

Basic Mathematics

Brackets

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence at simplifying brackets.

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1. Brackets (Introduction)

Quantities are enclosed within brackets to indicate that they are to be treated as a single entity. If we wish to subtract, say, $3a - 2b$ from $4a - 5b$ then we do this as follows.

Example 1

$$\begin{aligned} \text{(a)} \quad (4a - 5b) - (3a - 2b) &= 4a - 5b - 3a - (-2b) \\ &= 4a - 5b - 3a + 2b \\ &= 4a - 3a - 5b + 2b \\ &= a - 3b. \end{aligned}$$

and similarly

$$\begin{aligned} \text{(b)} \quad (7x + 5y) - (2x - 3y) &= 7x + 5y - 2x - (-3y) \\ &= 7x + 5y - 2x + 3y \\ &= 7x - 2x + 5y + 3y \\ &= 5x + 8y. \end{aligned}$$

When there is more than one bracket it is usually necessary to begin with the inside bracket and work outwards.

Example 2

Simplify the following expressions by removing the brackets.

(a) $3a - c + (5a - 2b - [3a - c + 2b])$,

(b) $-\{3y - (2x - 3y) + (3x - 2y)\} + 2x$.

Solution

(a) We have

$$\begin{aligned}3a - c + (5a - 2b - [3a - c + 2b]) &= 3a - c + (5a - 2b - 3a + c - 2b) \\ &= 3a - c + (2a - 4b + c) \\ &= 3a - c + 2a - 4b + c \\ &= 3a + 2a - 4b - c + c \\ &= 5a - 4b.\end{aligned}$$

(b) Similarly we have

$$\begin{aligned} -\{3y - (2x - 3y) + (3x - 2y)\} + 2x &= -\{3y - 2x + 3y + 3x - 2y\} + 2x \\ &= -\{3y + 3y - 2y + 3x - 2x\} + 2x \\ &= -\{4y + x\} + 2x \\ &= -4y - x + 2x \\ &= x - 4y. \end{aligned}$$

EXERCISE 1. Remove the **brackets** from each of the following expressions and **simplify** as far as possible. (Click on **green** letters for solutions.)

- (a) $x - (y - z) + x + (y - z) - (z + x),$
- (b) $2x - (5y + [3z - x]) - (5x - [y + z]),$
- (c) $(3/a) + b + (7/a) - 2b,$
- (d) $a - (b + [c - \{a - b\}]).$

2. Distributive Rule

A quantity outside a bracket multiplies *each* of the terms inside the bracket. This is known as the **distributive rule**.

Example 3

$$(a) \quad 3(x - 2y) = 3x - 6y.$$

$$(b) \quad 2x(x - 2y + z) = 2x^2 - 4xy + 2xz.$$

$$(c) \quad 7y - 4(2x - 3) = 7y - 8x + 12.$$

This is a relatively simple rule but, as in all mathematical arguments, a great deal of care must be taken to proceed correctly.

EXERCISE 2. Remove the **brackets** and **simplify** the following expressions. (Click on **green** letters for solution.)

$$(a) \quad 5x - 7x^2 - (2x)^2$$

$$(b) \quad (3y)^2 + x^2 - (2y)^2$$

$$(c) \quad 3a + 2(a + 1)$$

$$(d) \quad 5x - 2x(x - 1)$$

$$(e) \quad 3xy - 2x(y - 2)$$

$$(f) \quad 3a(a - 4) - a(a - 2)$$

In the case of *two* brackets being multiplied together, to simplify the expression first choose *one* bracket as a single entity and multiply this into the other bracket.

Example 4 For each of the following expressions, multiply out the brackets and simplify as far as possible.

$$(a) (x + 5)(x + 2), \quad (b) (3x - 2)(2y + 3).$$

Solution

$$\begin{aligned}(a) \quad (x + 5)(x + 2) &= (x + 5)x + (x + 5)2 \\ &= x(x + 5) + 2(x + 5) \\ &= x^2 + 5x + 2x + 10 \\ &= x^2 + 7x + 10.\end{aligned}$$

$$\begin{aligned}(b) \quad (3x - 2)(2y + 3) &= (3x - 2)2y + (3x - 2)3 \\ &= 2y(3x - 2) + 3(3x - 2) \\ &= 6xy - 4y + 9x - 6.\end{aligned}$$

Try this short quiz.

