

## Basic Mathematics

# Solving Simple Equations

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence at solving simple equations.

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## 1. Introduction

In this section we shall look at some simple equations and the methods used to find their solution. There are four basic rules:

**Rule 1** An equal quantity may be added to both sides of an equation.

**Rule 2** An equal quantity may be subtracted from both sides of an equation.

**Rule 3** An equal quantity may multiply both sides of an equation.

**Rule 4** An equal, *non-zero* quantity may divide both sides of an equation.

The application of these rules is illustrated in the following examples.

**Example 1** Solve the equations

$$(a) \quad 3x - 8 = x + 10, \quad (b) \quad \frac{x}{2} = -6.$$

**Solution**

(a) By Rule 1 we may add 8 to both sides:

$$3x - 8 + 8 = x + 10 + 8 \quad \text{i.e.} \quad 3x = x + 18.$$

By Rule 2 we may subtract  $x$  from both sides:

$$3x - x = x + 18 - x \quad \text{i.e.} \quad 2x = 18.$$

Finally, by Rule 4 we may divide both sides by 2 giving  $x = 9$ .

(b) By Rule 3 we may multiply both sides by 2,

$$\left(\frac{2}{1}\right) \times \left(\frac{x}{2}\right) = 2 \times (-6) \quad \text{i.e.} \quad x = -12.$$

It is always good to check that the solution is correct by substituting the value into both sides of the equation. In **Example 1 (a)**, by substituting  $x = 9$  into the left hand side of the equation we see that

$$3x - 8 = 3 \times 9 - 8 = 19.$$

Substituting  $x = 9$  into the right hand side of the equation gives

$$x + 10 = 9 + 10 = 19.$$

Since both sides of the equation are equal when  $x = 9$ , it is a correct solution. In this case it is the only solution to the equation but it is important to note that some equations have more than one solution.

**EXERCISE 1.** Solve each of the following equations. (Click on green letters for solutions.)

(a)  $3x = 18,$

(c)  $-2x = -10$

(e)  $5x - 3x - 12x = 29 - 2 - 7$

(b)  $7x = -14$

(d)  $28x = 35$

(f)  $-\frac{x}{5} = 3$

Try the following short quizzes.

**Quiz** Which of the following is the solution to the equation

$$8x + 5x - 3x = 17 - 9 + 22?$$

(a) 2

(b) -2

(c) 3

(d) -3

**Quiz** Which of the following is the solution to the equation

$$x - 13x = 3x - 6?$$

(a)  $\frac{2}{5}$

(b)  $-\frac{1}{5}$

(c)  $\frac{1}{3}$

(d)  $-\frac{6}{17}$

## 2. Further Equations

We are now ready to move on to slightly more sophisticated examples.

**Example 2** Find the solution to the equation

$$5(x - 3) - 7(6 - x) = 24 - 3(8 - x) - 3$$

**Solution**

Removing the brackets from both sides first and then simplifying:

$$5(x - 3) - 7(6 - x) = 24 - 3(8 - x) - 3$$

$$5x - 15 - 42 + 7x = 24 - 24 + 3x - 3$$

$$5x + 7x - 15 - 42 = 3x - 3$$

$$12x - 57 = 3x - 3.$$

Adding 57 to both sides:

$$12x = 3x - 3 + 57 = 3x + 54$$

Subtracting  $3x$  from both sides:

$$12x - 3x = 54 \quad \text{or} \quad 9x = 54 \quad \text{giving} \quad x = 6.$$

**EXERCISE 2.** Find the solution to each of the following equations. (Click on green letters for solutions.)

(a)  $2x + 3 = 16 - (2x - 3)$

(b)  $8(x - 1) + 17(x - 3) = 4(4x - 9) + 4$

(c)  $15(x - 1) + 4(x + 3) = 2(7 + x)$

**Quiz** Which of the following is the solution to the equation

$$5x - (4x - 7)(3x - 5) = 6 - 3(4x - 9)(x - 1)?$$

(a)  $-2$

(b)  $-1$

(c)  $2$

(d)  $4$



When fractions occur we can sometimes transform the equation to one that does not involve fractions.

**Example 3** Find the solution to the equation

$$(4x/5) - (7/4) = (x/5) + (x/4).$$

**Solution**

The least common multiple of the denominators in the equation is  $4 \times 5 = 20$  and we proceed as follows:

$$\begin{aligned}20 \left( \frac{4x}{5} - \frac{7}{4} \right) &= 20 \left( \frac{x}{5} + \frac{x}{4} \right) \\ \frac{20}{1} \cdot \frac{4x}{5} - \frac{20}{1} \cdot \frac{7}{4} &= \frac{20}{1} \cdot \frac{x}{5} + \frac{20}{1} \cdot \frac{x}{4} \\ 16x - 35 &= 4x + 5x \\ 16x - 35 &= 9x.\end{aligned}$$

