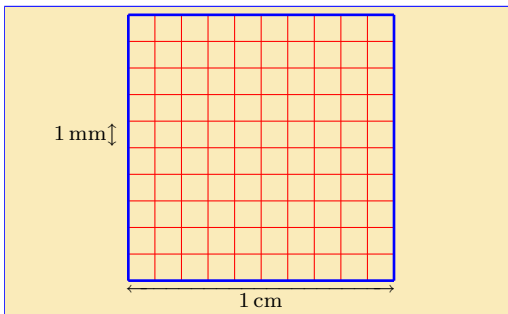
EXERCISE 1. Express the following quantities in the units requested (click on the **green** letters for the solutions).

- (a) One year in **seconds**. (b) 0.016 miles in **kilometres**.
(c) A speed of 10 miles per hour (d) One million pounds in
in **kilometres per hour**. **pennies**.

2. Powers of Units

It is important to take care with powers of units.

Example 2 Consider the **area** of the square drawn below. From the package on **Dimensional Analysis** we know that this has dimensions of length squared (L^2). Its numerical value depends on the units used.



Two possible ways of expressing the area are **either**:

$(1 \text{ cm}) \times (1 \text{ cm}) = 1 \text{ cm}^2$ **or** $(10 \text{ mm}) \times (10 \text{ mm}) = 100 \text{ mm}^2$.

In general, different units are linked by a **conversion factor** and an area given in one unit may be expressed in another unit by multiplying the area by the square of the conversion factor.

Since $1 \text{ cm} = 10 \text{ mm}$, the **conversion factor is 10**. So an area of one square centimetre is $10^2 = 100$ square millimetres.

Now try these two short quizzes:

Quiz Which of the following is the **area** of the square whose sides are 1 cm long in the **SI units of square metres**?

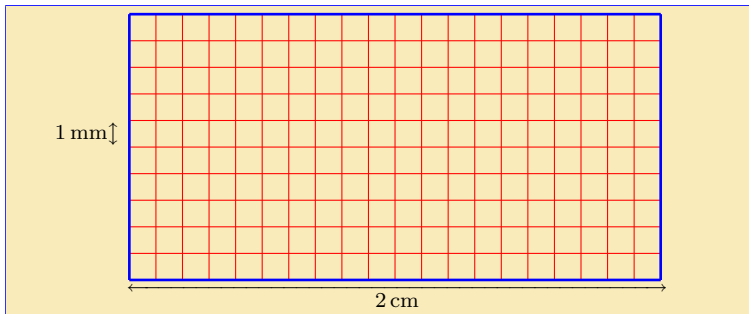
- (a) 0.01 m^2 (b) 100 m^2 (c) 0.0001 m^2 (d) $10,000 \text{ m}^2$

Quiz An **area** of a square mile is a mile times a mile. Recalling that $8 \text{ km} \approx 5 \text{ miles}$, what is this in square kilometres?

- (a) $\frac{25}{64} \text{ km}^2$ (b) $\frac{64}{25} \text{ km}^2$ (c) $\frac{16}{10} \text{ km}^2$ (d) $\frac{10}{16} \text{ km}^2$

Note that we multiply the area by the square of the conversion factor. See also the following example.

Example 3 Consider the **area** of the rectangle drawn below.



Its area is given by the product of the sides. This is **either**:

$$\text{area} = (2 \text{ cm}) \times (1 \text{ cm}) = 2 \text{ cm}^2$$

or

$$\text{area} = (20 \text{ mm}) \times (10 \text{ mm}) = 2 \times 10^2 \text{ mm}^2 = 200 \text{ mm}^2$$

Note the square of the conversion factor in the last line.

In general an area of $A \text{ cm}^2$ can be expressed in square millimetres as

$$\begin{aligned} A \text{ cm}^2 &= A \times (10 \text{ mm})^2 \\ &= A \times 10^2 \text{ mm}^2 \\ &= 100A \text{ mm}^2 \end{aligned}$$

Similarly a **volume** (whose **dimensions are L^3**) can be converted from one set of units to another by multiplying by the **cube of the appropriate conversion factor**.

Quiz What is a **volume** of 3 cm^3 expressed in cubic millimetres?

