

Straight Line Graphs

R Horan & M Lavelle

The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence at sketching graphs of linear functions.

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The full range of these packages and some instructions, should they be required, can be obtained from our web page [Mathematics Support Materials](#).

1. Straight Line Graphs (Introduction)

A general linear function has the form $y = mx + c$, where m and c are constants.

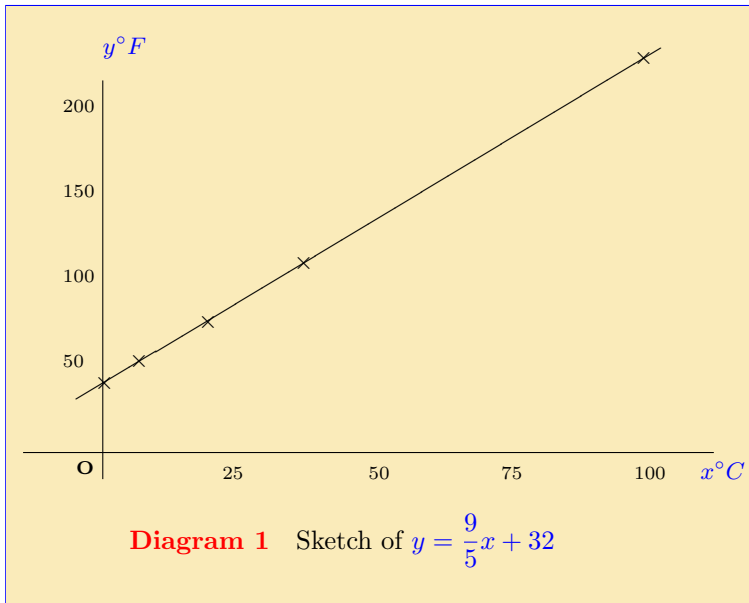
Example 1 If x is the temperature in $^{\circ}C$ and y the temperature in $^{\circ}F$ then there is a simple rule relating the values of x and y . The table illustrates this rule for various values of x and y .

$x(^{\circ}C)$	$y(^{\circ}F)$	
0	32	freezing point of water
10	50	temperature on a cold day
25	77	temperature on a warm day
37	98.6	blood temperature
100	212	boiling point of water

The general rule is

$$y = \frac{9}{5}x + 32,$$

so that $m = 9/5$ and $c = 32$ in this case. A graph of this relationship is shown on the next page.



Example 2 A straight line passes through the two points $P(x, y)$ and $Q(x, y)$ with coordinates $P(0, 2)$ and $Q(1, 5)$. Find the equation of this straight line.

Solution The general equation of a straight line is $y = mx + c$. Since the line passes through the points P , with coordinates $x = 0, y = 2$, and Q , with coordinates $x = 1, y = 5$, these coordinates must satisfy this equation, i.e.

$$2 = m \times 0 + c$$

$$5 = m \times 1 + c$$

(See the package on **simultaneous equations**.) Solving these equations gives $c = 2$ and $m = 3$, i.e. the line is $y = 3x + 2$.

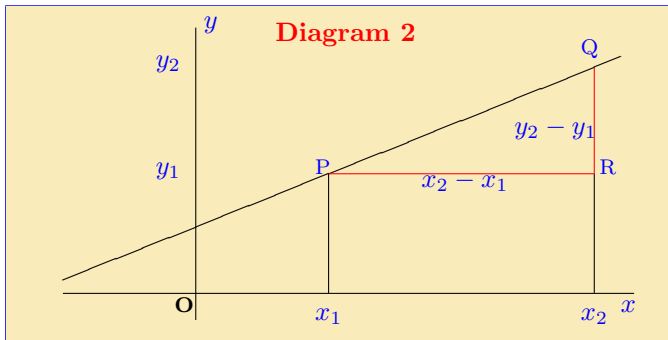
EXERCISE 1. In each of the following find the equation of the straight line through the given pairs of points. (Click on the **green** letters for solution.)

(a) The points $P(0, -3)$ and $Q(2, 1)$.

(b) The points $P(0, 4)$ and $Q(1, 3)$.

2. Gradient of a Straight Line

The gradient of a straight line is defined as follows. Suppose that two points P and Q , on the line, have coordinates $P(x_1, y_1)$ and $Q(x_2, y_2)$.



The **gradient** of the line is (see diagram above)

$$\text{gradient} = \frac{RQ}{PR} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Example 3 From the table given in **example 1**, find the gradient of the line giving the relationship between $x^{\circ}C$ and $y^{\circ}F$.

