

Simultaneous Equations

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The aim of this package is to provide a short self assessment programme for students who are learning how to solve simultaneous equations.

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1. Two Equations and Two Unknowns

Many scientific problems lead to **simultaneous equations** containing quantities which need to be calculated. The simplest case is two simultaneous equations in two unknowns, say x and y .

Example 1 To start to see how we can solve such relations, consider

$$\begin{aligned}4x + y &= 9 \\ 3x &= 6\end{aligned}$$

There are two unknown variables x and y . However, the bottom equation only involves x and is solved by $x = 2$. We can then substitute this into the top equation to find

$$\begin{aligned}4 \times 2 + y &= 9 \\ y &= 9 - 8 \\ y &= 1\end{aligned}$$

The full solution is therefore $x = 2, y = 1$.

EXERCISE 1. Solve the following pairs of simultaneous equations (Click on the **green** letters for the solutions.)

(a)
$$\begin{aligned}x + y &= 3 \\x &= 2\end{aligned}$$

(b)
$$\begin{aligned}4x - y &= 10 \\y &= 2\end{aligned}$$

(c)
$$\begin{aligned}z - x &= 2 \\2x &= -2\end{aligned}$$

(d)
$$\begin{aligned}3t + 2s &= 0 \\s + 1 &= 2\end{aligned}$$

Quiz What value of y solves the following pair of equations?

$$\begin{aligned}x + 2y &= 10 \\x &= -2\end{aligned}$$

- (a) 12 (b) 4 (c) 8 (d) 6

2. Simultaneous Equations

More generally both equations may involve both unknowns.

Example 2 Consider

$$x + y = 4 \quad (1)$$

$$x - y = 2 \quad (2)$$

Now **add** the left hand side of (1) to the left hand side of (2) and the right hand side of (1) to the right hand side of (2). The y 's **cancel** and we get an equation for x alone

$$x + y + x - y = 4 + 2$$

$$2x = 6$$

which implies that $x = 3$. We can now **insert this into (1)** and so obtain:

$$3 + y = 4, \quad \Rightarrow \quad y = 4 - 3 = 1.$$

In other words the full solution is $x = 3, y = 1$

It is easy to **check** that you have the correct solution to simultaneous equations: by **substituting** your answers back into the original equations. We have already used (1) to find y , so let's check that (2) is correctly solved: we get $x - y = 3 - 1 = 2 \checkmark$

Always make such a check!

Example 2 illustrates the central idea of the method which is to combine the two equations so as to get a single equation for one variable and then use this to find the other unknown.

EXERCISE 2. Solve the following pairs of equations (Click on the **green** letters for the solutions.)

(a) $x + y = 5$

$$x - y = 1$$

(b) $4x + 3y = 7$

$$x - 3y = -2$$

Example 3 Consider

$$x + 2y = 4 \quad (1)$$

$$x + y = 3 \quad (2)$$

Subtracting these equations yields an equation in y , i.e., (1)-(2) gives

$$x + 2y - (x + y) = 4 - 3$$

$$y = 1$$

Reinserting this result into (2) gives $x + 1 = 3$, so we obtain $x = 2$.

Check the results by substituting them into (1)!

Quiz Solve the following simultaneous equations and select the correct result:

$$3x + 3y = 0$$

$$2x + 3y = 1$$

(a) $x = 0, \quad y = 0$

(b) $x = -1, \quad y = 1$

(c) $x = 0, \quad y = 1$

(d) $x = 3, \quad y = 2$

3. A Systematic Approach

The **first step** in solving a system of two simultaneous equations is to **eliminate one of the variables**. This can be done by making the coefficient of x the same in each equation.

Example 4 Consider

$$3x + 2y = 4 \quad (1)$$

$$2x + y = 3 \quad (2)$$

If we multiply (1) by 2, and (2) by 3, then we get

$$6x + 4y = 8$$

$$6x + 3y = 9$$

We see that the **coefficient of x is now the same in each equation!** Subtracting them cancels (‘eliminates’) x and we can solve the simultaneous equations using the methods described above. Let us now work through an example.

Example 5 Consider the equations

$$5x + 3y = 7 \quad (1)$$

$$4x + 5y = 3 \quad (2)$$

Multiply (1) by 4 (which is the coefficient of x in (2)) and also multiply (2) by 5 (the coefficient of x in (1)).

$$20x + 12y = 28 \quad (3)$$

$$20x + 25y = 15 \quad (4)$$

The coefficient of x is now the same in both equations. **Subtracting (4)–(3) eliminates x :**

$$25y - 12y = 15 - 28, \quad \Rightarrow \quad 13y = -13$$

i.e., we have $y = -1$. Substituting this into (1) gives

$$5x - 3 = 7, \quad \Rightarrow \quad 5x = 10$$

so that $x = 2$. Now check that $x = 2, y = -1$ by substitution into (2)!