Quiz To which of the following does the expression

$$(2x - 1)(x + 4)$$

simplify?

(a) $2x^2 - 2x + 4$

(b) $2x^2 - 7x + 4$

(c) $2x^2 + 7x - 4$

(d) $2x^2 + 2x - 4$

3. FOIL

When it comes to expanding a bracket like $(a + c)(x + y)$ there is a simple way to remember all of the terms. This is the word **FOIL**, and stands for

take products of the

First **O**utside **I**nside **L**ast

This is illustrated in the following.

Example 5

$$(a + c)(x + y) = \overset{\mathbf{F}}{a}x + \overset{\mathbf{O}}{a}y + \overset{\mathbf{I}}{c}x + \overset{\mathbf{L}}{c}y .$$

These terms are the products of the pairs highlighted below.

$$\underbrace{(a + c)}_{\mathbf{F}}(x + y), \underbrace{(a + c)(x + y)}_{\mathbf{O}}, \underbrace{(a + c)}_{\mathbf{I}}(x + y), (a + \underbrace{c)(x + y)}_{\mathbf{L}}.$$

There are two other brackets that are worth remembering. These are

$(x + y)^2$, which is a *complete square*, and

$(x + y)(x - y)$, which is a *difference of two squares*.

These are included in the following exercises.

EXERCISE 3. Remove the **brackets** from each of the following expressions using **FOIL**.

(a) $(x + y)^2$

(c) $(x + 4)(x + 5)$

(e) $(2y + 1)(y - 3)$

(b) $(x + y)(x - y)$

(d) $(y + 1)(y + 3)$

(f) $2(x - 3)^2 - 3(x + 1)^2$

Quiz To which of the following expressions does $9 - (x - 3)^2$ simplify?

(a) $-x^2$

(b) $6x - x^2$

(c) $18 - x^2$

(d) $6x + x^2$

4. Quiz on Brackets

Begin Quiz In each of the following, remove the brackets, simplify the expression and choose the solution from the options given.

1. $(a + 2m)(a - m)$
- (a) $a^2 - am - 2m^2$ (b) $a^2 + am - 2m^2$
(c) $a^2 + 2m^2 - am$ (d) $a^2 + 2am + 2m^2$
2. $(3b - a)(2a + 3b)$
- (a) $6b^2 + a^2 - 3ab$ (b) $9b^2 + 3ab - 2a^2$
(c) $9b^2 + 9ab - 3b^2$ (d) $6b^2 + 3ab - a^2$
3. $(2x + 1)^2 - (x + 3)^2$
- (a) $x^2 - 8$ (b) $x^2 - 2x - 8$
(c) $3x^2 - 8$ (d) $3x^2 - 2x - 8$
4. $3(x + 2)^2 - (x - 2)^2$
- (a) $2x^2 + 16x + 8$ (b) $2x^2 + 16$
(c) $4x^2 + 8x + 16$ (d) $4x^2 - 16$

End Quiz

Solutions to Exercises

Exercise 1(a)

$$\begin{aligned} & x - (y - z) + x + (y - z) - (z + x) \\ = & x - y + z + x + y - z - z - x \\ = & x + x - x - y + y + z - z - z \\ = & x - z. \end{aligned}$$

Click on green square to return



Exercise 1(b)

$$\begin{aligned} & 2x - (5y + [3z - x]) - (5x - [y + z]) \\ = & 2x - (5y + 3z - x) - (5x - y - z) \\ = & 2x - 5y - 3z + x - 5x + y + z \\ = & 2x + x - 5x - 5y + y - 3z + z \\ = & -2x - 4y - 2z. \end{aligned}$$

Click on green square to return



Exercise 1(c)

$$\begin{aligned}\frac{3}{a} + b + \frac{7}{a} - 2b &= \frac{3}{a} + \frac{7}{a} + b - 2b \\ &= \frac{3+7}{a} - b \\ &= \frac{10}{a} - b.\end{aligned}$$

Click on green square to return



Exercise 1(d)

$$\begin{aligned} a - (b + [c - \{a - b\}]) &= a - (b + [c - a + b]) \\ &= a - (b + c - a + b) \\ &= a - (2b + c - a) \\ &= a - 2b - c + a \\ &= 2a - 2b - c. \end{aligned}$$

Click on green square to return



Exercise 2(a)

First note that $(2x)^2 = (2x) \times (2x) = 4x^2$.