Solution The boiling point, Q , of water is $100^{\circ}C$ or $212^{\circ}F$, i.e. $Q(100, 212)$. The freezing point of water is $0^{\circ}C$ or $32^{\circ}F$, i.e. $P(0, 32)$. The gradient is therefore

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}.$$

The equation was $y = (9/5)x + 32$. Comparing this with the general equation $y = mx + c$ shows that m is the value of the gradient.

EXERCISE 2. Find the gradient of the line through the points P, Q with the following coordinates. (Click on the green letters for solution.)

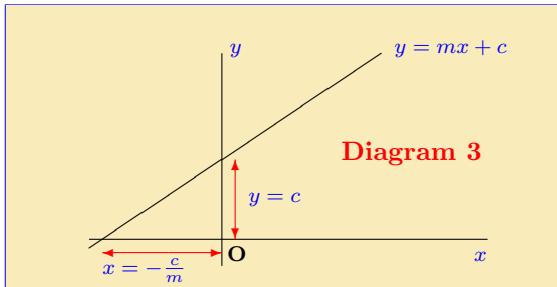
- (a) $P(3, 9), Q(2, 3)$ (b) $P(-1, 2), Q(2, -1)$ (c) $P(1, 2), Q(4, 3)$

3. Intercepts of a Straight Line

By putting $x = 0$ into the equation $y = mx + c$, the point where the straight line crosses the y axis is found to be $y = c$. This is known as the *intercept on the y axis*. The *intercept on the x axis*, i.e. when $y = 0$, is at

$$\begin{aligned}0 &= mx + c \\ -c &= mx \\ -c/m &= x.\end{aligned}$$

The x and y
intercepts.



Example 4 By rearranging the equation $3y - 2x - 5 = 0$, show that it is a straight line and find its gradient and intercept. Sketch the line.

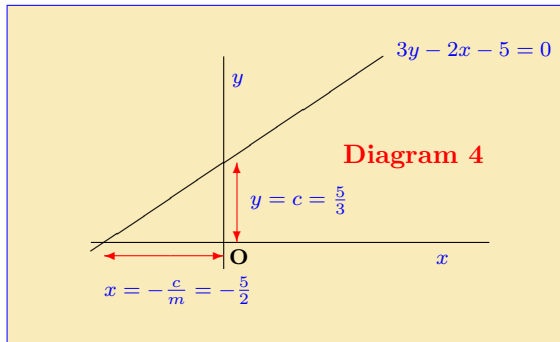
Solution Rearranging the equation,

$$3y - 2x - 5 = 0$$

$$3y = 2x + 5$$

$$y = \left(\frac{2}{3}\right)x + \left(\frac{5}{3}\right)$$

(Equation of a straight line
with $m = 2/3$ and $c = 5/3$.)



4. Positive and Negative Gradients

If a line has gradient $m = 1$ then, *providing that the scales are the same for both axes*, it makes an angle of 45° with the positive x -axis. If $m > 1$ then the gradient is steeper. If $0 < m < 1$ then the line makes an angle between 0° and 45° with the positive x -axis.

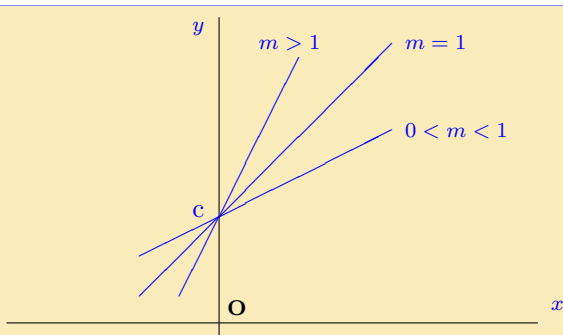
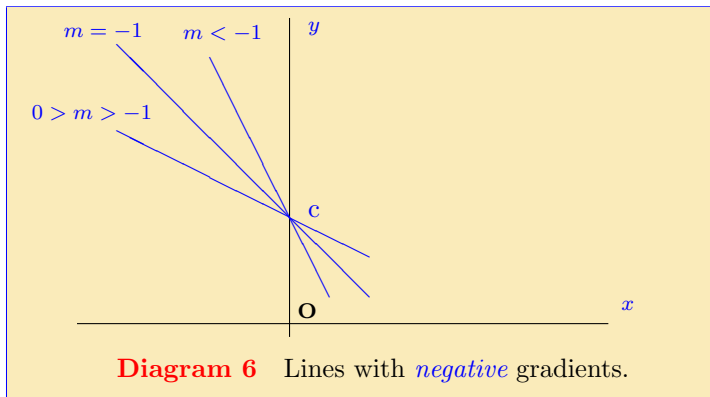


Diagram 5 Lines with *positive* gradients.

