

# Trigonometry 1

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic understanding of some trigonometric functions.

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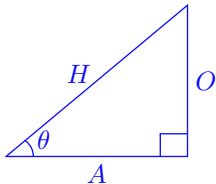
## 1. Trigonometry (Introduction)

In the right angled triangle shown in **diagram 1**,  $O$  is the side *opposite* the angle  $\theta$ ,  $A$  is the side *adjacent* to the angle  $\theta$  and  $H$ , the side opposite the right angle, is the *hypotenuse* of the triangle. The three trigonometric functions dealt with in this package are the *sine*, *cosine* and *tangent* functions:

$$\sin \theta = \frac{O}{H}$$

$$\tan \theta = \frac{O}{A}$$

$$\cos \theta = \frac{A}{H}$$



**Diagram 1**

One useful observation is the following relationship between the three functions:

$$\frac{\sin \theta}{\cos \theta} = \frac{O/H}{A/H} = \frac{O}{A} = \tan \theta.$$

## 2. Using the Sine Function

### Example 1

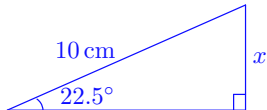
Find the length of the side marked  $x$  in **diagram 2**.

**Solution** Using the sine function

$$\sin 22.5^\circ = \frac{O}{H} = \frac{x}{10}$$

$$\therefore 10 \sin 22.5^\circ = 10 \times 0.383 = x$$

$$\text{i.e. } x = 3.83 \text{ cm}$$



**Diagram 2**

where the value of  $\sin 22.5^\circ$  was found using a calculator.

### Example 2

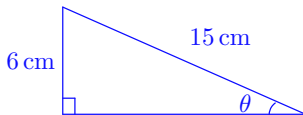
Find the value of the angle denoted by  $\theta$  in **diagram 3**.

**Solution** Using the sine function

$$\sin \theta = \frac{O}{H} = \frac{6}{15} = 0.4.$$

Using a calculator will show that

$$\theta \approx 23.58^\circ.$$



**Diagram 3**

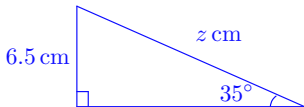
**Example 3**

Use the sine function to find  $z$  from **diagram 4**.

**Solution**

Using the sine function again:

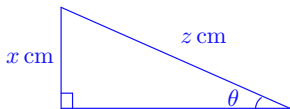
$$\begin{aligned}\sin 35^\circ &= \frac{O}{H} = \frac{6.5}{z} \\ \therefore z \times \sin 35^\circ &= z \times 0.574 = 6.5 \\ z &= \frac{6.5}{0.574} = 11.3 \text{ cm}\end{aligned}$$

**Diagram 4**

**EXERCISE 1.** In the exercises below, two of the three values of  $x, z, \theta$ , referring to **diagram 5**, are given. Find the value of the missing one. (Click on the green letters for solutions.)

(a)  $x = 5, z = 10,$     (b)  $x = 4, \theta = 47^\circ,$

(c)  $x = 10, \theta = 50^\circ.$

**Diagram 5**

### 3. Using the Cosine Function

#### Example 4

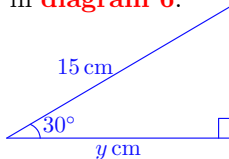
Use the cosine function to find the value of  $y$  in **diagram 6**.

#### Solution

$$\cos 30^\circ = \frac{A}{H} = \frac{y}{15}$$

$$\therefore 15 \times \cos 30^\circ = 15 \times 0.866 = y$$

$$\text{i.e. } y = 13.0 \text{ cm}$$



**Diagram 6**

#### Example 5

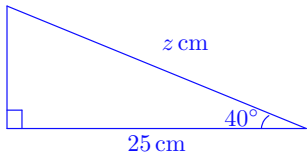
Use the cosine function to find the value of  $z$  in **diagram 7**.

#### Solution

$$\cos 40^\circ = \frac{A}{H} = \frac{25}{z}$$

$$\therefore z \times \cos 40^\circ = z \times 0.766 = 25$$

$$\begin{aligned} \text{i.e. } z &= \frac{25}{0.766} \\ &= 32.6 \text{ cm} \end{aligned}$$



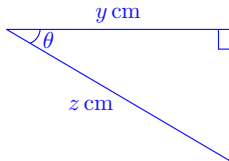
**Diagram 7**

**EXERCISE 2.** In the exercises below, two of the three values of  $y, z, \theta$ , referring to **diagram 8**, are given. Find the value of the missing one. (Click on the green letters for solutions.)

