

# Introduction to Vectors

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic understanding of vectors.

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The full range of these packages and some instructions, should they be required, can be obtained from our web page [Mathematics Support Materials](#).

# 1. Vectors (Introduction)

A vector is a combination of three things:

- a positive number called its *magnitude*,
- a *direction* in space,
- a *sense* making more precise the idea of direction.

Typically a vector is illustrated as a directed straight line.



**Diagram 1**

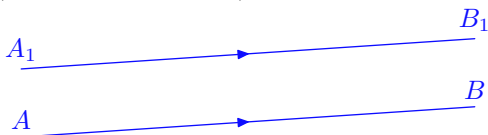
The vector in the above diagram would be written as  $\vec{AB}$  with:

- the direction of the arrow, from the point  $A$  to the point  $B$ , indicating the *sense* of the vector,
- the *magnitude* of  $\vec{AB}$  given by the length of  $AB$ .

The *magnitude* of  $\vec{AB}$  is written  $|\vec{AB}|$ .

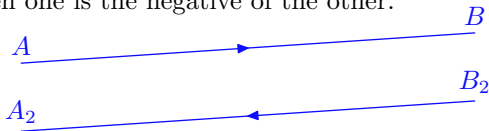
There are very many physical quantities which are best described as vectors; velocity, acceleration and force are all *vector* quantities.

Two vectors are *equal* if they have the same *magnitude*, the same *direction* (i.e. they are *parallel*) and the same *sense*.



**Diagram 2**

In **diagram 2** the vectors  $\vec{AB}$  and  $\vec{A_1B_1}$  are equal, i.e.  $\vec{AB} = \vec{A_1B_1}$ . If two vectors have the same length, are parallel but have *opposite* senses then one is the negative of the other.

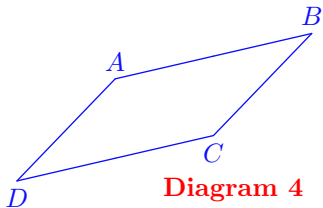


**Diagram 3**

In **diagram 3** the vectors  $\vec{AB}$  and  $\vec{B_2A_2}$  are of equal length, are parallel but are *opposite* in sense, so  $\vec{AB} = -\vec{B_2A_2}$ .

## Quiz

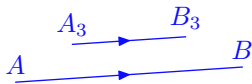
**Diagram 4** shows a parallelogram. Which of the following equations is the correct one?

**Diagram 4**

- (a)  $\vec{DA} = \vec{BC}$ ,    (b)  $\vec{AD} = -\vec{CB}$ ,    (c)  $\vec{AD} = \vec{CB}$ ,    (d)  $\vec{DA} = -\vec{CB}$ .

If two vectors are parallel, have the same sense but different magnitudes then one vector is a *scalar* (i.e. numeric) multiple of the other.

In **diagram 5** the vector  $\vec{AB}$  is parallel to  $\vec{A_3B_3}$ , has the same sense but is twice as long, so  $\vec{AB} = 2 \vec{A_3B_3}$ .

**Diagram 5**

In general *multiplying a vector by a positive number  $\lambda$  gives a vector parallel to the original vector, with the same sense but with magnitude  $\lambda$  times that of the original.* If  $\lambda$  is *negative* then the sense is reversed.

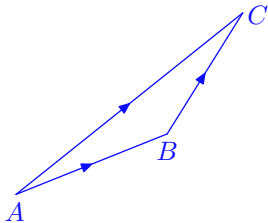
Thus from **diagram 5** for example,  $\vec{A_3B_3} = -\frac{1}{2} \vec{BA}$ .

## 2. Addition of Vectors

In **diagram 6** the three vectors given by  $\vec{AB}$ ,  $\vec{BC}$ , and  $\vec{AC}$ , make up the sides of a triangle as shown. Referring to this diagram, the law of addition, which is usually known as the *triangle law of addition*, is

$$\vec{AB} + \vec{BC} = \vec{AC} .$$

The vector  $\vec{AC}$  is called the *resultant vector*.



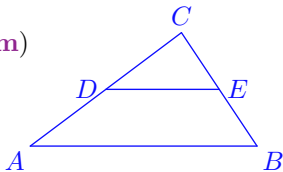
**Diagram 6**

Physical quantities which can be described as vectors satisfy such a law. One such example is the action of forces. If two forces are represented by the vectors  $\vec{AB}$  and  $\vec{BC}$  then the effect of applying *both of these forces together* is equivalent to a single force, the *resultant force*, represented by the vector  $\vec{AC}$ .