- (a) 300 mm^3 (b) $27,000 \text{ mm}^3$ (c) $3 \times 10^4 \text{ mm}^3$ (d) $3,000 \text{ mm}^3$

Quiz A **litre (l)** is a cubic decimetre, where $1 \text{ dm} = 0.1 \text{ m}$. Express 0.5 l in cubic metres.

- (a) $5 \times 10^{-4} \text{ m}^3$ (b) $1.25 \times 10^{-2} \text{ m}^3$ (c) 0.005 m^3 (d) $5,000 \text{ m}^3$

Similarly, with **negative powers** of a unit we **divide** by the appropriate power of the conversion factor. An example of this follows.

Example 4 To convert a density (dimension ML^{-3}) of 1 kg m^{-3} into kg dm^{-3} we use

$$\begin{aligned} 1 \text{ kg m}^{-3} &= 1 \frac{\text{kg}}{\text{m}^3} \\ &= 1 \frac{\text{kg}}{(10\text{dm})^3} \\ &= 1 \frac{\text{kg}}{1000\text{dm}^3} \\ &= 0.001 \text{ kg dm}^{-3} \end{aligned}$$

This may now be practised in the next quiz:

Quiz Express a **density** of $3 \times 10^2 \text{ kg m}^{-3}$ in terms of kilograms per cubic centimetre.

(a) $3 \times 10^{-4} \text{ kg cm}^{-3}$

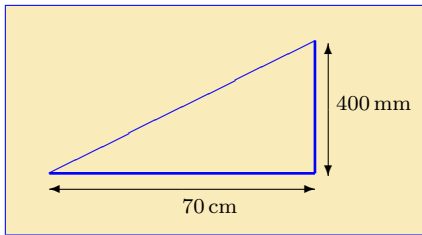
(b) $2.7 \times 10^{11} \text{ kg cm}^{-3}$

(c) $3 \times 10^8 \text{ kg cm}^{-3}$

(d) $3 \times 10^5 \text{ kg cm}^{-3}$

EXERCISE 2. Perform the following unit conversions (click on the **green** letters for the solutions).

- (a) Find the density $3 \times 10^2 \text{ kg m}^{-3}$ in **grams per cubic decimetre**.
- (b) Calculate the area of the triangle below in **square metres**.



- (c) The power in **Watts** of a device that uses 60 mJ (milli Joules) of energy in half a microsecond. (Power is the rate of conversion of energy with time. See the package on **Units**.)
- (d) A kilowatt hour is the energy used by a device with a power output of one kilowatt in one hour. Express this in **Joules**.

3. Equations and Units

Generally it is best in equations to express all quantities in SI units before performing calculations.

Example 4 The electron Volt (eV) is a widespread unit of energy in atomic and sub-atomic physics. It is related to the SI unit of energy, the Joule, by: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. The energy of a photon is related to its frequency ν by $E = h\nu$, where $h = 6.6 \times 10^{-34} \text{ Js}$. What is the frequency of a photon with energy 2.2 eV ?

The energy of the photon in SI units is $E = 2.2 \times 1.6 \times 10^{-19} = 3.52 \times 10^{-19} \text{ J}$. It follows that the photon's frequency is

$$\begin{aligned}\nu &= \frac{E}{h} \\ &= \frac{3.52 \times 10^{-19} \text{ J}}{6.6 \times 10^{-34} \text{ Js}} \\ &= \frac{3.52}{6.6} \times 10^{-19+34} \text{ s}^{-1} \\ \therefore \nu &= 5.3 \times 10^{14} \text{ s}^{-1}\end{aligned}$$

EXERCISE 3. Put the quantities below into SI units to perform the requested calculation (click on the **green** letters for the solutions).

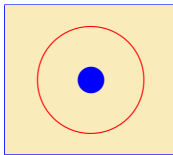
- (a) Ohm's law $V = IR$ links the voltage V across a resistance R to the current I flowing through it. Calculate the **voltage** across a $4.3\ \Omega$ (Ohm) resistor if a 4 mA (milli-Amp) current is measured.
- (b) The escape speed v_{esc} of a projectile from a planet is given by $v_{\text{esc}} = \sqrt{2gr}$ where r is the radius of the planet and g is the acceleration due to gravity on the surface of the planet. The Earth's radius is 6380 km and $g = 9.8\ \text{m s}^{-2}$. Calculate the **escape speed from the Earth**.
- (c) Air pressure P at sea level is roughly 100 kPa (kilo-Pascal). The height H of a mercury column in a barometer is related to the pressure by $P = H\rho g$ where ρ is the density of Mercury (14 Tonnes per cubic metre) and g is the acceleration due to gravity. Find the **height of the column of Mercury**.