The diagram below illustrates lines similar to those of **diagram 5** except with **negative** gradients. They are the mirror images of the straight lines which are shown in **diagram 5**, with the y axis acting as the mirror.



EXERCISE 4. In each of the following either the coordinates of two points, P, Q are given, or the coordinates of a single point R and a gradient m . In each case, find the equation of the line.

- (a) $P(1, 1), Q(2, -1)$, (b) $R(1, 2), m = 2$, (c) $P(-1, 2), Q(1, -3)$.
(d) $R(-2, 1), m = 4$. (e) $P(-1, 2), Q(-3, 3)$ (f) $P(1, 2), Q(-4, 7)$

Example 5 Two lines are described as follows: the first has gradient -1 and passes through the point $R(2, 1)$; the second passes through the two points with coordinates $P(2, 0)$ and $Q(0, 4)$. Find the equation of both lines and find the coordinates of their point of intersection.

Solution The first line has gradient $m = -1$ so it must be $y = (-1)x + c$, i.e. $y = -x + c$, for some c . Since the line passes through the point $R(2, 1)$ these values of x, y must satisfy the equation. Thus $1 = -(2) + c$, so $c = 3$. The first line therefore has equation $y = -x + 3$. For the second case both points lie on the line and so satisfy the equation. If the equation is $y = mx + c$ then putting these values into the equation gives

$$\begin{aligned}y &= mx + c \\0 &= 2m + c && \text{using the coordinates of } P \\4 &= c && \text{using the coordinates of } Q\end{aligned}$$

These equations yield $m = -2$ and $c = 4$. The second line thus has the equation $y = -2x + 4$. The equations of the two lines can now be rewritten as

$$\begin{aligned}y + x &= 3 && (1) \\y + 2x &= 4 && (2)\end{aligned}$$

which is a pair of simultaneous equations. Subtracting equation (1) from equation (2) gives $x = 1$ and substituting this into the first equation then yields $y = 2$. The point of intersection thus has coordinates $x = 1, y = 2$. (By substituting these coordinates into equation (2) and verifying that they satisfy the equation, it can be checked that this is also a point on the second line.)

5. Some Useful Facts

- Parallel lines have the same gradient (i.e. the same value of m). Thus, for example, the lines with equations $y = 3x + 7$ and $y = 3x - 2$ are parallel.
- Lines parallel to the x -axis (when $m = 0$) have equations of the form $y = k$, for some constant, k .
- Lines parallel to the y -axis have equations of the form $x = k$, for some constant, k .
- The larger the *absolute* value of m , the ‘steeper’ the slope of the line.
- If two lines intersect at right angles then the product of their gradients is -1 . The lines $y = -7x + 4$ and $y = (1/7)x + 5$, for example, intersect each other at right angles.

6. Quiz on Straight Lines

Begin Quiz In each of the following, choose the solution from the options given.

- The straight line through $P(-5, 4)$ and $Q(2, -3)$.
(a) $2x - y = -14$ (b) $-x + 2y = 17$
(c) $-x + y - 1 = 0$ (d) $x + y + 1 = 0$
- The gradient m and intercept c of $-2x + 3y + 6 = 0$.
(a) $m = 2/3, c = -2$ (b) $m = -2/3, c = 2$
(c) $m = 3/2, c = -3$ (d) $m = -3/2, c = 3$
- The straight line with gradient $m = -3$ passing through $R(-1, 3)$.
(a) $-3x + y = 0$ (b) $2y - 6x = 4$
(c) $y - 3x = 1$ (d) $y + 3x = 0$
- The point of intersection of the lines $2x + y = 1$ and $3x - 2y = 5$.
(a) $(-1, 1)$ (b) $(1, -1)$ (c) $(3, -5)$ (d) $(2, -3)$

End Quiz

Solutions to Exercises

Exercise 1(a) The general equation of a straight line is $y = mx + c$. Since the line passes through the points P and Q , the coordinates of both points must satisfy this equation. The point P has coordinates $x = 0, y = -3$ and the point Q has coordinates $x = 2, y = 1$. These satisfy the pair of simultaneous equations

$$\begin{aligned}-3 &= m \times 0 + c \\ 1 &= m \times 2 + c\end{aligned}$$

Solving these equations gives $c = -3$ and $m = 2$, i.e. the line is $y = 2x - 3$.