One further vector is required, the *zero vector*. This has *no direction* and *zero magnitude*. It will be written as  $\mathbf{0}$ .

**Example 1 (The mid-points theorem)**

Let  $ABC$  be a triangle and let  $D$  be the midpoint of  $AC$  and  $E$  be the midpoint of  $BC$ . Prove that  $DE$  is parallel to  $AB$  and half its length i.e.  $|AB| = 2|DE|$ .

**Diagram 7****Proof**

Since  $D$  is the midpoint of  $\vec{AC}$ , it follows that  $\vec{AC} = 2 \vec{DC}$ . Similarly  $\vec{CB} = 2 \vec{CE}$ . Then

$$\begin{aligned}\vec{AC} + \vec{CB} &= 2 \vec{DC} + 2 \vec{CE} \\ &= 2(\vec{DC} + \vec{CE}).\end{aligned}$$

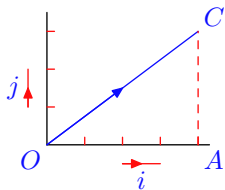
Now  $\vec{AC} + \vec{CB} = \vec{AB}$  and  $\vec{DC} + \vec{CE} = \vec{DE}$ .

Substituting these into the equation above gives  $\vec{AB} = 2 \vec{DE}$ .

Since these are vectors,  $AB$  must be parallel to  $DE$  and the length of  $AB$  is twice that of  $DE$ , i.e.  $|\vec{AB}| = 2|\vec{DE}|$ .

### 3. Component Form of Vectors

The diagram shows a vector  $\vec{OC}$  at an angle to the  $x$  axis. Take  $\mathbf{i}$  to be a vector of length 1 (called a *unit vector*) parallel to the  $x$  axis and in the positive direction, and  $\mathbf{j}$  to be a vector of length 1 (another *unit vector*) parallel to the  $y$  axis and in the positive direction.



**Diagram 8**

From **diagram 8**,  $\vec{OC} = \vec{OA} + \vec{AC}$ . The vector  $\vec{OA}$  is parallel to the vector  $\mathbf{i}$  and four times its length so  $\vec{OA} = 4\mathbf{i}$ . Similarly  $\vec{AC} = 3\mathbf{j}$ . Thus the vector  $\vec{OC}$  may be written as

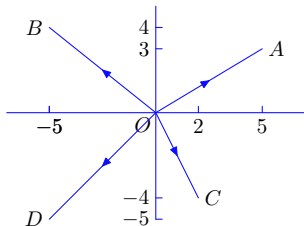
$$\vec{OC} = 4\mathbf{i} + 3\mathbf{j}.$$

This is known as the *2-dimensional component form* of the vector. In general every vector can be written in component form. This package will consider only 2-dimensional vectors.



**EXERCISE 1.** From **diagram 9**, write down the component form of the following vectors: (Click on the **green** letters for solutions.)

- (a)  $\vec{OA}$ ,            (b)  $\vec{OB}$ ,  
(c)  $\vec{OC}$ ,            (d)  $\vec{OD}$ ,



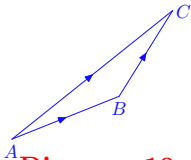
**Diagram 9**

In this package, the following properties of vectors are used.

- To add two or more vectors in component form, add the corresponding components.
- To multiply a vector in component form by a scalar, multiply each of the components by the scalar.
- If a vector in component form is  $a\mathbf{i} + b\mathbf{j}$  then its magnitude is  $\sqrt{a^2 + b^2}$ . (*Pythagoras' theorem*)

**Example 3**

If  $\vec{AB} = 2\mathbf{i} + 2\mathbf{j}$  and  $\vec{BC} = \mathbf{i} + 2\mathbf{j}$ , prove that the magnitude of  $\vec{AC}$  is 5.

**Diagram 10****Proof**

The three vectors form three sides of a triangle (see **diagram 10** which is NOT to scale) so

$$\vec{AC} = \vec{AB} + \vec{BC} = (2\mathbf{i} + 2\mathbf{j}) + (\mathbf{i} + 2\mathbf{j})$$

Thus  $|\vec{AC}| = \sqrt{3^2 + 4^2} = 5$ .