adding  $35$  to both sides and subtracting  $9x$  from both sides leads to

$$7x = 35 \quad \text{so} \quad x = 5 \text{ is the solution to the equation.}$$

**EXERCISE 3.** Find the solution to each of the following equations. (Click on green letters for solutions.)

(a)  $5x - 6(x - 5) = 2(x + 5) + 5(x - 4)$

(b)  $(x + 15)(x - 3) - (x^2 - 6x + 9) = 30 - 15(x - 1)$

(c)  $(x - 2)/2 + (x + 10)/9 = 5$

**Quiz** Which of the following is the solution to the equation

$$(x - 4)/7 = (x - 10)/5?$$

(a) 11

(b) -10

(c) 19

(d) 25

### 3. Quiz on Equations

**Begin Quiz** In each of the following, solve the equation and choose the solution from the options given.

1.  $4(x + 2)/5 = 7 + 5x/13$

(a) 5

(b) 13

(c) -5

(d) -13

2.  $(x + 20)/9 + 3x/7 = 6$

(a) 9

(b) 7

(c) 5

(d) 2

3.  $(x + 35)/6 - (x + 7)/9 = (x + 21)/4$

(a) -5

(b) 2

(c) 4

(d) -1

4.  $(x + 1)(2x + 1) = (x + 3)(2x + 3) - 14$

(a) 1

(b) -1

(c) 2

(d) -2

**End Quiz**

## Solutions to Exercises

### Exercise 1(a)

Dividing both sides by 3 gives

$$\frac{3x}{3} = \frac{18}{3}$$

or

$$x = 6.$$

Click on green square to return



**Exercise 1(b)**

Dividing both sides by 7 gives

$$\frac{7x}{7} = -\frac{14}{7}$$

or

$$x = -2.$$

Click on green square to return



**Exercise 1(c)**

Dividing both sides by  $-2$  gives

$$\frac{-2x}{-2} = \frac{-10}{-2}$$

or

$$x = 5.$$

Click on green square to return



**Exercise 1(d)**

Here 7 is the highest common factor of 28 and 35. First let us divide both sides by this.

$$28x = 35$$

$$\frac{28x}{7} = \frac{35}{7}$$

$$4x = 5.$$

Now divide both sides by 4.

$$\frac{4x}{4} = \frac{5}{4}$$

$$x = \frac{5}{4}.$$

The solution is thus  $x = 5/4$ .

Click on green square to return



**Exercise 1(e)**

First let us simplify both sides. The left hand side is

$$5x - 3x - 12x = 5x - 15x = -10x.$$

The right hand side is

$$29 - 2 - 7 = 29 - 9 = 20.$$

The original equation is thus

$$-10x = 20$$

and the solution to this is obtained by dividing both sides of the equation by  $-10$ .

$$\frac{-10x}{-10} = \frac{20}{-10},$$

so that

$$x = -2.$$

Click on green square to return





**Exercise 1(f)**

In this case we must *multiply* both sides by 5.

$$\begin{aligned} -\frac{x}{5} &= 3 \\ -\frac{5 \times x}{5} &= 5 \times 3 \\ -x &= 15 \\ x &= -15, \end{aligned}$$

and the solution in this case is  $x = -15$ .

Click on green square to return



**Exercise 2(a)**

$$\begin{aligned}2x + 3 &= 16 - (2x - 3) \\ &= 16 - 2x + 3 \\ &= 19 - 2x\end{aligned}$$

Now add  $2x$  to both sides and subtract  $3$  from both sides

$$\begin{aligned}2x + 3 &= 19 - 2x \\ 4x + 3 &= 19 \\ 4x &= 19 - 3 \\ 4x &= 16\end{aligned}$$

and the solution is  $x = 4$ . This can be checked by putting  $x = 4$  in both sides of the first equation above and noting that each side will have the value 11.

Click on green square to return



**Exercise 2(b)**

$$8(x - 1) + 17(x - 3) = 4(4x - 9) + 4$$

$$8x - 8 + 17x - 51 = 16x - 36 + 4$$

$$25x - 59 = 16x - 32$$

$$25x - 16x - 59 = -32$$

$$9x - 59 = -32$$

$$9x = 59 - 32$$

$$9x = 27$$

$$x = 3.$$

Inserting  $x = 3$  into the equation we can check that both sides have the value 16.