Click on the green square to return



Exercise 1(b) The general equation of a straight line is $y = mx + c$. Since the line passes through the points P and Q , the coordinates of both points must satisfy this equation. The point P has coordinates $x = 0, y = 4$ and the point Q has coordinates $x = 1, y = 3$. These satisfy the pair of simultaneous equations

$$4 = m \times 0 + c$$

$$3 = m \times 1 + c$$

Solving these equations gives $c = 4$ and $m = -3$, i.e. the line is $y = -3x + 4$.

Click on the green square to return



Exercise 2(a) For $P(3, 9)$, $Q(2, 3)$, the gradient is given by

$$\begin{aligned}\text{gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - 9}{2 - 3} \\ &= \frac{-6}{-1} \\ &= 6,\end{aligned}$$

so that $m = 6$ in this case.

Click on the green square to return



Exercise 2(b) For $P(-1, 2)$, $Q(2, -1)$ the gradient is given by

$$\begin{aligned}\text{gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 2}{2 - (-1)} \\ &= -1,\end{aligned}$$

so that $m = -1$ in this case. We shall interpret the **negative** gradient later in this package.

Click on the green square to return



Exercise 2(c) For $P(1, 2)$, $Q(4, 3)$ the gradient is given by

$$\begin{aligned}\text{gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - 2}{4 - 1} \\ &= \frac{1}{3},\end{aligned}$$

so that $m = 1/3$ in this case.

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Exercise 3(a) For the equation $2y - 2x + 3 = 0$,

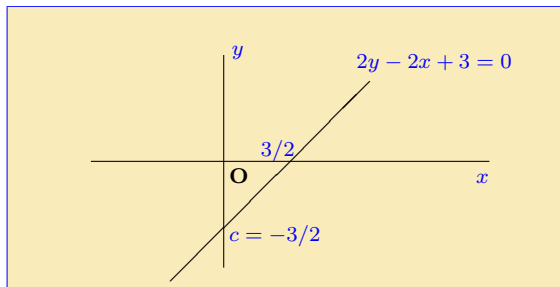
$$2y - 2x + 3 = 0$$

$$2y = 2x - 3$$

$$y = x - \frac{3}{2}$$

so that $m = 1$ and $c = -3/2$.

The intercept on the x axis is $-c/m = -(-3/2)/1 = 3/2$.



Click on the green square to return



Exercise 3(b) For the equation $3y - 5x + 6 = 0$,

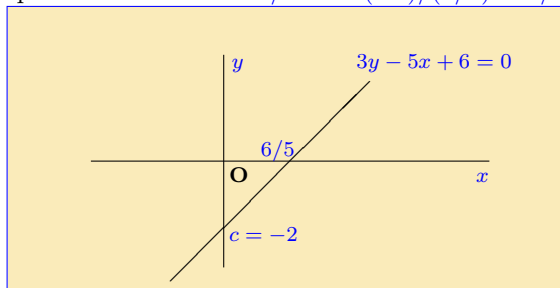
$$3y - 5x + 6 = 0$$

$$3y = 5x - 6$$

$$y = \frac{5}{3}x - 2$$

so that $m = 5/3$ and $c = -2$.

The intercept on the x axis is $-c/m = -(-2)/(5/3) = 6/5$.



Click on the green square to return



Exercise 3(c) For the equation $2y + 4x + 3 = 0$,

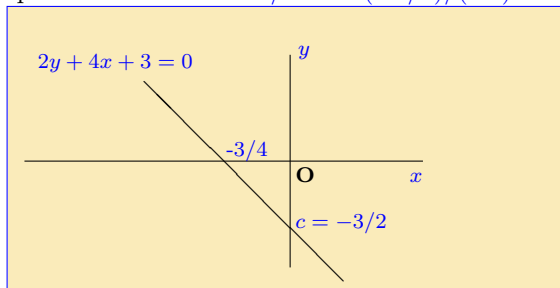
$$2y + 4x + 3 = 0$$

$$2y = -4x - 3$$

$$y = -2x - \frac{3}{2}$$

so that $m = -2$ and $c = -3/2$.

The intercept on the x axis is $-c/m = -(-3/2)/(-2) = -3/4$.



Click on the green square to return



Exercise 4(a) Let the line be $y = mx + c$. Since both $P(1, 1)$ and $Q(2, -1)$ lie on the line, both sets of coordinates must satisfy the equation. Thus we have

$$1 = m \times 1 + c \quad \text{using the coordinates of } P$$

$$-1 = m \times 2 + c \quad \text{using the coordinates of } Q$$

or $m + c = 1$

$$2m + c = -1.$$

This is a set of simultaneous equations which can be solved to give $m = -2$ and $c = 3$. (See the package on **simultaneous equations** for the technique for solving them.) The required equation is thus

$$y = -2x + 3.$$

Substituting the coordinates for P and then Q into this equation will confirm that this line passes through both of these points.