(a)  $y = 5, z = 10,$

(b)  $y = 4, \theta = 47^\circ,$

(c)  $z = 12, \theta = 50^\circ.$



**Diagram 8**

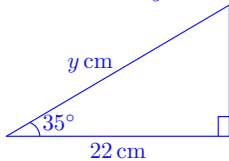
**Quiz** Referring to **diagram 9**, which is the value of  $y$ ?

(a) 18.0,

(b) 26.9 ,

(c) 38.4,

(d) 12.6.



**Diagram 9**

## 4. Using the Tangent Function

### Example 6

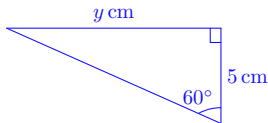
Use the tangent function to find the value of  $y$  in **diagram 10**.

### Solution

$$\tan 60^\circ = \frac{O}{A} = \frac{y}{5}$$

$$\therefore 5 \times \tan 60^\circ = 5 \times 1.732 = y$$

$$\text{i.e. } y = 8.7 \text{ cm}$$



**Diagram 10**

### Example 7

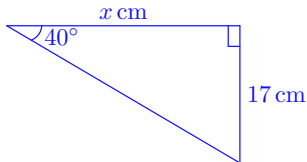
In **diagram 11**, use the tangent function to find the value of  $x$ .

### Solution

$$\tan 40^\circ = \frac{O}{A} = \frac{17}{x}$$

$$\therefore x \times \tan 40^\circ = x \times 0.839 = 17$$

$$\text{i.e. } x = \frac{17}{0.839} = 20.3$$



**Diagram 11**

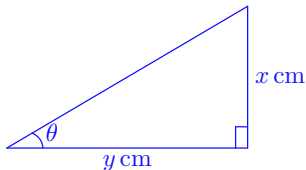


**EXERCISE 3.** In the exercises below, two of the three values of  $x, y, \theta$ , referring to **diagram 12**, are given. Find the value of the missing one. (Click on the green letters for solutions.)

(a)  $x = 5, y = 10,$

(b)  $y = 4, \theta = 25^\circ,$

(c)  $x = 12, \theta = 56^\circ.$



**Diagram 12**

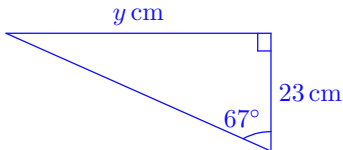
**Quiz** Referring to **diagram 13**, which is the value of  $y$ ?

(a) 33.6,

(b) 54.2,

(c) 67.2,

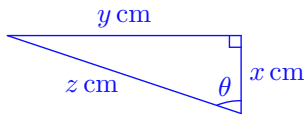
(d) 38.7.



**Diagram 13**

## 5. Quiz on Trigonometric Functions

The following questions all refer to **diagram 14**.



**Diagram 14**

Begin Quiz

1. If  $y = 13$  cm and  $z = 22$  cm, find  $\theta$ .

- (a)  $18^\circ$ ,      (b)  $43^\circ$       (c)  $35^\circ$ ,      (d)  $36^\circ$ .

2. If  $y = 30$  cm and  $\theta = 25^\circ$ , find  $x$ .

- (a) 69 cm,      (b) 64 cm,      (c) 38 cm,      (d) 16 cm.

3. If  $x = 15$  cm and  $\theta = 25^\circ$ , find  $z$ .

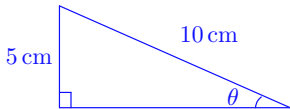
- (a) 24 cm,      (b) 17 cm,      (c) 25 cm,      (d) 14 cm.

End Quiz

## Solutions to Exercises

### Exercise 1(a)

To find the value of the angle  $\theta$  in the diagram, we use the sine function:



$$\sin \theta = \frac{5}{10} = 0.5.$$

Using a calculator will show that

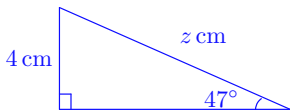
$$\theta = 30^\circ.$$

Click on the green square to return



**Exercise 1(b)**

To find  $z$ , the length of the *hypotenuse* of the triangle, we use the sine function:



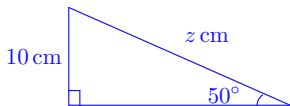
$$\begin{aligned}\sin 47^\circ &= \frac{4}{z} \\ \therefore z \times \sin 47^\circ &= z \times 0.732 = 4 \\ z &= \frac{4}{0.732} = 5.5 \text{ cm},\end{aligned}$$

where the value of  $\sin 47^\circ \approx 0.732$  was found using a calculator.  
Click on the green square to return



**Exercise 1(c)**

To find  $z$ , the length of the *hypotenuse* of the triangle, we use the sine function:



$$\begin{aligned}\sin 50^\circ &= \frac{10}{z} \\ \therefore z \times \sin 50^\circ &= z \times 0.766 = 10 \\ z &= \frac{10}{0.766} = 13.1 \text{ cm} .\end{aligned}$$

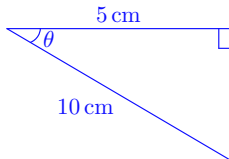
Here the value of  $\sin 50^\circ \approx 0.766$  was found using a calculator.