Click on green square to return



**Exercise 2(c)**

$$15(x - 1) + 4(x + 3) = 2(7 + x)$$

$$15x - 15 + 4x + 12 = 14 + 2x$$

$$19x - 3 = 2x + 14$$

$$19x - 2x - 3 = 14$$

$$17x - 3 = 14$$

$$17x = 14 + 3 = 17$$

$$x = 1$$

Inserting  $x = 1$  into the equation we can check that both sides have the value 16. [Click on green square to return](#) □

**Exercise 3(a)**

$$5x - 6(x - 5) = 2(x + 5) + 5(x - 4)$$

$$5x - 6x + 30 = 2x + 10 + 5x - 20$$

$$-x + 30 = 7x - 10$$

$$30 = x + 7x - 10$$

$$30 = 8x - 10$$

$$30 + 10 = 8x$$

$$8x = 40$$

$$x = 5$$

Click on green square to return



**Exercise 3(b)** First, using **FOIL**, we expand

$$(x + 15)(x - 3) = x^2 - 3x + 15x - 45 = x^2 + 12x - 45$$

Now we have

$$(x + 15)(x - 3) - (x^2 - 6x + 9) = 30 - 15(x - 1)$$

$$x^2 + 12x - 45 - x^2 + 6x - 9 = 30 - 15x + 15$$

$$18x - 54 = 45 - 15x$$

$$18x + 15x - 54 = 45$$

$$33x - 54 = 45$$

$$33x = 45 + 54$$

$$33x = 99$$

$$x = 3$$

Click on green square to return



**Exercise 3(c)**

This time we multiply both sides by  $2 \times 9$ .

$$\begin{aligned}\frac{(x-2)}{2} + \frac{(x+10)}{9} &= 5 \\ \frac{2 \times 9}{1} \times \frac{(x-2)}{2} + \frac{2 \times 9}{1} \times \frac{(x+10)}{9} &= 2 \times 9 \times 5 \\ 9(x-2) + 2(x+10) &= 90 \\ 9x - 18 + 2x + 20 &= 90 \\ 11x + 2 &= 90 \\ 11x &= 88 \\ x &= 8\end{aligned}$$

Click on green square to return



## Solutions to Quizzes

### Solution to Quiz:

Simplify both sides first:

$$\begin{aligned}8x + 5x - 3x &= 13x - 3x \\ &= 10x.\end{aligned}$$

$$\begin{aligned}17 - 9 + 22 &= 8 + 22 \\ &= 30.\end{aligned}$$

The equation to be solved is thus  $10x = 30$  and this clearly has solution  $x = 3$ .

End Quiz



**Solution to Quiz: 1**

$$\begin{aligned}x - 13x &= 3x - 6 \\-12x &= 3x - 6. \\0 &= 12x + 3x - 6 \\15x - 6 &= 0 \\15x &= 6 \\x &= \frac{6}{15} \\&= \frac{2}{5}.\end{aligned}$$

End Quiz

**Solution to Quiz:** First expand the brackets separately using **FOIL** (see the package on [Brackets](#)):

$$\begin{aligned}(4x - 7)(3x - 5) &= \overset{\mathbf{F}}{12x^2} - \overset{\mathbf{O}}{20x} - \overset{\mathbf{I}}{21x} + \overset{\mathbf{L}}{35} \\ &= 12x^2 - 41x + 35.\end{aligned}$$

$$\begin{aligned}(4x - 9)(x - 1) &= \overset{\mathbf{F}}{4x^2} - \overset{\mathbf{O}}{4x} - \overset{\mathbf{I}}{9x} + \overset{\mathbf{L}}{9} \\ &= 4x^2 - 13x + 9.\end{aligned}$$

These can now be substituted, *carefully*, into the equation:

$$\begin{aligned}5x - [(4x - 7)(3x - 5)] &= 6 - 3[(4x - 9)(x - 1)] \\ 5x - [12x^2 - 41x + 35] &= 6 - [4x^2 - 13x + 9] \\ 5x - 12x^2 + 41x - 35 &= 6 - 4x^2 + 13x - 9 \\ -12x^2 + 46x - 35 &= -4x^2 + 19x - 3.\end{aligned}$$

Notice the extra pair of *square* brackets in the first equation above. These are to emphasise that the *negative* sign multiplies *all* of the parts inside the  $[\ ]$ brackets. The procedure now follows in an obvious

manner. Add  $12x^2$  to both sides, subtract  $39x$  from both sides then add  $35$  to both sides:

$$-12x^2 + 46x - 35 = -12x^2 + 39x - 21$$

$$46x - 35 = 39x - 21$$

$$46x - 39x - 35 = -21$$

$$46x - 39x = 35 - 21$$

$$7x = 14$$

$$x = 2.$$

End Quiz

**Solution to Quiz:**

The highest common factor of the denominators is  $5 \times 7 = 35$ . Multiplying both sides of the equation by this

$$\frac{35}{1} \times \frac{(x-4)}{7} = \frac{35}{1} \times \frac{(x-10)}{5}$$

$$5(x-4) = 7(x-10)$$

$$5x - 20 = 7x - 70$$

$$5x - 20 + 70 = 7x - 70 + 70$$

$$5x + 50 = 7x$$

$$50 = 7x - 5x = 2x$$

$$x = 25$$

so that  $x = 25$  is the solution. This can be checked by putting this value into the original equation and showing that each side will have the value 3.

End Quiz