Basic Mathematics

Indefinite Integration

R Horan & M Lavelle

The aim of this package is to provide a short self assessment programme for students who want to be able to calculate basic indefinite integrals.

Last Revision Date: June 7, 2004 Version 1.0

Table of Contents

The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Anti-Derivatives

If $f = \frac{dF}{dx}$, we call F the anti-derivative (or indefinite integral) of f.

Example 1 If $f(x) = x$, we can find its anti-derivative by realising that for $F(x) = \frac{1}{2}x^2$

$$\frac{dF}{dx} = \frac{d}{dx}(\frac{1}{2}x^2) = \frac{1}{2} \times 2x = x = f(x)$$

Thus $F(x) = \frac{1}{2}x^2$ is an anti-derivative of $f(x) = x$.

However, if C is a constant:

$$\frac{d}{dx}(\frac{1}{2}x^2 + C) = \frac{1}{2} \times 2x = x$$

since the derivative of a constant is zero. The **general anti-derivative** of x is thus $\frac{1}{2}x^2 + C$ where C can be *any* constant.

Note that you should always check an anti-derivative F by differentiating it and seeing that you recover f.

Quiz Using $\frac{d(x^n)}{dx} = nx^{n-1}$, select an anti-derivative of x^6

(a) $6x^5$ (b) $\frac{1}{5}x^5$ (c) $\frac{1}{7}x^7$ (d) $\frac{1}{6}x^7$

In general the anti-derivative or integral of x^n is:

$$\text{If } f(x) = x^n\text{, then } F(x) = \frac{1}{n+1}x^{n+1} \quad \text{for } n \neq -1$$

N.B. this rule does not apply to $1/x = x^{-1}$. Since the derivative of $\ln(x)$ is $1/x$, the anti-derivative of $1/x$ is $\ln(x)$ – see later.

Also **note** that since $1 = x^0$, the rule says that the anti-derivative of 1 is x. This is correct since the derivative of x is 1.

We will now introduce two important properties of integrals, which follow from the corresponding rules for derivatives.

If a is any constant and $F(x)$ is the anti-derivative of $f(x)$, then

$$\frac{d}{dx}\left(aF(x)\right) = a\frac{d}{dx}F(x) = af(x)\,.$$

Thus

$$aF(x) \text{ is the anti-derivative of } af(x)$$

Quiz Use this property to select the general anti-derivative of $3x^{\frac{1}{2}}$ from the choices below.

(a) $2x^{\frac{3}{2}} + C$ (b) $\frac{3}{2}x^{-\frac{1}{2}} + C$ (c) $\frac{9}{2}x^{\frac{3}{2}} + C$ (d) $6\sqrt{x} + C$

If $\dfrac{dF}{dx} = f(x)$ and $\dfrac{dG}{dx} = g(x)$, from the sum rule of differentiation

$$\frac{d}{dx}(F+G) = \frac{d}{dx}F + \frac{d}{dx}G = f(x) + g(x).$$

(See the package on the **product and quotient rules.**) This leads to the sum rule for integration:

> If $F(x)$ is the anti-derivative of $f(x)$ and $G(x)$ is the anti-derivative of $g(x)$, then $F(x) + G(x)$ is the anti-derivative of $f(x) + g(x)$.

Only one arbitrary constant C is needed in the anti-derivative of the sum of two (or more) functions.

Quiz Use this property to find the general anti-derivative of $3x^2 - 2x^3$.

(a) C (b) $x^3 - \frac{1}{2}x^4 + C$ (c) $\frac{3}{2}x^3 - \frac{2}{3}x^4 + C$ (d) $x^3 + \frac{2}{3}x + C$

We now introduce the integral notation to represent anti-derivatives.

2. Indefinite Integral Notation

The notation for an anti-derivative or indefinite integral is:

$$\text{if} \quad \frac{dF}{dx} = f(x)\,, \quad \text{then} \quad \int f(x)\,dx = F(x) + C$$

Here $\int$ is called the integral sign, while dx is called the measure and C is called the integration constant. We read this as "the integral of f of x with respect to x" or "the integral of f of x dx".