Quiz To eliminate x from the following simultaneous equations, what should you multiply them by?

$$3x - 2y = 7$$

$$4x - 5y = 7$$

- (a) 7 & 7 (b) 4 & 3 (c) 3 & -2 (d) 3 & 4

Quiz To eliminate x from the simultaneous equations

$$7x + 3y = 13$$

$$-2x + 5y = 8$$

you can multiply (1) by -2 and (2) by 7 . Which of the following equations for y will this procedure eventually yield?

(a) $29y = 82$

(b) $29y = 30$

(c) $41y = 82$

(d) $7y = -56$

EXERCISE 3. Solve the following equations by first eliminating x .

$$\begin{aligned} \text{(a)} \quad 3x + 4y &= 10 \\ 2x + 5y &= 9 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 3x - 2y &= 9 \\ -x + 3y &= -3 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 2x - y &= 5 \\ 3x + 4y &= 2 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 5x + 7t &= 8 \\ 7x - 4t &= 25 \end{aligned}$$

Quiz Choose the solution of the following simultaneous equations

$$\begin{aligned} \frac{1}{2}x + 2y &= 3 \\ 2x + 3y &= 7 \end{aligned}$$

$$\text{(a)} \quad x = \frac{1}{2}, \quad y = 2 \qquad \text{(b)} \quad x = -\frac{1}{2}, \quad y = 2$$

$$\text{(c)} \quad x = 4, \quad y = 0 \qquad \text{(d)} \quad x = 2, \quad y = 1$$

4. Final Quiz

Begin Quiz Choose the solutions from the options given.

1. If $x + y = 1$ and $x - y = 3$, what are x and y ?

(a) $x = 2, y = -1$

(b) $x = -1, y = 2$

(c) $x = 2, y = 2$

(d) $x = 4, y = 1$

2. To eliminate x from the following equations, $ax + 2y = 4$ and $3x - 2ay = -17$, what do need to multiply them by?

(a) 4 & -17

(b) a & 3

(c) 3 & a

(d) 2 & $-2a$

3. Solve $3x + 2y = 1$ and $2x + 3y = -1$.

(a) $x = 3, y = -4$

(b) $x = 5, y = 3$

(c) $x = -3, y = 5$

(d) $x = 1, y = -1$

4. For $2x - 3y = 1$ and $3x - 2y = 4$, find x and y .

(a) $x = 2, y = 1$

(b) $x = -1, y = -1$

(c) $x = 1, y = 2$

(d) $x = 3, y = -2$

End Quiz

Solutions to Exercises

Exercise 1(a) We have

$$x + y = 3$$

$$x = 2$$

Substituting $x = 2$ into $x + y = 3$ we obtain:

$$2 + y = 3$$

$$y = 3 - 2$$

$$\Rightarrow y = 1$$

The solution is thus $x = 2, y = 1$.

Click on the **green** square to return



Exercise 1(b) We have

$$4x - y = 10$$

$$y = 2$$

Substituting $y = 2$ into $4x - y = 10$ yields

$$4x - 2 = 10$$

$$4x = 12$$

$$x = 3$$

The solution is thus $x = 3$, $y = 2$.

Click on the **green** square to return



Exercise 1(c) We have

$$z - x = 2$$

$$2x = -2$$

From $2x = -2$ we have that $x = -1$. Inserting this into $z - x = 2$ we find

$$z - (-1) = 2$$

$$z + 1 = 2$$

$$z = 1$$

The solution is thus $x = -1$, $z = 1$.

Click on the **green** square to return



Exercise 1(d) We have

$$3t + 2s = 0$$

$$s + 1 = 2$$

From $s + 1 = 2$, we have $s = 1$ and this can be inserted into $3t + 2s = 0$ to give

$$3t + 2 = 0$$

$$3t = -2$$

$$t = -\frac{2}{3}$$

The solution is thus $s = 1, t = -\frac{2}{3}$.

Click on the **green** square to return



Exercise 2(a) We have the equations

$$x + y = 5$$

$$x - y = 1$$

and adding them yields

$$2x = 6$$

so $x = 3$. This can now be inserted into the first equation to give

$$3 + y = 5$$

$$y = 2$$

The solution is thus $x = 3$, $y = 2$.

These results can be **checked** by inserting them into the second equation

$$x - y = 3 - 2 = 1 \checkmark$$

Click on the **green** square to return



Exercise 2(b) We have the equations

$$\begin{aligned}4x + 3y &= 7 \\ x - 3y &= -2\end{aligned}$$

and adding them yields

$$\begin{aligned}4x + 3y + x - 3y &= 7 - 2 \\ 5x &= 5 \\ x &= 1\end{aligned}$$

Substituting $x = 1$ into the first equation yields

$$\begin{aligned}4 + 3y &= 7 \\ 3y &= 3 \\ y &= 1\end{aligned}$$

This can now be [checked](#) by substitution into $x - 3y = 1 - 3 = -2$ ✓
Click on the [green](#) square to return □

Exercise 3(a) We have the equations

$$3x + 4y = 10 \quad (1)$$

$$2x + 5y = 9 \quad (2)$$

and multiplying the first equation by 2 and the second by 3 yields:

$$6x + 8y = 20 \quad (3)$$

$$6x + 15y = 27 \quad (4)$$

The coefficient of x is now the same and subtracting (3) from (4) yields an equation in y alone.

$$15y - 8y = 27 - 20$$

$$7y = 7$$

so $y = 1$. Inserting this into (1) yields $3x + 4 = 10$, which implies that $3x = 6$ and so $x = 2$. Check $x = 2, y = 1$ by substitution into (2)!