[Click on the green square to return](#)



**Exercise 2(a)**

To find the value of the angle  $\theta$  in the triangle given here, use the cosine function:



$$\cos \theta = \frac{5}{10} = \frac{1}{2}.$$

Using a calculator one finds that

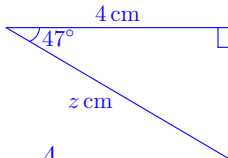
$$\theta = 60^\circ.$$

Click on the green square to return



**Exercise 2(b)**

To find  $z$ , the length of the *hypotenuse* of the triangle given in the picture, we use the cosine function:



$$\cos 47^\circ = \frac{4}{z}$$

$$\therefore z \times \cos 47^\circ = z \times 0.682 = 4$$

$$\begin{aligned} \text{i.e. } z &= \frac{4}{0.682} \\ &= 5.9 \text{ cm} \end{aligned}$$

Click on the green square to return



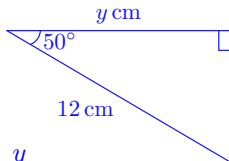
**Exercise 2(c)**

To find  $y$ , the length of the side *adjacent* to the angle  $\theta = 50^\circ$  in the triangle given here, we use the cosine function:

$$\cos 50^\circ = \frac{y}{12}$$

$$\therefore 12 \times \cos 50^\circ = 12 \times 0.643 = y$$

$$\text{i.e. } y = 7.7 \text{ cm}$$



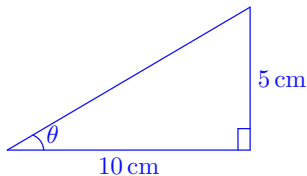
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**Exercise 3(a)**

To find the value of the angle  $\theta$  of the triangle given in the picture, we use the tangent function:



$$\tan \theta = \frac{5}{10} = 0.5$$

Using a calculator will show that

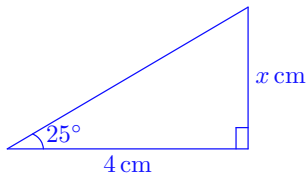
$$\theta \approx 27^\circ.$$

Click on the green square to return



**Exercise 3(b)**

To find  $x$ , the length of the side *opposite* the angle  $\theta = 25^\circ$  of the triangle given in the picture, we use the tangent function:



$$\tan 25^\circ = \frac{x}{4}$$

$$\therefore 4 \times \tan 25^\circ = 4 \times 0.466 = x$$

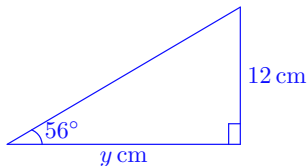
$$\text{i.e. } x = 1.9 \text{ cm}$$

Click on the green square to return



**Exercise 3(c)**

To find  $y$ , the length of the side *adjacent* to the angle  $\theta = 56^\circ$  of the triangle given in the picture, we use the tangent function:



$$\tan 56^\circ = \frac{12}{y}$$

$$\therefore y \times \tan 56^\circ = y \times 1.483 = 12$$

$$\text{i.e. } y = \frac{12}{1.483} = 8.1 \text{ cm}$$

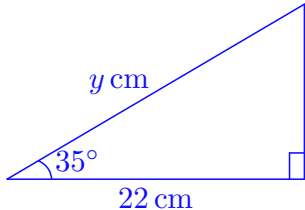
Click on the green square to return



## Solutions to Quizzes

### Solution to Quiz:

To find  $y$ , the length of the *hypotenuse* of the triangle given in the picture, we use the cosine function:



$$\cos 35^\circ = \frac{22}{y}$$

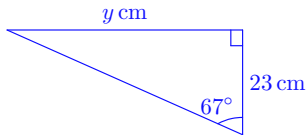
$$\therefore y \times \cos 35^\circ = y \times 0.819 = 22$$

$$\text{i.e. } y = \frac{22}{0.819} = 26.9 \text{ cm}$$

End Quiz

**Solution to Quiz:**

To find  $y$ , the length of the side *opposite* to the angle  $67^\circ$  of the triangle given in the picture, we use the tangent function:



$$\tan 67^\circ = \frac{y}{23}$$

$$\therefore 23 \times \tan 67^\circ = 23 \times 2.356 = y$$

$$\text{i.e. } y = 54.2 \text{ cm}$$

End Quiz