In other words $\int f(x)\,dx$ means the general anti-derivative of $f(x)$ *including* an integration constant.

Example 2 To calculate the integral $\int x^4\,dx$, we recall that the anti-derivative of x^n for $n \neq -1$ is $x^{n+1}/(n+1)$. Here $n = 4$, so we have

$$\int x^4\,dx = \frac{x^{4+1}}{4+1} + C = \frac{x^5}{5} + C$$

Quiz Select the correct result for the indefinite integral $\int \frac{1}{\sqrt{x}}\, dx$

(a) $-\frac{1}{2}x^{-\frac{3}{2}} + C$ (b) $2\sqrt{x} + C$ (c) $\frac{1}{2}x^{\frac{1}{2}} + C$ (d) $\frac{2}{\sqrt{x^2}} + C$

The previous rules for anti-derivatives may be expressed in integral notation as follows.

The integral of a function multiplied by any constant a is:

$$\int af(x)dx = a \int f(x)dx$$

The sum rule for integration states that:

$$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$$

To be able to integrate a greater number of functions, it is convenient first to recall the derivatives of some simple functions:

y	$\sin(ax)$	$\cos(ax)$	e^{ax}	$\ln(x)$
$\dfrac{dy}{dx}$	$a\cos(ax)$	$-a\sin(ax)$	$a\ e^{ax}$	$\dfrac{1}{x}$

Exercise 1. From the above table of derivatives calculate the indefinite integrals of the following functions: (click on the **green** letters for the solutions)

(a) $\sin(ax)$,

(b) $\cos(ax)$,

(c) e^{ax} ,

(d) $\dfrac{1}{x}$

These results give the following table of indefinite integrals (the integration constants are omitted for reasons of space):

$y(x)$	x^n $(n \neq -1)$	$\sin(ax)$	$\cos(ax)$	e^{ax}	$\frac{1}{x}$
$\int y(x) dx$	$\frac{1}{n+1} x^{n+1}$	$-\frac{1}{a}\cos(ax)$	$\frac{1}{a}\sin(ax)$	$\frac{1}{a}\, e^{ax}$	$\ln(x)$

Exercise 2. From the above table, calculate the following integrals: (click on the **green** letters for the solutions)

(a) $\int x^7\, dx\,,$ (b) $\int 2\sin(3x)\, dx\,,$

(c) $\int 4\cos(2x)\, dx\,,$ (d) $\int 15\, e^{-5s}\, ds\,,$

(e) $\int \frac{3}{w}\, dw\,,$ (f) $\int (\, e^s + e^{-s})\, ds\,.$

Quiz Select the indefinite integral of $4\sin(5x) + 5\cos(3x)$.

(a) $20\cos(5x) - 15\sin(3x) + C$ (b) $4\sin(\frac{5x^2}{2}) + 5\cos(\frac{3x^2}{2}) + C$

(c) $-\frac{2}{3}\cos(5x) + \frac{5}{4}\sin(3x) + C$ (d) $-\frac{4}{5}\cos(5x) + \frac{5}{3}\sin(3x) + C$

Exercise 3. It may be shown that

$$\frac{d}{dx}\left[x(\ln(x) - 1)\right] = \ln(x)\,.$$

(See the package on the **product and quotient rules** of differentiation.) From this result and the properties reviewed in the package on **logarithms** calculate the following integrals: (click on the **green** letters for the solutions)

(a) $\int \ln(x)\,dx\,,$ (b) $\int \ln(2x)\,dx\,,$

(c) $\int \ln(x^3)\,dx\,,$ (d) $\int \ln(3x^2)\,dx\,.$

Hint expressions like ln(2) are constants!

3. Fixing Integration Constants

Example 3 Consider a rocket whose velocity in metres per second at time t seconds after launch is $v = bt^2$ where $b = 3\,\mathrm{ms}^{-3}$. If at time $t = 2\,\mathrm{s}$ the rocket is at a position $x = 30\,\mathrm{m}$ away from the launch position, we can calculate its position at time $t\,\mathrm{s}$ as follows.