Click on the **green** square to return



Exercise 3(b) We have the equations

$$3x - 2y = 9 \quad (1)$$

$$-x + 3y = -3 \quad (2)$$

Multiplying the first equation by -1 and the second by 3 yields

$$-3x + 2y = -9 \quad (3)$$

$$-3x + 9y = -9 \quad (4)$$

and subtracting (4) from (3) gives $-7y = 0$, so that $y = 0$.

Inserting this into (1) yields $x = 3$. The solution, $x = 3$, $y = 0$, should be checked by substitution into (2): $-x + 3y = -3 + 0 \checkmark$

Click on the **green** square to return



Exercise 3(c) We have the equations

$$2x - y = 5 \quad (1)$$

$$3x + 4y = 2 \quad (2)$$

Multiplying (1) by 3 and (2) by 2 yields

$$6x - 3y = 15 \quad (3)$$

$$6x + 8y = 4 \quad (4)$$

and subtracting (4) from (3) gives $-11y = 11$, so $y = -1$.

Inserting this into the initial equation yields

$$2x + 1 = 5$$

$$2x = 4$$

$$x = 2$$

Now check that $x = 2$, $y = -1$, by substitution into (2)!

Click on the **green** square to return



Exercise 3(d) We have the equations

$$5x + 7t = 8 \quad (1)$$

$$7x - 4t = 25 \quad (2)$$

Multiplying (1) by 7 and (2) by 5 yields

$$35x + 49t = 56 \quad (3)$$

$$35x - 20t = 125 \quad (4)$$

and subtracting (4) from (3) gives

$$49t + 20t = 56 - 125$$

$$69t = -69$$

So $t = -1$. Inserting this into (1) yields

$$5x - 7 = 8$$

$$5x = 15$$

so we get $x = 3$, $t = -1$. Check this by substitution into (2)!

Click on the **green** square to return



Solutions to Quizzes

Solution to Quiz: We are given

$$x + 2y = 10$$

$$x = -2$$

Substituting $x = -2$ into $x + 2y = 10$ yields

$$-2 + 2y = 10$$

$$2y = 12$$

$$y = 6$$

The solution is thus $x = -2, y = 6$.

End Quiz

Solution to Quiz: We are given

$$3x + 3y = 0 \quad (1)$$

$$2x + 3y = 1 \quad (2)$$

Subtracting these equations yields

$$\begin{aligned} 3x + 3y - (2x + 3y) &= 0 - 1 \\ x &= -1 \end{aligned}$$

This can now be substituted into (1) to yield

$$\begin{aligned} -3 + 3y &= 0 \\ 3y &= 3 \\ y &= 1 \end{aligned}$$

Check the solution, $x = -1$, $y = 1$, by substitution into (2).

End Quiz

Solution to Quiz: We have the equations

$$3x - 2y = 7 \quad (1)$$

$$4x - 5y = 7 \quad (2)$$

To eliminate x we have to multiply (1) by 4 and (2) by 3.

This procedure yields:

$$12x - 8y = 28 \quad (3)$$

$$12x - 15y = 21 \quad (4)$$

The x coefficient is then the same in each equation and so subtracting (4) from (3) indeed eliminates x . End Quiz

Solution to Quiz: We have

$$7x + 3y = 13 \quad (1)$$

$$-2x + 5y = 8 \quad (2)$$

Multiplication by -2 and 7 respectively yields

$$-14x - 6y = -26 \quad (3)$$

$$-14x + 35y = 56 \quad (4)$$

Subtracting (4) from (3) cancels the x 's and yields

$$-6y - 35y = -26 - 56$$

$$-41y = -82$$

$$41y = 82$$

This implies that $y = 2$ and on substitution into (1) we obtain $x = 1$. These answers can then be [checked](#) by substituting into (2).

End Quiz

Solution to Quiz: We have the equations

$$\frac{1}{2}x + 2y = 3 \quad (1)$$

$$2x + 3y = 7 \quad (2)$$

It is easiest here to multiply (1) by 4 and then subtract (2) from it. In this way we do not have unnecessary fractions. We find:

$$2x + 8y = 12 \quad (3)$$

$$2x + 3y = 7 \quad (4)$$

Subtracting them cancels the x 's and yields

$$5y = 5$$

$$y = 1$$

Substituting this into (3) yields $x = 2$. The solution, $x = 2$, $y = 1$, can be checked by substitution into (2).

End Quiz