Here are two further quizzes to practise on:

Quiz The energy E required to melt a mass m of a substance is given by $E = m\ell$ where ℓ is the specific latent heat of fusion (in J kg^{-1}). If 5 MJ is required to melt $2 \times 10^4 \text{g}$ of a solid, what is ℓ ?

- (a) 0.2J kg^{-1} (b) $250,000 \text{J kg}^{-1}$ (c) 2.5J kg^{-1} (d) 20J kg^{-1}

Quiz The area of a circle is πr^2 where r is its radius. Calculate the area of the region between the two circles below if the larger one has an area of $1.69\pi \text{ m}^2$ and the smaller one has a radius of 50 cm.



- (a) $1.44\pi \text{ m}^2$ (b) $0.194\pi \text{ m}^2$ (c) $2.5\pi \text{ m}^2$ (d) $1.44 \times 10^{-3}\pi \text{ m}^2$

4. Final Quiz

Begin Quiz Choose the solutions from the options given.

- The sunspot cycle takes place over 11 years. Roughly how many solar cycles take place **per millennium**?
(a) 110 (b) 11 (c) 91 (d) 181
- What is a cubic decimetre in terms of **cubic centimetres**?
(a) $1 \times 10^{-3} \text{ cm}^3$ (b) $1 \times 10^4 \text{ cm}^3$
(c) $1 \times 10^6 \text{ cm}^3$ (d) $1 \times 10^3 \text{ cm}^3$
- Express the area of a square of side $5 \mu\text{m}$ in **square metres**.
(a) $25 \mu\text{m}^2$ (b) $2.5 \times 10^{-12} \text{ m}^2$
(c) $2.5 \times 10^{-11} \text{ m}^2$ (d) $2.5 \times 10^{-13} \text{ m}^2$
- Find the current in **Amperes** through a 10Ω resistor with a 2 mV potential difference across it.
(a) $2 \times 10^{-3} \text{ A}$ (b) 500 A (c) 0.2 A (d) $2 \times 10^{-4} \text{ A}$

End Quiz

Solutions to Exercises

Exercise 1(a) To calculate how many seconds there are in a year, recall that there are 365 days each of which lasts 24 hours. In each hour there are 60 minutes each of 60 seconds duration. Thus we get

$$\begin{aligned}\text{No. of seconds in a year} &= 365 \times 24 \times 60 \times 60 \\ &= 31,536,000 \text{ s}\end{aligned}$$

There are 3.1536×10^6 seconds in a year. (Just over 30 million seconds.)

Click on the **green** square to return



Exercise 1(b) The distance **0.016 miles** is

$$\begin{aligned}0.016 \text{ miles} &= 0.016 \times \frac{8}{5} \text{ km} \\ &= 0.0256 \text{ miles}\end{aligned}$$

Click on the **green** square to return



Exercise 1(c) A speed of 10 miles per hour is

$$\begin{aligned} 10 \text{ miles per hour} &= \frac{10 \text{ miles}}{1 \text{ hour}} \\ &= \frac{10 \times \frac{8}{5} \text{ kilometres}}{1 \text{ hour}} \\ &= 16 \text{ kilometres per hour} \end{aligned}$$

Click on the **green** square to return



Exercise 1(d) One million pounds in pennies is given by

$$10^6 \times 100 = 10^8 \text{ pennies}$$

which is just one hundred million pennies.

Click on the green square to return



Exercise 2(a) To calculate the density $3 \times 10^2 \text{ kg m}^{-3}$ in **grams per cubic decimetre**. We need to use the **conversion factors**:

$$1 \text{ kg} = 10^3 \text{ g}$$

$$1 \text{ m} = 10 \text{ dm}$$

So we have

$$\begin{aligned} 3 \times 10^2 \text{ kg m}^{-3} &= 3 \times 10^2 \times \frac{10^3}{(10)^3} \\ &= 3 \times 10^2 \text{ g dm}^{-3} \end{aligned}$$

Click on the **green** square to return



Exercise 2(b) We need to calculate the **area of the triangle** with base 70 cm and height 400 mm. In **SI units** these lengths are 0.7 m and 0.4 m respectively.