Velocity is the derivative of position with respect to time: $v = \dfrac{dx}{dt}$, so it follows that x is the integral of v $(= bt^2\,\mathrm{ms}^{-1})$:

$$x = \int 3t^2\,dt = 3 \times \frac{1}{3}t^3 + C = t^3 + C$$

The information that $x = 30\,\mathrm{m}$ at $t = 2\,\mathrm{s}$, can be substituted into the above equation to find the value of C:

$$\begin{aligned} 30 &= 2^3 + C \\ 30 &= 8 + C \\ \textit{i.e.,} \qquad 22 &= C\,. \end{aligned}$$

Thus at time $t\,\mathrm{s}$, the rocket is at $x = t^3 + 22\,\mathrm{m}$ from the launch site.

Quiz If $y = \int 3x\,dx$ and at $x = 2$, it is measured that $y = 4$, calculate the integration constant.

(a) $C = 2$ (b) $C = 4$ (c) $C = -2$ (d) $C = 10$

Quiz Find the position of an object at time $t = 4\,\mathrm{s}$ if its velocity is $v = \alpha t + \beta\,\mathrm{ms}^{-1}$ for $\alpha = 2\,\mathrm{ms}^{-2}$ and $\beta = 1\,\mathrm{ms}^{-1}$ and its position at $t = 1\,\mathrm{s}$ was $x = 2\,\mathrm{m}$.

(a) $12\,\mathrm{m}$ (b) $24\,\mathrm{m}$ (c) $0\,\mathrm{m}$ (d) $20\,\mathrm{m}$

Quiz Acceleration a is the rate of change of velocity v with respect to time t, i.e., $a = \dfrac{dv}{dt}$.

If a ball is thrown upwards on the Earth, its acceleration is constant and approximately $a = -10\,\mathrm{m\,s}^{-2}$. If its initial velocity was $3\,\mathrm{ms}^{-1}$, when does the ball stop moving upwards (i.e., at what time is its velocity zero)?

(a) $0.3\,\mathrm{s}$ (b) $1\,\mathrm{s}$ (c) $0.7\,\mathrm{s}$ (d) $0.5\,\mathrm{s}$

4. Final Quiz

Begin Quiz Choose the solutions from the options given.

1. Which of the following is an anti-derivative with respect to x of $f(x) = 2\cos(3x)$?

(a) $2x\cos(3x)$ (b) $-6\sin(3x)$ (c) $\frac{2}{3}\sin(3x)$ (d) $\frac{2}{3}\sin(\frac{3}{2}x^2)$

2. What is the integral with respect to x of $f(x) = 11\exp(10x)$?

(a) $\frac{11}{10}\exp(10x) + C$ (b) $11\exp(5x^2) + C$
(c) $\exp(11x) + C$ (d) $110\exp(10x) + C$

3. If the speed of an object is given by $v = bt^{-\frac{1}{2}}\,\mathrm{ms}^{-1}$ for $b = 1\,\mathrm{ms}^{-\frac{1}{2}}$, what is its position x at time $t = 9\,\mathrm{s}$ if the object was at $x = 3\,\mathrm{m}$ at $t = 1\,\mathrm{s}$?

(a) $x = 7\,\mathrm{m}$ (b) $x = 11\,\mathrm{m}$ (c) $x = 4\,\mathrm{m}$ (d) $x = 0\,\mathrm{m}$

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a) To calculate the indefinite integral $\int \sin(ax)\, dx$ let us use the table of derivatives to find the function whose derivative is $\sin(ax)$.