**NB** Vectors are often printed as boldface lower case letters such as **a**.

**EXERCISE 2.** If  $\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$ ,  $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$  and  $\mathbf{c} = \mathbf{i} - 2\mathbf{j}$ , calculate:

- |                                   |                                     |  |
|-----------------------------------|-------------------------------------|--|
| (a) $\mathbf{a} + \mathbf{b}$ ,   | (b) $\mathbf{b} + \mathbf{c}$ ,     | (c) $\mathbf{a} + \mathbf{b} + \mathbf{c}$ , |
| (d) $\mathbf{a} + 2\mathbf{b}$ ,  | (e) $2\mathbf{b} - 3\mathbf{a}$ ,   | (f) $ \mathbf{a} $ ,                         |
| (g) $ \mathbf{a} + \mathbf{b} $ , | (h) $ \mathbf{a}  +  \mathbf{b} $ , | (i) $ 2\mathbf{a} - \mathbf{b} $ ,           |

**Example 4** Two vectors are  $\vec{AB} = \mathbf{i} + \mathbf{j}$  and  $\vec{CD} = 2\mathbf{i} + 3\mathbf{j}$ . Find

- (a) The value of  $\lambda$  such that  $\lambda \vec{AB} + \vec{CD}$  is parallel to  $\mathbf{i}$ ,  
 (b) The value of  $\lambda$  such that  $\lambda \vec{AB} + \vec{CD}$  is parallel to  $4\mathbf{i} + 3\mathbf{j}$ .

**Solution** First find  $\lambda \vec{AB} + \vec{CD}$  in component form.

$$\begin{aligned} \lambda \vec{AB} + \vec{CD} &= \lambda(\mathbf{i} + \mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) \\ &= (\lambda\mathbf{i} + \lambda\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) \\ &= (\lambda + 2)\mathbf{i} + (\lambda + 3)\mathbf{j}. \end{aligned}$$

(a) If  $\lambda \vec{AB} + \vec{CD}$  is parallel to  $\mathbf{i}$  then the  $\mathbf{j}$  component must be zero, i.e.  $\lambda + 3 = 0$ . Thus  $\lambda = -3$  and we have  $-3 \vec{AB} + \vec{CD} = -\mathbf{i}$ .

(b) If  $\lambda \vec{AB} + \vec{CD}$  is parallel to  $4\mathbf{i} + 3\mathbf{j}$  then there is a number  $\kappa$  such that

$$\begin{aligned} (\lambda + 2)\mathbf{i} + (\lambda + 3)\mathbf{j} &= \kappa(4\mathbf{i} + 3\mathbf{j}) \\ \therefore (\lambda + 2)\mathbf{i} + (\lambda + 3)\mathbf{j} &= 4\kappa\mathbf{i} + 3\kappa\mathbf{j} \\ \text{so } \lambda + 2 = 4\kappa \quad \text{and} \quad \lambda + 3 = 3\kappa. \end{aligned}$$

$$\begin{aligned}\text{Then} \quad \frac{\lambda + 2}{\lambda + 3} &= \frac{4\kappa}{3\kappa} = \frac{4}{3} \\ \therefore 3(\lambda + 2) &= 4(\lambda + 3) \\ 3\lambda + 6 &= 4\lambda + 12 \\ 6 - 12 &= 4\lambda - 3\lambda \\ \text{i.e.} \quad \lambda &= -6,\end{aligned}$$

and the vector is  $-6(\mathbf{i} + \mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = -4\mathbf{i} - 3\mathbf{j} = -(4\mathbf{i} + 3\mathbf{j})$ .

**Quiz** If  $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ ,  $\mathbf{b} = -3\mathbf{i} + 2\mathbf{j}$  and  $\mathbf{c} = 2\mathbf{i} - \mathbf{j}$ , which of the following vectors is parallel to the resultant of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ , i.e.  $\mathbf{a} + \mathbf{b} + \mathbf{c}$ ?

- (a)  $-2\mathbf{i} - 6\mathbf{j}$ ,      (b)  $2\mathbf{i} - 6\mathbf{j}$ ,      (c)  $2\mathbf{i} + 8\mathbf{j}$ ,      (d)  $2\mathbf{i} - 8\mathbf{j}$ .

**Quiz** If  $\mathbf{a} = \mathbf{i} + \mathbf{j}$  and  $\mathbf{b} = \mathbf{i} - \mathbf{j}$ , for which of the following values of  $\lambda$  is the vector  $\lambda\mathbf{a} + \mathbf{b}$  parallel to  $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$ ?