[Click on the green square to return](#)



Exercise 4(b) Since $m = 2$, the equation must have the form $y = 2x + c$ and only the value of c remains to be found. The line passes through $R(1, 2)$ so the coordinates of this point must satisfy the equation. Thus

$$y = 2x + c$$

$$2 = 2 \times 1 + c \quad \text{using the coordinates of } R$$

giving $c = 0$. The equation of the line is now

$$y = 2x.$$

Click on the green square to return



Exercise 4(c) Let the line be $y = mx + c$. Since both $P(-1, 2)$ and $Q(1, -3)$ lie on the line, both sets of coordinates must satisfy the equation. Thus we have

$$2 = m \times (-1) + c \quad \text{using the coordinates of } P$$

$$-3 = m \times 1 + c \quad \text{using the coordinates of } Q$$

or $-m + c = 2$

$$m + c = -3.$$

This is a set of simultaneous equations which can be solved to give $m = -5/2$ and $c = -1/2$. (See the package on **simultaneous equations** for the technique for solving them.) The required equation is thus

$$y = -\frac{5}{2}x - \frac{1}{2}.$$

Substituting the coordinates for P and then Q into this equation will confirm that this line passes through both of these points.

[Click on the green square to return](#)



Exercise 4(d) Since $m = 4$, the equation must have the form $y = 4x + c$ and only c remains to be found. The line passes through $R(-2, 1)$ so the coordinates of this point must satisfy the equation. Thus

$$y = 4x + c$$

$$1 = 4 \times (-2) + c \quad \text{using the coordinates of } R$$

$$\text{or } -8 + c = 1$$

$$c = 9,$$

and the equation of the line is

$$y = 4x + 9.$$

Click on the green square to return



Exercise 4(e) Let the line be $y = mx + c$. The coordinates $P(-1, 2)$ and $Q(-3, 3)$ both lie on the line so both sets of coordinates must satisfy the equation. We have

$$2 = m \times (-1) + c \quad \text{using the coordinates of } P$$

$$3 = m \times (-3) + c \quad \text{using the coordinates of } Q$$

or $-m + c = 2$

$$-3m + c = 3.$$

This set of simultaneous equations can be solved to give $m = -1/2$ and $c = 3/2$. (See the package on **simultaneous equations** for the technique for solving them.) The required equation is thus

$$y = -\frac{1}{2}x + \frac{3}{2}.$$

Substituting the coordinates for P and then Q into this equation will confirm that this line passes through both of these points.

[Click on the green square to return](#)



Exercise 4(f) Let the line be $y = mx + c$. Both $P(1, 2)$ and $Q(-4, 7)$ lie on the line so both sets of coordinates must satisfy the equation. Thus

$$2 = m \times (1) + c \quad \text{using the coordinates of } P$$

$$7 = m \times (-4) + c \quad \text{using the coordinates of } Q$$

$$\text{or } m + c = 2$$

$$-4m + c = 7.$$

The solution to this set of simultaneous equations is found to be $m = -1$ and $c = 3$. (See the package on **simultaneous equations** for the technique for solving them.) The equation of the line is thus

$$y = -x + 3.$$

Substituting the coordinates for P and then Q into this equation will confirm that this line passes through both of these points.

Click on the green square to return



Solutions to Quizzes

Solution to Quiz:

The line crosses the x axis when $y = 0$. Putting this into the equation of the line, $3x + y + 3 = 0$, gives

$$\begin{aligned}3x + 0 + 3 &= 0 \\3x &= -3 \\x &= -1.\end{aligned}$$

Thus $P(-1, 0)$ is the first point.

The line crosses the y axis when $x = 0$. Putting this into the equation of the line,

$$\begin{aligned}3 \times (0) + y + 3 &= 0 \\y + 3 &= 0 \\y &= -3.\end{aligned}$$

Thus $Q(0, -3)$ is the second point.

End Quiz

Solution to Quiz:

The equation $6x + 2y = 3$ is rearranged as follows:

$$\begin{aligned}6x + 2y &= 3 \\2y &= -6x + 3 \\y &= -\frac{6}{2}x + \frac{3}{2} \\&= -3x + \frac{3}{2}\end{aligned}$$

so $m = -3$ and $c = 3/2$.

End Quiz