From the table one can see that if $y = \cos(ax)$, then its derivative with respect to x is

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax), \quad \text{so} \quad \frac{d}{dx}\left(-\frac{1}{a}\cos(ax)\right) = \sin(ax)\,.$$

Thus one can conclude

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + C\,.$$

Click on the **green** square to return

□

Exercise 1(b) We have to find the indefinite integral of $\cos(ax)$. From the table of derivatives we have

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax), \quad \text{so} \quad \frac{d}{dx}\left(\frac{1}{a}\sin(ax)\right) = \cos(ax)\,.$$

This implies

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C\,.$$

Click on the **green** square to return

□

Exercise 1(c) We have to find the integral of e^{ax}. From the table of derivatives

$$\frac{d}{dx}\left(e^{ax}\right) = a\ e^{ax}, \quad \text{so} \quad \frac{d}{dx}\left(\frac{1}{a}e^{ax}\right) = \ e^{ax}\,.$$

Thus the indefinite integral of e^{ax} is

$$\int e^{ax}\,dx = \frac{1}{a}\ e^{ax} + C\,.$$

Click on the **green** square to return □

Exercise 1(d) We need to find the function whose derivative is $\frac{1}{x}$. From the table of derivatives we see that the derivative of $\ln(x)$ with respect to x is

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}\,.$$

This implies that

$$\int \frac{1}{x}\,dx = \ln(x) + C\,.$$

Click on the **green** square to return □

Exercise 2(a) We want to calculate $\int x^7\, dx$. From the table of indefinite integrals, for any $n \neq -1$,

$$\int x^n dx = \frac{1}{n+1} x^{n+1} .$$

In the case of $n = 7 (\neq -1)$,

$$\begin{aligned} \int x^7 dx &= \frac{1}{7+1} \times x^{7+1} + C \\ &= \frac{1}{8} x^8 + C . \end{aligned}$$

Checking this:

$$\frac{d}{dx}\left(\frac{1}{8}x^8 + C\right) = \frac{1}{8}\frac{d}{dx}\ x^8 = \frac{1}{8} \times 8\ x^7 = x^7 .$$

Click on the **green** square to return

□

Exercise 2(b) To calculate the integral $\int 2\sin(3x)\,dx$ we use the formula

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)\,.$$

In our case $a = 3$. Thus we have

$$\begin{aligned}\int 2\sin(3x)dx &= 2\int \sin(3x)dx = 2\times\left(-\frac{1}{3}\cos(3x)\right) + C\\ &= -\frac{2}{3}\cos(3x) + C\,.\end{aligned}$$

Checking:

$$\frac{d}{dx}\left(-\frac{2}{3}\cos(3x) + C\right) = -\frac{2}{3}\frac{d}{dx}\cos(3x) = -\frac{2}{3}\times(-3\sin(3x)) = 2\sin(3x).$$

Click on the **green** square to return

□

Exercise 2(c) To calculate the integral $\int 4\cos(2x)\,dx$ we use the formula

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax)\,,$$

with $a = 2$. This yields

$$\begin{aligned}\int 4\cos(2x)dx &= 4\int \cos(2x)dx \\ &= 4\times\left(\frac{1}{2}\sin(2x)\right) \\ &= 2\sin(2x) + C\,.\end{aligned}$$

It may be checked that

$$\frac{d}{dx}\left(2\sin(2x)+C\right) = 2\frac{d}{dx}\sin(2x) = 2\times(2\cos(2x)) = 4\cos(2x)\,.$$

Click on the **green** square to return

□

Exercise 2(d) To find the integral $\int 15\ e^{-5s}\,ds$ we use the formula

$$\int e^{ax}dx = \frac{1}{a}e^{ax}$$

with $a = -5$. This gives

$$\begin{aligned}\int 15e^{-5s}ds &= 15\int e^{-5s}ds \\ &= 15\times\left(-\frac{1}{5}\ e^{-5s}\right) \\ &= -3\ e^{-5s} + C\,,\end{aligned}$$

and indeed

$$\frac{d}{ds}\left(-3\ e^{-5s} + C\right) = -3\frac{d}{ds}e^{-5s} = -3\times\left(-5e^{-5s}\right) = 15e^{-5s}.$$