- (a)  $\lambda = \frac{1}{5}$ ,      (b)  $\lambda = -\frac{1}{5}$ ,      (c)  $\lambda = 5$ ,      (d)  $\lambda = -5$ .

## 4. Quiz on Vectors

Choose the correct option for each of the following.

### Begin Quiz

1. If  $\mathbf{a} = -2\mathbf{i} + 4\mathbf{j}$ ,  $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j}$ ,  $\mathbf{c} = 4\mathbf{i} + 5\mathbf{j}$  then  $\mathbf{a} + \mathbf{b} + \mathbf{c}$  is  
(a)  $-5\mathbf{i} - 7\mathbf{j}$ , (b)  $5\mathbf{i} - 7\mathbf{j}$ , (c)  $-5\mathbf{i} + 7\mathbf{j}$ , (d)  $5\mathbf{i} + 7\mathbf{j}$ .
2. If  $\mathbf{u} = -2\mathbf{i} + 4\mathbf{j}$ ,  $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$ ,  $\mathbf{w} = 4\mathbf{i} + 6\mathbf{j}$  then  $|\mathbf{u} + \mathbf{v} + \mathbf{w}|$  is  
(a) 5, (b) 13, (c) 4, (d) 15.
3. If  $\mathbf{u} = -\mathbf{i} + 3\mathbf{j}$  and  $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$ , then  $\lambda\mathbf{u} + \mathbf{v}$  is parallel to  $\mathbf{w} = -\mathbf{i} + 4\mathbf{j}$  if  $\lambda$  is  
(a)  $-6$ , (b)  $6$ , (c)  $-5$ , (d)  $5$ .

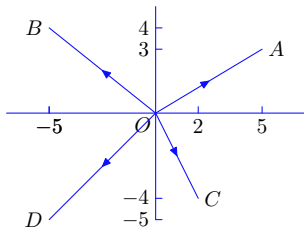
### End Quiz

## Solutions to Exercises

### Exercise 1(a)

For the vector  $\vec{OA}$  shown on the diagram the component in the direction given by the unit vector  $\mathbf{i}$  is 5 and the component in the direction  $\mathbf{j}$  is 3. Therefore the 2-dimensional vector  $\vec{OA}$  is, in component form, written as

$$\vec{OA} = 5\mathbf{i} + 3\mathbf{j}.$$



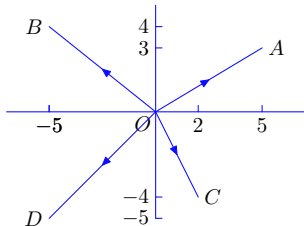
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**Exercise 1(b)**

The vector  $\vec{OB}$  shown on the diagram has the component  $-5$  in the  $\mathbf{i}$  direction while the component in the  $\mathbf{j}$  direction is  $4$ . Thus the 2-dimensional vector  $\vec{OB}$  in component form is written as

$$\vec{OB} = -5\mathbf{i} + 4\mathbf{j}.$$



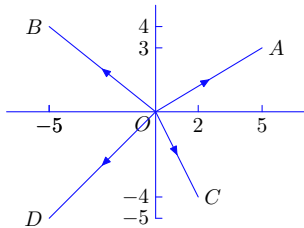
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**Exercise 1(c)**

For the vector  $\vec{OC}$  shown on the diagram the component in the direction given by the unit vector  $\mathbf{i}$  is  $2$  while the component in the direction given by  $\mathbf{j}$  is  $-4$ . Therefore the component form of the 2-dimensional vector  $\vec{OC}$  is

$$\vec{OC} = 2\mathbf{i} - 4\mathbf{j}.$$



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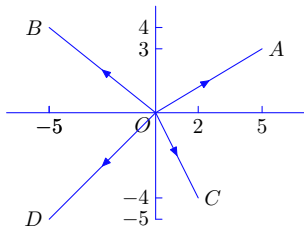




**Exercise 1(d)**

For the vector  $\vec{OD}$  shown on the diagram the component in the direction given by the unit vector  $\mathbf{i}$  is  $-5$  and the component in the direction given by  $\mathbf{j}$  is also  $-5$ . The component form of the 2-dimensional vector  $\vec{OD}$  is therefore

$$\vec{OD} = -5\mathbf{i} - 5\mathbf{j}.$$



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**Exercise 2(a)**

The sum of the two vectors

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \quad \text{and} \quad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$$

is found by summing up the corresponding components of each vector.  
Thus

$$\mathbf{a} + \mathbf{b} = (-\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = (-1 + 2)\mathbf{i} + (3 + 3)\mathbf{j} = \mathbf{i} + 6\mathbf{j}.$$