The formula for the area of a triangle is

$$\text{area} = \frac{1}{2} \text{base} \times \text{height}$$

So the **area** is $\frac{1}{2} \times 0.7 \times 0.4 = \mathbf{0.14 \text{ m}^2}$.

Click on the **green** square to return



Exercise 2(c) We want to find the power in **Watts** of a device that uses 60 mJ (milli Joules) of energy in half a microsecond. In **SI units**

$$60 \text{ mJ} = 60 \times 10^{-3} \text{ J} = 6 \times 10^{-2} \text{ J}$$

$$0.5 \mu\text{s} = 0.5 \times 10^{-6} \text{ s} = 5 \times 10^{-7} \text{ s}$$

We recall that

$$\text{Power} = \frac{\text{energy}}{\text{time}}$$

so the power is

$$\begin{aligned} \text{Power} &= \frac{6 \times 10^{-2} \text{ (J)}}{5 \times 10^{-7} \text{ (s)}} \\ &= 1.2 \times 10^{-2+7} \text{ J s}^{-1} \\ &= 1.2 \times 10^5 \text{ W} \end{aligned}$$

where we recall that a **Watt** is a **Joule per second**.

Click on the **green** square to return



Exercise 2(d) To express a kilowatt hour in Joules, note that a kilowatt hour is the energy used in one hour by a device with a power consumption. In SI units an hour is 3,600 s and a kilowatt is 1,000 W. So we have

$$\text{energy} = 1,000 \times 3,600 = 3.6 \times 10^6 \text{ J}$$

where we again use that $1 \text{ J} = 1 \text{ W s}$.

Click on the green square to return



Exercise 3(a) We want to find the **voltage** across a $4.3\ \Omega$ resistor if a $4\ \text{mA}$ (milli-Amp) current is measured. In **SI units** the current is $I = 4 \times 10^{-3}\ \text{A}$ and the resistance is $R = 4.3\ \Omega$. From Ohm's law, $V = IR$, so

$$V = 4 \times 10^{-3}\ (\text{A}) \times 4.3\ \Omega = 1.72 \times 10^{-2}\ \text{V}$$

where we use that **$1\ \text{A}\ \Omega = 1\ \text{V}$** .

Click on the **green** square to return



Exercise 3(b) We need to find the **escape speed** of a projectile from the Earth: $v_{\text{esc}} = \sqrt{2gr}$.

In **SI units** $g = 9.8 \text{ m s}^{-2}$ and $r = 6380 \times 1000 = 6.38 \times 10^6 \text{ m}$. Substituting these values into the equation gives:

$$\begin{aligned}v_{\text{esc}} &= \sqrt{2 \times 9.8 \text{ m s}^{-2} \times 6.38 \times 10^6 \text{ m}} \\ &= \sqrt{1.25 \times 10^8 \text{ m}^2 \text{ s}^{-2}} \\ &= 1.1 \times 10^4 \text{ m s}^{-1}\end{aligned}$$

Click on the **green** square to return



Exercise 3(c) We wish to calculate the height of a column of Mercury in a barometer if the air pressure, P , is 100 kPa. We use

$$P = H\rho g \quad \text{or} \quad H = \frac{P}{\rho g}$$

where $g = 9.8 \text{ m s}^{-2}$ and $\rho = 14 \text{ Tonnes per cubic metre}$. In SI units $\rho = 14 \times 1000 = 1.4 \times 10^4 \text{ kg m}^{-3}$ and $P = 100 \times 1000 = 10^5 \text{ Pa}$. This gives

$$\begin{aligned} H &= \frac{10^5 \text{ Pa}}{1.4 \times 10^4 \text{ kg m}^{-3} \times 9.8 \text{ m s}^{-2}} \\ &= \frac{1}{1.4 \times 9.8} \times 10^{5-4} \times \text{N m}^{-2} \text{kg}^{-1} \text{m}^3 \text{m}^{-1} \text{s}^2 \\ &= 0.73 \text{ N kg}^{-1} \text{s}^2 \end{aligned}$$

Now $1 \text{ N} = 1 \text{ kg ms}^{-2}$, so therefore the height is $H = 0.73 \text{ m}$.