Click on the **green** square to return

□

Exercise 2(e) To find the integral $\int \frac{3}{w}\,dw$ we use the formula

$$\int \frac{1}{x}\,dx = \ln(x)\,.$$

Thus we have

$$\begin{aligned}\int \frac{3}{w}\,dw = \int 3 \times \frac{1}{w}\,dw &= 3\int \frac{1}{w}\,dw \\ &= 3\ln(w) + C\,.\end{aligned}$$

This can be checked as follows

$$\frac{d}{dw}\left(3\ln(w)+C\right) = 3\frac{d}{dw}\ln(w) = 3 \times \frac{1}{w} = \frac{3}{w}\,.$$

Click on the **green** square to return □

Exercise 2(f) To find the integral $\int(e^s + e^{-s})\, ds$ we use the sum rule for integrals, rewriting it as the sum of two integrals

$$\int(e^s + e^{-s})\, ds = \int e^s\, ds + \int e^{-s}\, ds$$

and then use

$$\int e^{ax}\, dx = \frac{1}{a}\, e^{ax}.$$

Take $a = 1$ in the first integral and $a = -1$ in the second integral. This implies

$$\begin{aligned} \int(e^s + e^{-s})\, ds &= \int e^s\, ds + \int e^{-s}\, ds \\ &= e^s + \left(\frac{1}{-1}\right) e^{-s} + C \\ &= e^s - e^{-s} + C\,. \end{aligned}$$

Click on the **green** square to return □

Exercise 3(a) To calculate the indefinite integral $\int \ln(x)\, dx$ we have to find the function whose derivative is $\ln(x)$. We are given

$$\frac{d}{dx}\left[x(\ln(x) - 1)\right] = \ln(x)\,.$$

This implies

$$\int \ln(x)\, dx \quad = \quad x\left[\ln(x) - 1\right] + C\,.$$

This can be checked by differentiating $x\left[\ln(x) - 1\right] + C$ using the product rule. (See the package on the **product and quotient rules** of differentiation.)

Click on the **green** square to return

□

Exercise 3(b) To calculate the indefinite integral $\int \ln(2x)\, dx$ we recall the following property of **logarithms**

$$\ln(ax) = \ln(a) + \ln(x)$$

and then use the integral $\int \ln(x)\, dx = x\,[\ln(x) - 1] + C$ calculated in Exercise 3(a). This gives

$$\begin{aligned}
\int \ln(2x)\, dx &= \int (\ln(2) + \ln(x))\ dx \\
&= \ln(2) \times \int 1\, dx + \int \ln(x)\, dx \\
&= x \ln(2) + x\,(\ln(x) - 1) + C \\
&= x\,(\ln(2) + \ln(x) - 1) + C \\
&= x\,(\ln(2x) - 1) + C\,.
\end{aligned}$$

In the last line we used $\ln(2) + \ln(x) = \ln(2x)$.

Click on the **green** square to return

□

Exercise 3(c) To calculate the indefinite integral $\int \ln(x^3)\,dx$ we first recall from the package on **logarithms** that

$$\ln(x^n) = n\ln(x)$$

and the integral

$$\int \ln(x)\,dx = x\,[\ln(x) - 1] + C$$

calculated in Exercise 3(a). This all gives

$$\begin{aligned} \int \ln(x^3)\,dx &= \int (3\ln(x))\ dx \\ &= 3 \times \int \ln(x)\,dx \\ &= 3x\,(\ln(x) - 1) + C\,. \end{aligned}$$

Click on the **green** square to return □

Exercise 3(d) Using the rules from the package on **logarithms**, $\ln(3x^2)$ may be simplified

$$\ln(3x^2) = \ln(3) + \ln(x^2) = \ln(3) + 2\ln(x)\,.$$

Thus

$$\begin{aligned}\int \ln(3x^2)\,dx &= \int \left(\ln(3) + 2\ln(x)\right)\,dx \\ &= \ln(3) \times \int 1\,dx + 2 \times \int \ln(x)\,dx \\ &= \ln(3)x + 2x\,[\ln(x) - 1] + C \\ &= x\,[\ln(3) + 2\ln(x) - 2] + C \\ &= x\left[\ln(3x^2) - 2\right] + C\,,\end{aligned}$$

where the final expression for $\ln(3x^2)$ is obtained by using the rules of logarithms.