Click on the green square to return



**Exercise 2(b)**

The sum of the two vectors

$$\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} \quad \text{and} \quad \mathbf{c} = \mathbf{i} - 2\mathbf{j}$$

is found by adding the corresponding components of each vector. Thus

$$\mathbf{b} + \mathbf{c} = (2\mathbf{i} + 3\mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) = (2 + 1)\mathbf{i} + (3 - 2)\mathbf{j} = 3\mathbf{i} + \mathbf{j}.$$

Click on the green square to return



**Exercise 2(c)**

To find the sum of the three vectors

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j}, \quad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} \quad \text{and} \quad \mathbf{c} = \mathbf{i} - 2\mathbf{j},$$

add the corresponding components of each vector. The resulting vector is thus

$$\begin{aligned} \mathbf{a} + \mathbf{b} + \mathbf{c} &= (-\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \\ &= (-1 + 2 + 1)\mathbf{i} + (3 + 3 - 2)\mathbf{j} = 2\mathbf{i} + 4\mathbf{j}. \end{aligned}$$

Click on the green square to return



**Exercise 2(d)**

To find the sum  $\mathbf{a} + 2\mathbf{b}$  with

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \quad \text{and} \quad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j},$$

first find the vector  $2\mathbf{b}$ :

$$2\mathbf{b} = 2 \times (2\mathbf{i} + 3\mathbf{j}) = 4\mathbf{i} + 6\mathbf{j}.$$

The vector  $\mathbf{a} + 2\mathbf{b}$  is now found by adding the corresponding components of each vector. The resulting vector is thus

$$\begin{aligned} \mathbf{a} + 2\mathbf{b} &= (-\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} + 6\mathbf{j}) \\ &= (-1 + 4)\mathbf{i} + (3 + 6)\mathbf{j} = 3\mathbf{i} + 9\mathbf{j}. \end{aligned}$$

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**Exercise 2(e)**

To find the vector  $2\mathbf{b} - 3\mathbf{a}$  with

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \quad \text{and} \quad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j},$$

first find the vectors  $2\mathbf{b}$  and  $3\mathbf{a}$ :

$$2\mathbf{b} = 2 \times (2\mathbf{i} + 3\mathbf{j}) = 4\mathbf{i} + 6\mathbf{j},$$

$$3\mathbf{a} = 3 \times (-\mathbf{i} + 3\mathbf{j}) = -3\mathbf{i} + 9\mathbf{j},$$

The vector  $2\mathbf{b} - 3\mathbf{a}$  is now easily found by subtracting the components of these vectors:

$$\begin{aligned} 2\mathbf{b} - 3\mathbf{a} &= (4\mathbf{i} + 6\mathbf{j}) - (-3\mathbf{i} + 9\mathbf{j}) \\ &= (4 + 3)\mathbf{i} + (6 - 9)\mathbf{j} = 7\mathbf{i} - 3\mathbf{j}. \end{aligned}$$

Click on the green square to return



**Exercise 2(f)**

The magnitude of the vector

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$$

is given by

$$|\mathbf{a}| = \sqrt{(-1)^2 + 3^2} = \sqrt{1 + 9} = \sqrt{10}.$$

Click on the green square to return



**Exercise 2(g)**

To find the magnitude of the vector  $\mathbf{a} + \mathbf{b}$ , first find the sum of the two vectors

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \quad \text{and} \quad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j}.$$

The resulting vector is

$$\mathbf{a} + \mathbf{b} = (-\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = (-1 + 2)\mathbf{i} + (3 + 3)\mathbf{j} = \mathbf{i} + 6\mathbf{j}.$$

The magnitude of this vector is given by

$$|\mathbf{a} + \mathbf{b}| = \sqrt{1^2 + 6^2} = \sqrt{37}.$$

Click on the green square to return





**Exercise 2(h)**

To find  $|\mathbf{a}| + |\mathbf{b}|$ , first find the magnitude of each of the vectors  $\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$  and  $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$ .