$$\begin{aligned}5x - 7x^2 - (2x)^2 &= 5x - 7x^2 - 4x^2 \\ &= 5x - 11x^2\end{aligned}$$

Click on green square to return



Exercise 2(b)

$$\begin{aligned}(3y)^2 + x^2 - (2y)^2 &= 9y^2 + x^2 - 4y^2 \\ &= 9y^2 - 4y^2 + x^2 \\ &= 5y^2 + x^2\end{aligned}$$

Click on green square to return



Exercise 2(c)

$$\begin{aligned}3a + 2(a + 1) &= 3a + 2a + 2 \\ &= 5a + 2\end{aligned}$$

Click on green square to return



Exercise 2(d)

$$\begin{aligned}5x - 2x(x - 1) &= 5x - 2x^2 + 2x \\ &= 7x - 2x^2\end{aligned}$$

Click on green square to return



Exercise 2(e)

$$\begin{aligned}3xy - 2x(y - 2) &= 3xy - 2xy + 4x \\ &= xy + 4x\end{aligned}$$

Click on green square to return



Exercise 2(f)

$$\begin{aligned}3a(a - 4) - a(a - 2) &= 3a^2 - 12a - a^2 + 2a \\ &= 3a^2 - a^2 + 2a - 12a \\ &= 2a^2 - 10a\end{aligned}$$

Click on green square to return



Exercise 3(a)

$$\begin{aligned}(x + y)^2 &= (x + y)(x + y) \\ &= x^2 + xy + yx + y^2 \quad \text{using FOIL} \\ &= x^2 + 2xy + y^2\end{aligned}$$

This is an **IMPORTANT** result and should be committed to memory. Here x is the *first* member of the the bracket and y is the *second*. The rule for the *square* of $(x + y)$, i.e. $(x + y)^2$ is

$$\begin{array}{ccccccc}x^2 & + & & 2xy & + & & y^2 \\ \text{(square the first)} & + & \text{(twice the product)} & + & \text{(square the last)}\end{array}$$

Click on green square to return



Exercise 3(b)

Using **FOIL** again:

$$\begin{aligned}(x + y)(x - y) &= x^2 - xy + yx - y^2 \\ &= x^2 - y^2\end{aligned}$$

The form of the solution is the reason for the name **difference of two squares**. This is another important result and is worth committing to memory.

Click on green square to return



Exercise 3(c)

Using **FOIL**:

$$\begin{aligned}(x + 4)(x + 5) &= x^2 + 5x + 4x + 20 \\ &= x^2 + 9x + 20\end{aligned}$$

Click on green square to return



Exercise 3(d)Using **FOIL**:

$$\begin{aligned}(y + 1)(y + 3) &= y^2 + 3y + y + 3 \\ &= y^2 + 4y + 3\end{aligned}$$

Click on green square to return



Exercise 3(e)Using **FOIL**:

$$\begin{aligned}(2y + 1)(y - 3) &= 2y^2 - 6y + y - 3 \\ &= 2y^2 - 5y - 3\end{aligned}$$

Click on green square to return



Exercise 3(f)

This one is best done in parts. First we have

$$(x - 3)^2 = x^2 - 6x + 9$$

and

$$(x + 1)^2 = x^2 + 2x + 1$$

Thus

$$\begin{aligned} 2(x - 3)^2 - 3(x + 1)^2 &= 2(x^2 - 6x + 9) - 3(x^2 + 2x + 1) \\ &= 2x^2 - 12x + 18 - 3x^2 - 6x - 3 \\ &= 2x^2 - 3x^2 - 12x - 6x + 18 - 3 \\ &= -x^2 - 18x + 15 \end{aligned}$$

Click on green square to return



Solutions to Quizzes

Solution to Quiz:

$$\begin{aligned}(2x - 1)(x + 4) &= (2x - 1)x + (2x - 1)4 \\ &= (2x^2 - x) + (8x - 4) \\ &= 2x^2 - x + 8x - 4 \\ &= 2x^2 + 7x - 4\end{aligned}$$

End Quiz

Solution to Quiz:

First note that $(x - 3)^2 = x^2 - 6x + 9$, so

$$\begin{aligned}9 - (x - 3)^2 &= 9 - (x^2 - 6x + 9) \\ &= 9 - x^2 + 6x - 9 \\ &= 9 - 9 - x^2 + 6x \\ &= -x^2 + 6x = 6x - x^2\end{aligned}$$

End Quiz