Click on the **green** square to return □

Solutions to Quizzes

Solution to Quiz: To find an anti-derivative of x^6 first calculate the derivative of $F(x) = \frac{1}{7}x^7$. Using the basic formula

$$\frac{d}{dx}x^n = nx^{n-1}$$

with $n = 7$

$$\begin{aligned} \frac{dF}{dx} &= \frac{d}{dx}\left(\frac{1}{7}x^7\right) && (1) \\ &= \frac{1}{7}\frac{d}{dx}\left(x^7\right) && (2) \\ &= \frac{1}{7} \times 7\,x^{7-1} && (3) \\ &= x^6\,. && (4) \end{aligned}$$

This result shows that the function $F(x) = \frac{1}{7}x^7 + C$ is the general anti-derivative of $f(x) = x^6$. End Quiz

Solution to Quiz: To find the general anti-derivative of $3x^{\frac{1}{2}}$, recall that for constant a the anti-derivative of $af(x)$ is $aF(x)$, where $F(x)$ is the anti-derivative of $f(x)$.

Thus the anti-derivative of $3x^{\frac{1}{2}}$ is $3 \times \left(\text{the anti-derivative of } x^{\frac{1}{2}}\right)$.

To calculate the anti-derivative of $x^{\frac{1}{2}}$ we recall the anti-derivative of $f(x) = x^n$ is $F(x) = \frac{1}{n+1}x^{n+1}$ for $n \neq -1$. In our case $n = \frac{1}{2}$ and therefore this result can be used. The anti-derivative of $x^{\frac{1}{2}}$ is thus

$$\frac{1}{\frac{1}{2}+1}\, x^{(\frac{1}{2}+1)} = \frac{1}{3/2}x^{3/2} = 1 \times \frac{2}{3}\, x^{\frac{3}{2}} = \frac{2}{3}x^{\frac{3}{2}}\,.$$

Thus the general anti-derivative of $3x^{\frac{1}{2}}$ is $3 \times \frac{2}{3}x^{\frac{3}{2}} + C = 2x^{\frac{3}{2}} + C$.

This result may be checked by differentiating $F(x) = 2x^{3/2} + C$.

End Quiz

Solution to Quiz: To find the general anti-derivative of $3x^2 - 2x^3$, we use the sum rule for anti-derivatives. The anti-derivative of $3x^2 - 2x^3$ is $\big(\text{anti-derivative of } 3x^2\big) - \big(\text{anti-derivative of } 2x^3\big)$. Since the anti-derivative of $f(x) = x^n$ is $F(x) = \frac{1}{n+1}x^{n+1}$ for $n \neq -1$, for $n = 2$:

$$\text{anti-derivative of } x^2 = \frac{1}{2+1}x^{2+1} = \frac{1}{3}x^3\,.$$

Thus the anti-derivative of $3x^2$ is

$$3 \times \big(\text{anti-derivative of } x^2\big) = 3\, \times\, \frac{1}{3}x^3 = x^3.$$

Similarly the anti-derivative of $2x^3$ is

$$2 \times \big(\text{anti-derivative of } x^3\big) = 2\, \times\, \frac{1}{3+1}x^{3+1} = \frac{1}{2}x^4.$$

Putting these results together we find that the general anti-derivative of $3x^2 - 2x^3$ is

$$F(x) = x^3 - \frac{1}{2}x^4 + C\,,$$

which may be confirmed by differentiation. End Quiz

Solution to Quiz: To calculate the indefinite integral

$$\int \frac{1}{\sqrt{x}}\, dx = \int \frac{1}{x^{1/2}}\, dx = \int x^{-1/2}\, dx$$

we recall the basic result, that the anti-derivative of $f(x) = x^n$ is $F(x) = \frac{1}{n+1}x^{n+1}$ for $n \neq -1$. In this case $n = -\frac{1}{2}$ and so

$$\begin{aligned}
\int x^{-1/2}\, dx &= \frac{1}{-\frac{1}{2}+1}\, x^{(-\frac{1}{2}+1)} + C = \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} + C \\
&= 1 \times \frac{2}{1} x^{\frac{1}{2}} + C = 2x^{\frac{1}{2}} + C \\
&= 2\sqrt{x} + C\,,
\end{aligned}$$

where we recall that dividing by a fraction is equivalent to multiplying by its inverse (see the package on **fractions**). End Quiz