The magnitude of the vector  $\mathbf{a}$  is

$$|\mathbf{a}| = \sqrt{(-1)^2 + 3^2} = \sqrt{10}.$$

The magnitude of the vector  $\mathbf{b}$  is

$$|\mathbf{b}| = \sqrt{2^2 + 3^2} = \sqrt{13}.$$

Therefore

$$|\mathbf{a}| + |\mathbf{b}| = \sqrt{10} + \sqrt{13}.$$

Click on the green square to return



**Exercise 2(i)**

To find  $|2\mathbf{a} - \mathbf{b}|$ , first find  $2\mathbf{a} - \mathbf{b}$ . The vector  $\mathbf{a}$  in component form is given as

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$$

so the component form of the vector  $2\mathbf{a}$  is

$$2\mathbf{a} = 2 \times (-1)\mathbf{i} + 2 \times 3\mathbf{j} = -2\mathbf{i} + 6\mathbf{j}.$$

The difference between  $2\mathbf{a}$  and  $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$  is the vector

$$2\mathbf{a} - \mathbf{b} = (-2\mathbf{i} + 6\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) = (-2 - 2)\mathbf{i} + (6 - 3)\mathbf{j} = -4\mathbf{i} + 3\mathbf{j}.$$

The magnitude of the resulting vector  $2\mathbf{a} - \mathbf{b}$  is therefore

$$|2\mathbf{a} - \mathbf{b}| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5.$$

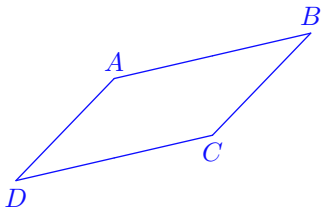
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## Solutions to Quizzes

### Solution to Quiz:

According to the diagram shown opposite the magnitudes of the vectors  $\vec{AD}$  and  $\vec{CB}$  are equal, but the direction of the vector  $\vec{AD}$  is from the point  $A$  to the point  $D$ , while the direction of the vector  $\vec{CB}$  is opposite, from the point  $B$  to the point  $C$ . Therefore  $\vec{AD} = -\vec{CB}$ .



If checked, the other solutions will be found to be false.

End Quiz

**Solution to Quiz:**

In order to determine which of the vectors is parallel to the resultant of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ , the resultant must first be calculated.

The resultant of the three vectors

$$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}, \quad \mathbf{b} = -3\mathbf{i} + 2\mathbf{j} \quad \text{and} \quad \mathbf{c} = 2\mathbf{i} - \mathbf{j}.$$

is

$$\begin{aligned} \mathbf{a} + \mathbf{b} + \mathbf{c} &= (2\mathbf{i} + 3\mathbf{j}) + (-3\mathbf{i} + 2\mathbf{j}) + (2\mathbf{i} - \mathbf{j}) \\ &= (2 - 3 + 2)\mathbf{i} + (3 + 2 - 1)\mathbf{j} = \mathbf{i} + 4\mathbf{j}. \end{aligned}$$

Next note that the vector  $2\mathbf{i} + 8\mathbf{j}$  given in the answer **(c)** can be written as

$$2\mathbf{i} + 8\mathbf{j} = 2 \times (\mathbf{i} + 4\mathbf{j}) = 2(\mathbf{a} + \mathbf{b} + \mathbf{c}),$$

so the resultant is parallel to the vector  $2\mathbf{i} + 8\mathbf{j}$ .

End Quiz

**Solution to Quiz:** To find the value of  $\lambda$  for which  $\lambda\mathbf{a} + \mathbf{b}$  parallel to  $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$ , first calculate the former. If  $\mathbf{a} = \mathbf{i} + \mathbf{j}$  and  $\mathbf{b} = \mathbf{i} - \mathbf{j}$  then

$$\lambda\mathbf{a} + \mathbf{b} = \lambda(\mathbf{i} + \mathbf{j}) + (\mathbf{i} - \mathbf{j}) = (\lambda + 1)\mathbf{i} + (\lambda - 1)\mathbf{j}.$$

If this vector is parallel to the vector  $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$  then there is a number  $k$  such that

$$(\lambda + 1)\mathbf{i} + (\lambda - 1)\mathbf{j} = k(2\mathbf{i} - 3\mathbf{j}).$$

This holds when  $\lambda + 1 = 2k$  and  $\lambda - 1 = -3k$ .

Multiply the first equation by 3

$$3\lambda + 3 = 6k,$$

and the second one by 2

$$2\lambda - 2 = -6k.$$

Now add the left and right sides of these equations to obtain:

$$5\lambda + 1 = 0, \quad \text{thus} \quad \lambda = -\frac{1}{5}.$$

End Quiz