Click on the green square to return



Solutions to Quizzes

Solution to Quiz: We want to express an area of 1 cm^2 in square metres.

One metre is 100 cm, so $1 \text{ cm} = 10^{-2} \text{ m}$. Thus the **conversion factor is 10^{-2}** .

$$\begin{aligned}\therefore 1 \text{ cm}^2 &= (10^{-2})^2 \text{ m}^2 \\ &= 1 \times 10^{-4} \text{ m}^2\end{aligned}$$

A square centimetre is a ten thousandth part of a square metre.

End Quiz

Solution to Quiz: We want to express an area of 1 mile^2 in square kilometres.

One mile is $\frac{8}{5}$ km, so the **conversion factor is $\frac{8}{5}$** . We thus obtain

$$\begin{aligned} 1 \text{ mile}^2 &= \left(\frac{8}{5}\right)^2 \text{ km}^2 \\ &= \frac{64}{25} \text{ km}^2 \end{aligned}$$

End Quiz

Solution to Quiz: We want to express a volume of 3 cm^3 in cubic millimetres.

One centimetre is 10 mm, so the conversion factor is 10. We thus obtain

$$\begin{aligned} 3 \text{ cm}^3 &= 3 \times (10)^3 \text{ mm}^3 \\ &= 3,000 \text{ mm}^3 \end{aligned}$$

End Quiz

Solution to Quiz: We want to express a volume of 0.5 l in cubic metres. A litre is 1 dm^3 and $1 \text{ dm} = 0.1 \text{ m}$, i.e., the conversion factor is 0.1. Thus

$$\begin{aligned} 0.5 \text{ l} &= 0.5 \text{ dm}^3 \\ &= 0.5 \times (0.1)^3 \text{ m}^3 \\ &= 5 \times 10^{-1} \times 10^{-3} \text{ m}^3 \\ &= 5 \times 10^{-4} \text{ m}^3 \end{aligned}$$

End Quiz

Solution to Quiz: We have to express a density of $3 \times 10^2 \text{ kg m}^{-3}$ in terms of kilograms per cubic centimetre.

A metre is 100 cm, so the **conversion factor is 100**. One cubic metre is thus $(100)^3 = 10^6 \text{ cm}^3$. Therefore

$$\begin{aligned} 3 \times 10^2 \text{ kg m}^{-3} &= \frac{3 \times 10^2 \text{ kg}}{1 \text{ m}^3} \\ &= \frac{3 \times 10^2 \text{ kg}}{1 \times 10^6 \text{ cm}^3} \\ &= 3 \times 10^2 \times 10^{-6} \text{ kg cm}^{-3} \\ &= 3 \times 10^{-4} \text{ kg cm}^{-3} \end{aligned}$$

End Quiz

Solution to Quiz: We are given $E = m\ell$ and need to calculate ℓ given that $E = 5 \text{ MJ}$ and $m = 2 \times 10^4 \text{ g}$.

Rearranging the equation we obtain: $\ell = E/m$. We also need to express E and m in SI units:

$$E = 5 \times 10^6 \text{ J}$$

$$m = 2 \times 10^4 \times 10^{-3} \text{ kg} = 20 \text{ kg}$$

Thus we have

$$\ell = \frac{E}{m} = \frac{5 \times 10^6 \text{ (J)}}{20 \text{ (kg)}} = 2.5 \times 10^5 \text{ J kg}^{-1}$$

End Quiz

Solution to Quiz: The area of the disk between the two circles is given by the difference of their areas. The larger has an area of $1.69\pi \text{ m}^2$ while the smaller has a radius of 50 cm. In SI units the radius is 0.5 m. Such a circle has area $\pi r^2 = \pi(0.5)^2 \text{ m}^2 = 0.25\pi \text{ m}^2$. Thus the difference is

$$\text{area of difference} = 1.69\pi - 0.25\pi = 1.44\pi \text{ m}^2$$

End Quiz