Solution to Quiz: To evaluate $\int (4\sin(5x) + 5\cos(3x))\,dx$ we use the sum rule for indefinite integrals to rewrite the integral as the sum of two integrals. Using

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) \quad \text{and} \quad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

we get

$$\begin{aligned}
\int (4\sin(5x) + 5\cos(3x))\,dx &= 4\int \sin(5x)\,dx + 5\int \cos(3x)\,dx \\
&= 4 \times (-\frac{1}{5})\,\cos(5x) + 5 \times \frac{1}{3}\sin(3x) + C \\
&= -\frac{4}{5}\,\cos(5x) + \frac{5}{3}\,\sin(3x) + C\,.
\end{aligned}$$

This can be checked by differentiation. End Quiz

Solution to Quiz: If $y = \int 3x\,dx$ and at $x = 2$, $y = 4$ then

$$\begin{aligned} y = \int 3x\,dx &= 3 \times \int x\,dx \\ &= 3 \times \frac{1}{2}x^{1+1} + C \\ &= \frac{3}{2}x^2 + C \end{aligned}$$

is the general solution. Substituting $x = 2$ and $y = 4$ into the above equation, the value of C is obtained

$$\begin{aligned} 4 &= \frac{3}{2} \times (2)^2 + C \\ 4 &= 6 + C \\ \text{i.e.,} \qquad C &= -2\,. \end{aligned}$$

Therefore, for all x, $y = \frac{3}{2}x^2 - 2$. End Quiz

Solution to Quiz:
We are told that $v = \alpha t + \beta$ with $\alpha = 2\text{ms}^{-2}$, $\beta = 1\text{ms}^{-1}$ and at $t = 1\text{s}$, $x = 2\text{m}$. Since x is the integral of v:

$$x = \int v\,dt = \int (2t+1)\,dt = 2 \times \int t\,dt + \int 1\,dt = t^2 + t + C\,.$$

The position at time $t = 1\,\text{s}$ was $x = 2\,\text{m}$ so these values may be substituted into the above equation to find C:

$$\begin{aligned} 2 &= 1^2 + 1 + C \\ 2 &= 2 + C \\ i.e., \qquad 0 &= C\,. \end{aligned}$$

Therefore, for all t, $x = t^2 + t + 0 = t^2 + t$. At $t = 4\,\text{s}$,

$$x = (4)^2 + 4 = 16 + 4 = 20 \text{ m}.$$

End Quiz

Solution to Quiz: We are given $a = \frac{dv}{dt} = -10\text{ms}^{-2}$ and initial velocity $v = 3\text{ms}^{-1}$, and want to find when the velocity is zero. Since $a = \frac{dv}{dt}$, velocity is the integral of acceleration, $v = \int a\, dt$. The acceleration of the ball is constant, $a = -10\text{ms}^{-2}$, so that

$$v = \int (-10)\ dt = -10 \times \int dt = -10t + C\,.$$

At $t = 0\,,\ v = 3\text{ms}^{-1}$, so these values may be substituted into the above equation to find the constant C:

$$\begin{aligned} 3 &= -10 \times 0 + C \\ 3 &= C\,. \end{aligned}$$

Thus $v = -10t + 3$ for this problem. Therefore if $v = 0$

$$\begin{aligned} 0 &= -10t + 3 \\ 10t &= 3\,, \qquad t = 3/10\,. \end{aligned}$$

The ball stops moving upwards at 0.3 s. End Quiz